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3D crack propagation with XFEM cohesive elements

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ABSTRACT

In case of brittle failure of metallic components or reinforced concrete structures, trajectories are often unknown in case of mixed mode loading. The XFEM cohesive model presented in this communication will address the issues of crack bifurcation and crack advance. A procedure in four steps is adopted: computation of the equilibrium state in the presence of cohesive forces with a given potential crack surface, detection of the updated crack front on the surface from the computed cohesive state, determination of bifurcation angles along the front, and update of the potential crack surface accordingly. The cohesive model that is used allows initial perfect adherence. It relies on the use of an appropriate XFEM space of Lagrange multipliers, on the use of a mortar formulation to write the cohesive law from quantities defined over this space in an appropriate manner, and finally on a lumping strategy leading to block-wise diagonal operators. The originality of the approach lies in the a posteriori computation of the crack advance speed that is naturally embedded in the cohesive model, while in most of the literature it is determined beforehand based on the stress state ahead of the front. Several numerical tests have been carried out in mixed mode I and II to reproduce 3D non planar crack paths and showed good accordance with previous results from the literature. Most situations are quasi-statics with load controlled strategies and extension to dynamics is also discussed.

INTRODUCTION

To evaluate the harmfulness of defaults detected in metallic components or concrete structures, EDF has developed advanced simulation tools such as Code_Aster, numerical software for finite element analyses in mechanical engineering. The class of phenomena aimed at in this communication concerns 3D crack propagation with a priori unknown trajectories, in fatigue but also in transient dynamics when unstable propagation occurs as in brittle rupture.

To tackle this issue, we propose to associate the extended finite element method [1] with the cohesive approach [2,3,4]. Cohesive zones are defined by surfaces extended tangentially from an existing crack surface. Hence the cohesive behaviour will separate naturally adherent zones from completely opened crack surfaces. An original implicit update of the crack front is performed. It requires a robust treatment of non regular interface laws combined with XFEM in Code_Aster [6].

In statics, it relies on the use of an appropriate XFEM space of Lagrange multipliers [5,6], on the use of a mortar formulation to write the cohesive law from quantities defined over this space in an appropriate manner, and finally on a lumping strategy leading to block-wise diagonal operators. Then, a directional criterion using information on the cohesive zone behind the crack front is applied. It allows us to propose a completely automated crack propagation procedure for which the trajectory is not known a priori that is compared to experimental results of the literature.

In dynamics, an initially perfectly adherent cohesive law treated implicitly with an explicit time scheme is used which allows an analytical determination of cohesive forces with an appropriate discretisation. The formulation is first validated on a conic DCB test specimen before being extended to industrial applications.

QUASI-STATIC PROPAGATION WITH X-FEM COHESIVE ELEMENTS

In most strategies using cohesive elements with crack propagation, crack advancement is given in an explicit way, information being provided by the stress field ahead of the crack front. We propose here an alternate solution for which the cohesive law is applied on a large potential surface of crack propagation ahead of the current crack front in its tangential direction. The advance of the crack front is then provided in an implicit manner, relying on the behaviour of the cohesive law to determine by post-processing the status of the points belonging to the potential surface of crack propagation. Equilibrium and behaviour law will then determine naturally the separation between adherent zones and open zones of the cohesive surface.

A necessary ingredient is the insertion of robust non regular interface laws within the XFEM framework. We can show for example that a cohesive law $\mathbf{t_c}([\mathbf{u}])$, $[\mathbf{u}]$ being the displacement gap on the interface and $\mathbf{t_c}$ the cohesive pressure distribution, with finite initial stiffness, such as the one proposed in figure 1, does not allow describing appropriately large adherent zones.

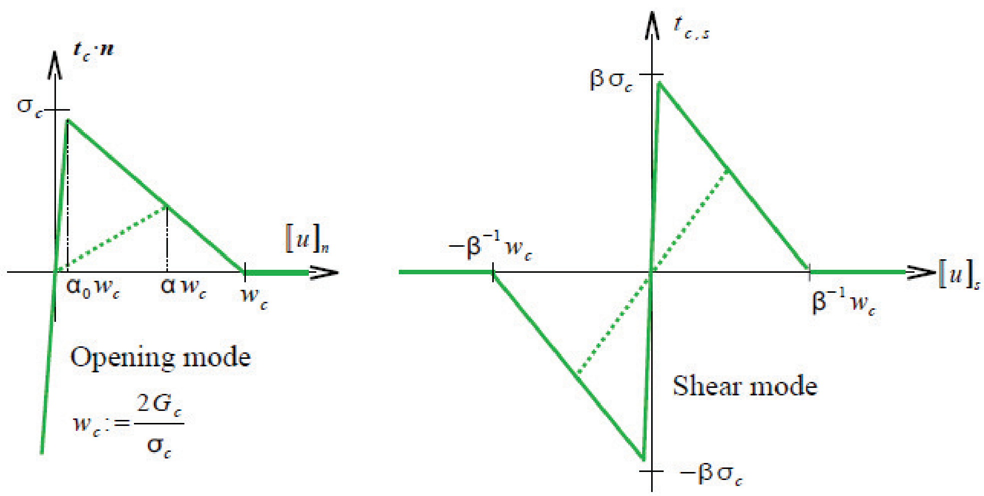

Figure 1. Penalised linear-softening mode-coupling cohesive law. Normal and tangential behaviours, with σ_c the tensile strength and G_c the fracture energy.

If the penalisation parameter α_0 is too low, the solution is not correct and non physical opening is observed in adherent zones (zone 1 of figure 3). Inversely, if the penalisation parameter α_0 is too high, then numerical oscillations are observed (zones 2 and 3 of figure 4) which lead to misevaluation of the regime (adherence or debonding). This can be observed on the inclusion debonding test whose geometry and boundary conditions are represented on figure 2.

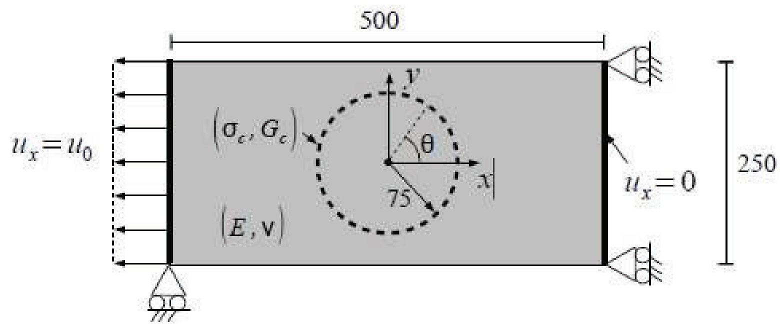

Figure 2. Inclusion debonding test.

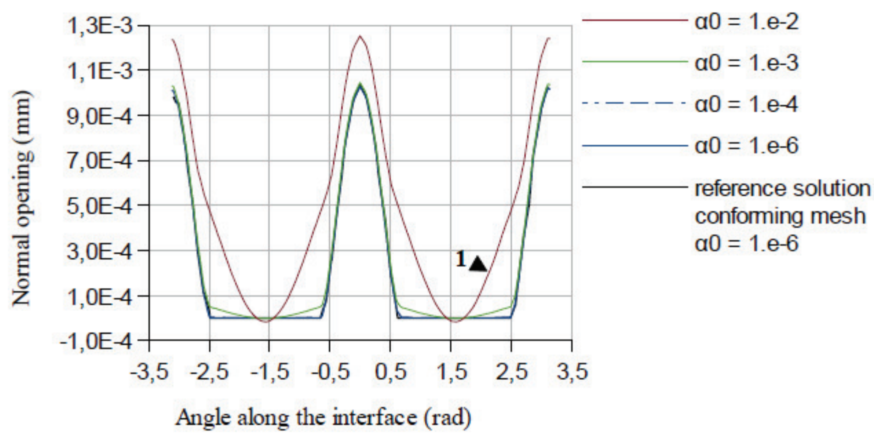

Figure 3. Normal opening for the inclusion debonding case.

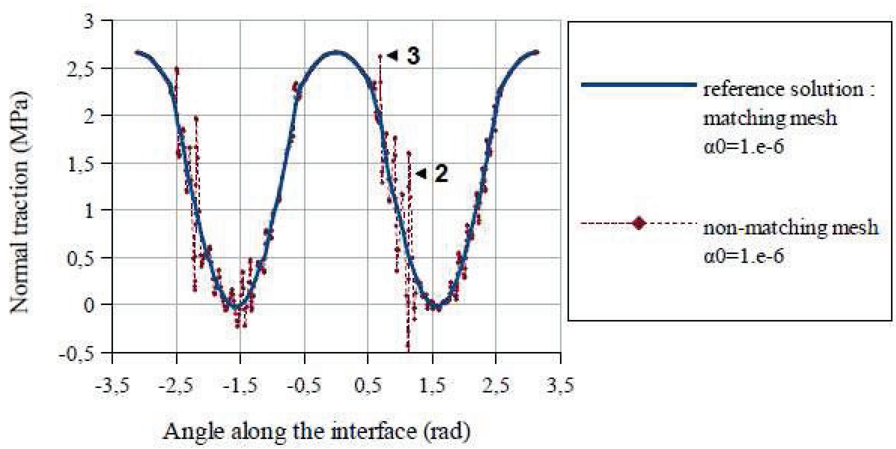

Figure 4. Normal tractions for the inclusion debonding case.

To take care of this inconvenient, several corrective solutions are applied:

- The introduction of an adapted Lagrange multiplier space $\boldsymbol{\lambda}$, adapted to XFEM [5,6]. When equilibrium is reached $\boldsymbol{\lambda}$ is similar to the contact pressure. So as to improve robustness and performance, a « mortar » formulation may be adopted to assess the consistency of this space.
- The use of an infinite initial stiffness in the cohesive law $\mathbf{t_c([u]; \lambda)}$ expressed in the formalism of an augmented Lagrangian formulation, such that $\alpha_0 = 0$ in figure 1.
- A diagonal expression of interface operators.

The efficiency of the approach is illustrated with the pull out test of the inclusion previously described. Each one of these modifications allows, apart from reducing drastically oscillations, a reduction of the number of iterations of the Newton algorithm used to solve the equilibrium, as can be seen on table 1.

	Classical formulation	Stable « mortar » formulation	
		Consistent operators	Blockwise diagonal operators
Penalized law	$t_c([u])$	$t_c(w)$	$t_c(w)$
Newton iterations	729	311	147
Mixed law	$t_c([u], \lambda)$	$t_c(w, \lambda)$	$t_c(w, \lambda)$
Newton iterations	63	43	15

Table 1. Evolution of the number of Newton iterations which are necessary to solve the debonding case in three time steps.

The capability of the method to determine exactly the crack extension is based on the introduction of the cohesive law on a large potential crack surface. Crack propagation obeys then the following rules described on figure 5.

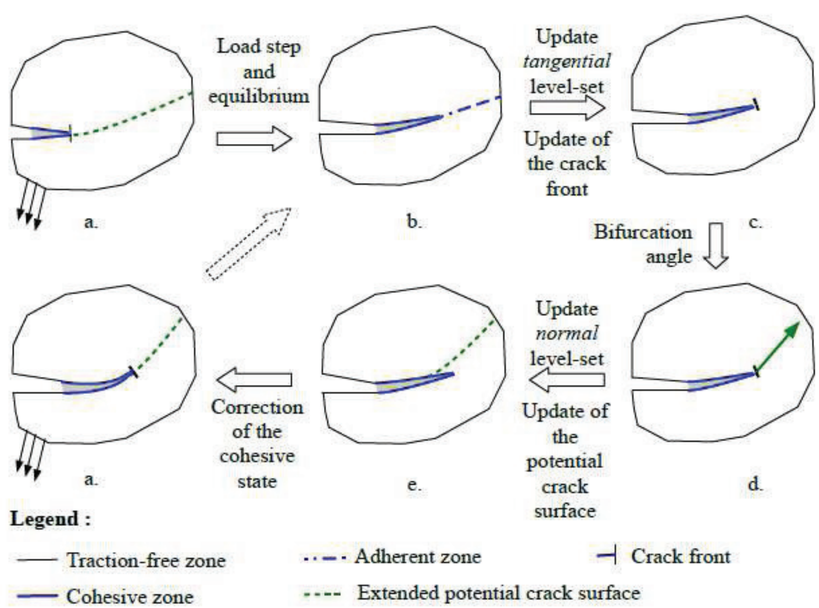

Figure 5. Crack propagation procedure.

This procedure can be summarized as follows:

- A large potential crack surface from previous time step is present (5a)
- Equilibrium is computed with the large potential crack surface (5b).
- The propagated crack front is localised by post-processing cohesive variables (5c).
- Propagation direction along the crack front is determined using criteria such as the maximum principal stress [7] (5d).
- A new potential crack surface is determined (5e).
- The Lagrange multiplier space is modified accordingly as well as the internal variables of the cohesive law (5a).
- A new equilibrium is then computed to obtain the new crack front and so on (5b).

In the localisation process of the crack front, a first rough crack front is determined using the intersection of the iso-zero of the internal variable α of the cohesive law with the faces of the mesh.

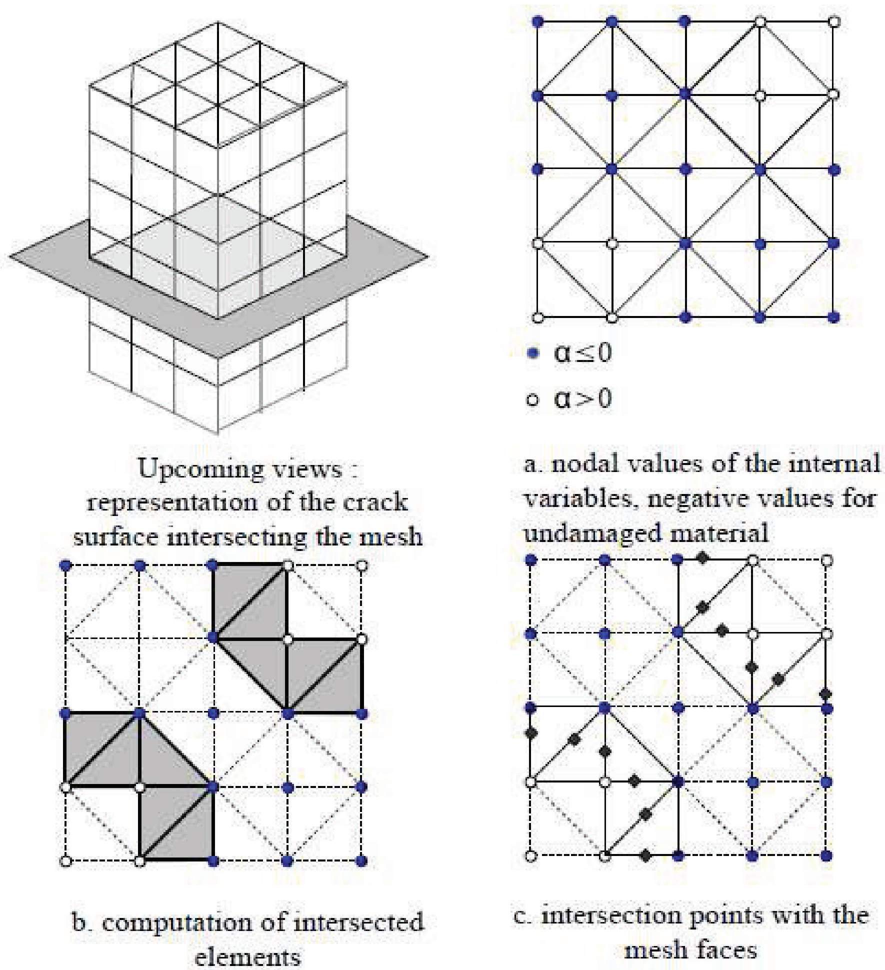

Figure 6. Computation of a rough crack front.

From the cloud of points obtained at the previous step, we reconstruct with a regularizing operator the advance of the previous crack front up to the regularized positions as illustrated on figure 7. This advance regularized field is provided to a propagation algorithm and provides a new crack front much more regular than the initial one. The crack front is obtained as the intersection of the iso-zero of a normal level set which characterizes the distance to the crack surface and the iso-zero of a tangent level set which characterizes the distance to the surface orthogonal to the crack surface at the crack front. Then level set update algorithms are applied once the position of the new crack front is determined from the regularized operator. In this paper we used a geometrical update [8] of the level sets characterising the position of the crack surface and its front.

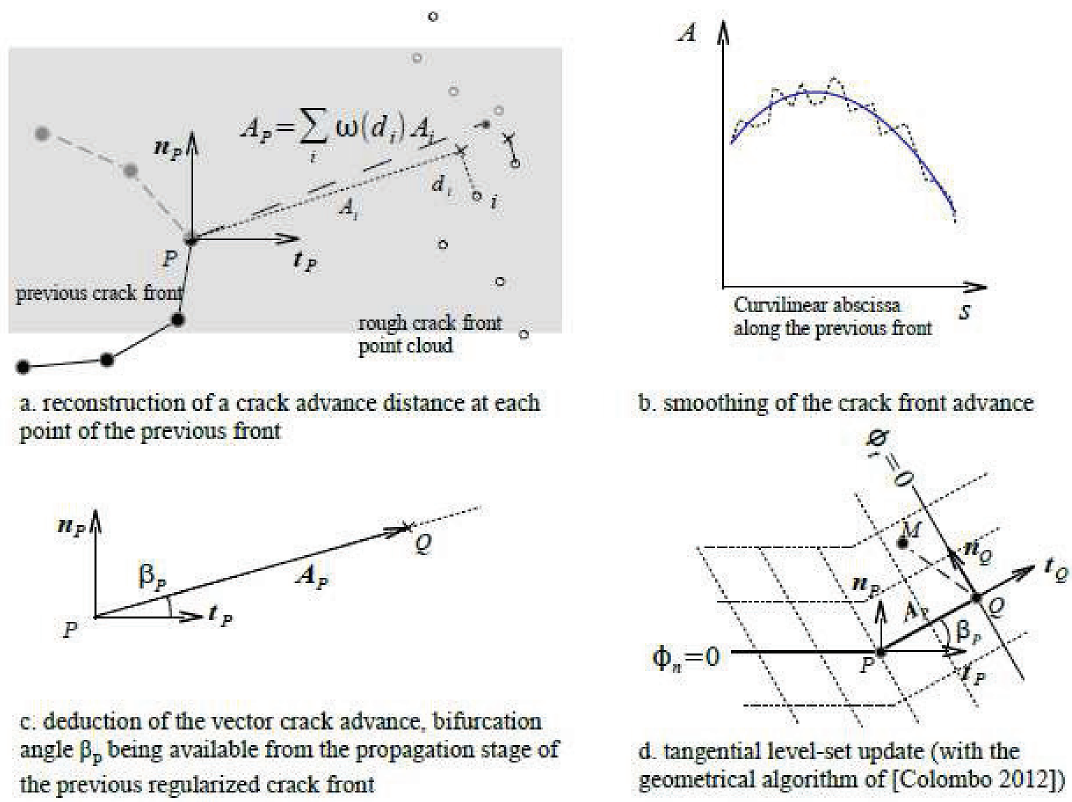

Figure 7. Reconstruction of a smoothed crack front.

The determination of the propagation crack angle is based on the criterion of the maximum of the circumferential stress provided by Erdogan and Sih [7], formulated in terms of stress intensity factors. Using stress intensity factors along cohesive zone models seems reasonable for concrete as suggested by [9,10] since experiments showed that the crack path, contrary to the load-deflection curve, is almost insensitive to the size of the cohesive zone, to the point where linear fracture mechanics is still valid. Moreover, since, with a cohesive zone, mechanical fields far away from the crack front have the same formulation than those of a free crack, we can define equivalent stress intensity factors [11]. We can show that these stress intensity factors can be deduced directly from the displacement gap and the cohesive stress using interface integrals [12]. There is therefore no need to construct contours surrounding the front and no need to compute auxiliary fields with this method.

This procedure is validated on three tests which can be found in the literature. The first one is the study of concrete crack propagation for a bracket specimen, described in [13]. Results obtained are in good agreement with the experimental ones, in terms of trajectory and load-displacement curve. It could still be improved using an exponential law for the cohesive zone rather than a linear or bilinear one. The second validation is performed with respect to a three point bending test of a Plexiglas specimen with a pre-crack at an angle with respect to the load direction [14,15]. In this case, only the crack trajectories are compared between the numerical simulations and the experiment, which show a good agreement. The last example focuses on the torsion of a parallelepiped concrete specimen pre-cracked at 45 degrees (see figure 8) [16]. The crack trajectory is complex, presented in figure 9, with an S shaped form. Once more the agreement between simulations and experiment in figure 10 is rather good from the point of view of trajectories and load-displacement curves, with a similar remark on the type of law used for the cohesive model.

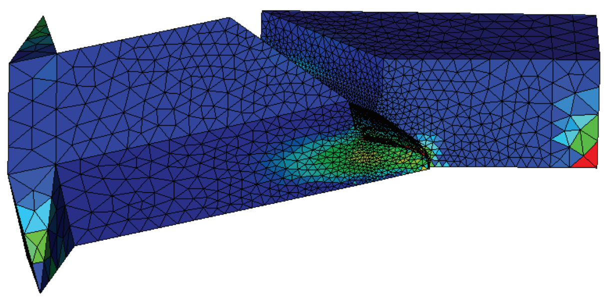

Figure 8. Prediction of a complex 3D trajectory for the torsion of a parallelepiped specimen pre-cracked at 45 degrees.

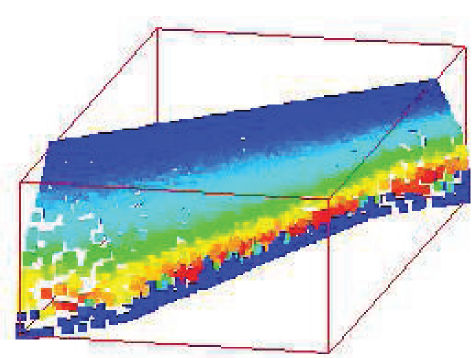

Figure 9. S-shaped computed crack path and normal cohesive tractions for Brokenshire's test.

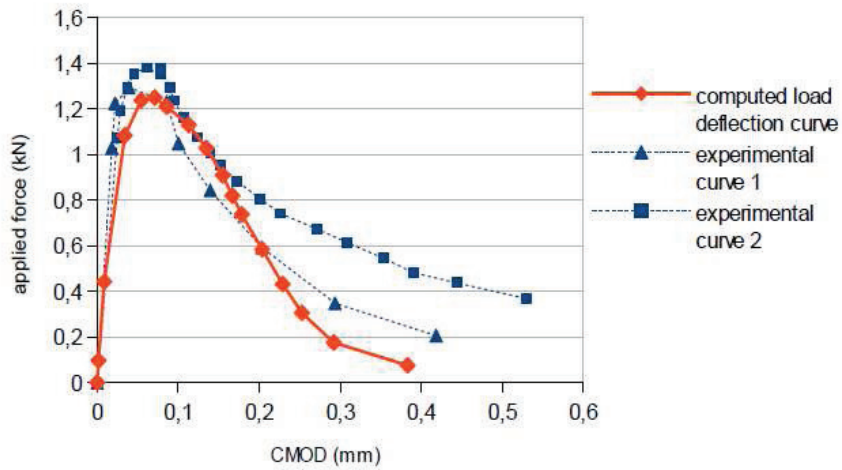

Figure 10. Load-deflection curve for Brokenshire's test.

DYNAMIC PROPAGATION WITH X-FEM COHESIVE ELEMENTS

Similarly to quasi-static propagation, we seek for a cohesive formulation to be applied on a large potential crack surface, which can describe correctly a partition in between large adherent zones and open ones.

Once more, we will consider a cohesive law with initial infinite stiffness. It will be expressed in an implicit way within an explicit time scheme, the one of centered difference. This approach was proposed by Doyen et al. [17] for interfaces fitting the structural mesh: we propose here its extension to XFEM. Applying the specific space described before for the cohesive force with diagonal interface operators, cohesive forces can be solved analytically from scalar equations.

The implementation is validated on a trapezoidal DCB specimen. In this experiment, a crack suddenly propagates dynamically before stopping inside the structure: its length when it stops is the quantity of interest to be studied. Several numerical simulations are performed with varying time steps as is often the case with explicit simulations: similar results are however obtained. It can be deduced from this result that the method is energetically reliable and allows determining correctly the length at arrest, even though some improvements could be performed.

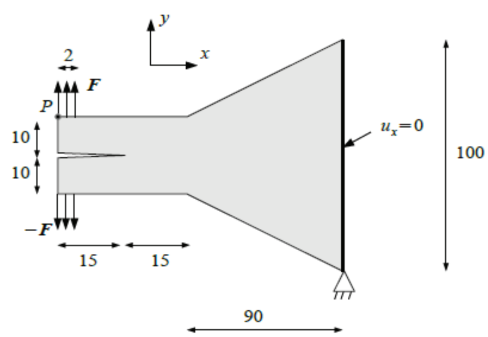

Figure 11. Trapezoidal Double Cantilever Beam. Load and Geometry.

For quasi-static loading, a load control technique can also applied to determine the critical load which triggers the unstable propagation. The one proposed by Lorentz [18] was chosen and extended to XFEM. It allows following instabilities during a quasi-static calculation, with the load intensity introduced as a new unknown in the system of equations to be solved, the maximal opening being prescribed in the cohesive zone. This method is equivalent to prescribing a maximal dissipation for which the load intensity is deduced. The crack length at arrest for the critical load obtained with this quasi-static method is smaller than the one predicted from the dynamic simulation, due to the fact that inertial effects are not taken into account (see figure 12).

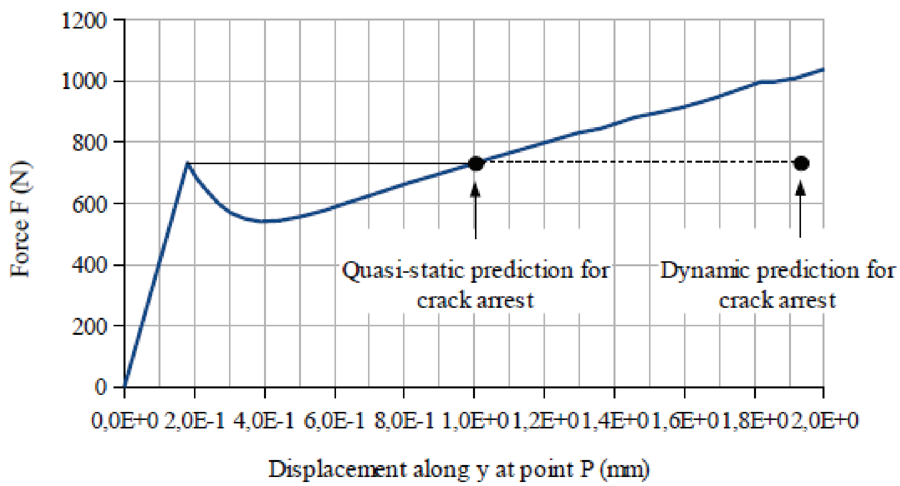

Figure 12. Load-deflection curve obtained with a path-following method and crack arrest for the Trapezoidal Double Cantilever Beam.

Finally, the different steps to determine the trajectory of the crack during propagation are adapted from quasi-statics to dynamics. Even if the first results obtained to describe an unstable propagation are encouraging, the methodology seems difficult to be applied in case of transient phenomena involving shocks: in this case, the crack front is not clearly defined, being diffuse, which will make its update more difficult. Moreover, it is not easy to estimate a priori the cohesive process zone size as in quasi-statics, which could complicate the choice of calculation parameters to follow the crack path.

CONCLUSION

An original method is presented to predict crack path and load-deflection curve of brittle materials under quasi-static loading, combining XFEM and cohesive zone models. Instead of specifying the location of the crack front of the next load increment in advance, the cohesive law is defined over a large surface, so that the location of the crack front results from the equilibrium. This demands a robust insertion of cohesive laws with an exact enforcement of adherence with XFEM. We think the methodology could be extended to other interfacial behaviour such as contact-friction. In case of instability, a load control strategy may be used, but do not allow to represent the crack propagation correctly and more importantly its arrest, since inertial effects are not taken into account. Finally, using an explicit dynamics approach allows recovering a correct behaviour.

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