A CDM Motivated Safety Factor for Creeping Materials and Structures

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1 INTRODUCTION

It is applicable in engineering design in creep conditions that the safety factor is determined on base of time-to-rupture calculation. This calculation assumes that the load remains constant within exploitation time. If not, the question arises how much safety factor calculated with respect to load is reduced by material deterioration process developed in creep conditions. It is necessary to distinguish between two safety factors: that connected with time-to-rupture, and another one connected with load. The above consideration is referred to a material point of a structure. If structure as a whole is considered, time-to-rupture has to be redefined taking into account spatial redistribution of damage caused by structural changes in structure material. As a result three periods of failure process have to be distinguished: time to the first macroscopic failure in a point t_I, time t_{II} at which deteriorated surface crosses a section of a structure, and finally time t_{III} when a structure collapses because of formation "a brittle hinges" [1] which transforms structure into a kinematically unstable mechanism. Thus, a safety factor is not only a function of stress and time, but also depends on structure and load configuration.

The present paper gives an analysis of all above factors consecutively. First the analysis limited to the point will be given, and next it will be expanded to the evaluation of the influence of spatial damage redistribution in a structure.

2 SAFETY FACTOR FOR A MATERIAL POINT

2.1. Definitions

The analysis has been based on the Kachanov theory of damage growth [5] which makes use of damage parameter ω with its critical value of ω=1 at failure.

In the following we will use an exertion factor W, which is inverse to safety factor. Let us specify the following exertion factors:

$$W_t = \frac{t}{t^*}, \tag{1}$$

where t is time of exploitation, t^* is time to failure at constant load and

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W_t denotes time-dependent exertion factor.

$$W = \frac{\sigma_e}{\overset{*}{\sigma}_e(0)}, \tag{2}$$

where σ_e is effective stress, $\overset{*}{\sigma}_e(0)$ is a value of former one causing instantaneous (time independent) failure, and Wdenotes stress-dependent exertion factor.

$$W_\sigma = \frac{\sigma_e}{\tilde{\sigma}_e}, \tag{3}$$

where $\tilde{\sigma}_e$is effective stress causing failure at exploitation time, and W_σ denotes modified stress-dependent exertion factor.

The above definitions are illustrated by Fig.1.

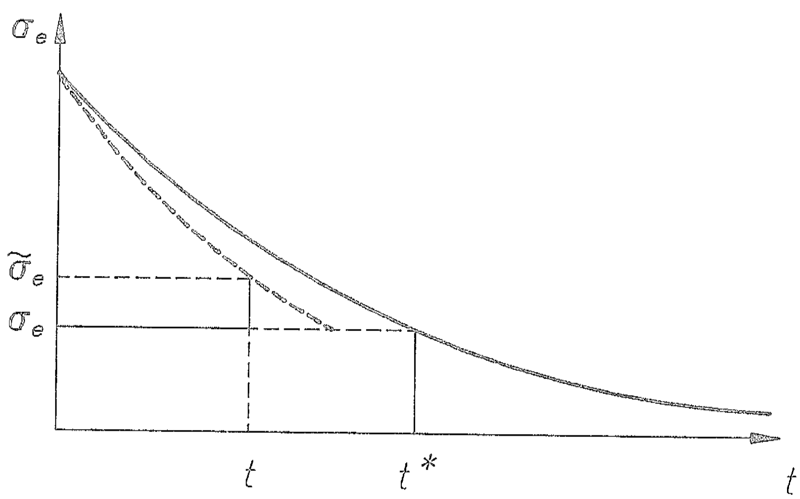

Fig.1.Applied stress and time to failure definitions

Relations between defined exertion factors depend on the form of damage kinetics law. Below we will consider the simple Kachaonv's theory and that modified by adding time-independent term.

2.2. The Kachanov theory of damage growth

According to this theory the damage kinetics law is:

$$\frac{d\omega}{dt} = A(\frac{\sigma_e}{1-\omega}) , \tag{4}$$

where σ_e is given by (according to [3]):

$$\sigma_e = \alpha\sigma_1 + (1-\alpha)\sigma_i \ , \tag{5}$$

and $$\sigma_i = \sqrt{\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2} \ ;$$

σ_1, σ_2 are main principal stress.

For $\omega=1$: $\quad t(\sigma_e)=t^*(\sigma_e) = \dfrac{1}{A(m+1)\sigma_e^m}$.

With $\sigma_e=R$: $\quad t^*(R) = \dfrac{1}{A(m+1)R^m} \overset{df}{=} t_1^*$,

where R is a stress causing failure under uniaxial state of stress.

Combining the above equations one can obtain:

$$t^*(\sigma_e) = \frac{t_1^* R^m}{\sigma_e^m} \ . \tag{6}$$

With $\alpha=0$ and

$$s \overset{df}{=} \frac{\sigma_e}{R} \tag{7}$$

we have from (5):

$$s = \sqrt{(s_1^2+s_2^2-s_1s_2)} \ , \tag{8}$$

where: $s_1 = \dfrac{\sigma_1}{R}$, $s_2 = \dfrac{\sigma_2}{R}$. Now the eq.(6) can be rewritten as:

$$t^*(\sigma_e) = \frac{t_1^*}{s^m} \ . \tag{9}$$

or

$$T^* = \frac{1}{s^m} \overset{df}{=} T_k^* \tag{10}$$

if the notation

$$\frac{t^*(\sigma_e)}{t_1^*} \overset{df}{=} T^* \tag{11}$$

is used.

T_k^* as a function of s_1 and s_2 is shown in Fig.2.

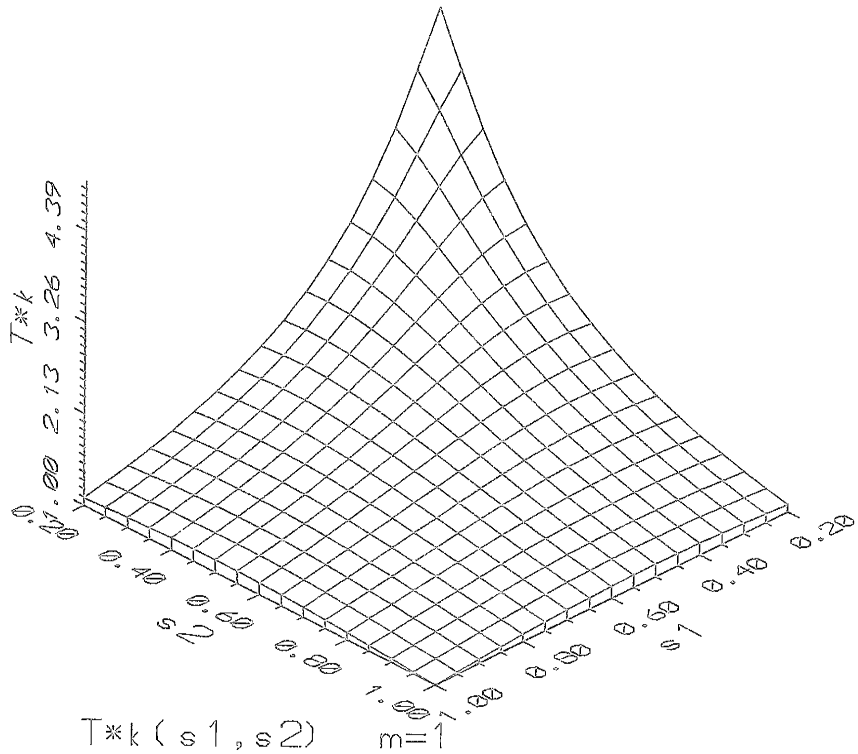

Fig.2. Time to failure due to Kachanov's theory

For $t < t^{*}(\sigma_e)$:

$$\omega(t) = 1-\left[1-\frac{t}{t^{*}(\sigma_e)}\right]^{\frac{1}{m+1}} . \qquad (12)$$

Now, we can calculate both: time dependent exertion factor denoted as W_{tk}, and stress dependent one denoted as W_{sk}.

According to definitions (1) and (9) we have:

$$W_{tk} = Ts^{m} \qquad (13)$$

where $$T = \frac{t}{t_1^{*}} . \qquad (14)$$

With the help of (10) one can put:

$$W_{tk} = \frac{T}{T_k^*} \ . \tag{15}$$

To calculate stress-dependent exertion factor W_{sk} we must find out the stress $\tilde{\sigma}_e$, causing the failure at given exploitation time, i.e. satisfying the condition:

$$t(\tilde{\sigma}_e) = t^*(\tilde{\sigma}_e) \ . \tag{16}$$

According to eq.(9):

$$t(\tilde{\sigma}_e) = \frac{t_1^*}{\tilde{s}^m} \ , \tag{17}$$

where $$\tilde{s} = \frac{\tilde{\sigma}_e}{R} \ . \tag{18}$$

Using definition (3) we have:

$$W_{sk} = \frac{s}{\tilde{s}} \ , \tag{19}$$

and making use of eq.(17):

$$W_{sk}^m = s^m \frac{t(\tilde{\sigma}_e)}{t_1^*} \ , \tag{20}$$

or with help of eq.(14):

$$W_{sk}^m = s^m T \ . \tag{21}$$

Comparison of eqs (13) and (21) yields:

$$W_{sk}^m = W_{tk} \ . \tag{22}$$

As $W_{tk}<1$ and $W_{sk}<1$, then for $m>1$ there is $W_{tk}<W_{sk}$. Thus, according to Kachanov's theory exertion depends quite paradoxically on acting stress, solely. In the following we suggest that this can be avoided by using a modified damage law.

2.3. Modified damage theory

In this paragraph a damage growth law which incorporates time-dependent and time-independent damage cumulation will be used:

$$\frac{d\omega}{dt} = A_o\left(\frac{\sigma_e}{1-\omega}\right)^{m_o}\frac{d\sigma_e}{dt} + A\left(\frac{\sigma_e}{1-\omega}\right)^{m} \qquad (23)$$

With $$A_o \overset{df}{=} \frac{1}{R_o^{m_o+1}} , \qquad (24)$$

and dimensionless stresses defined by eq.(8), and dimensionless time (14) we have:

$$\omega = 1-[(1-s^{m_o+1})^{\frac{m+1}{m_o+1}} - Ts^m]^{\frac{1}{m+1}}$$

or with help of eq.(10):

$$\omega = 1-[(1-s^{m_o+1})^{\frac{m+1}{m_o+1}} - \frac{T}{T_k^*}]^{\frac{1}{m+1}} \qquad (25)$$

By putting $\omega=1$ the time to failure T^* according to modified theory can be found:

$$T^* = T_k^*(1-s^{m_o+1})^{\frac{m+1}{m_o+1}} \qquad (26)$$

The function $T_m^*(s_1,s_2) = \frac{T^*}{T_k^*}$ is shown in Fig.3 for m_o=m=2.

According to definition given by eq.(1):

$$W_{tm} = \frac{T}{T_k^*}(1-s^{m_o+1})^{-\frac{m+1}{m_o+1}} \qquad (27)$$

The stress $\tilde{s}$ causing instantaneous failure at exploitation time t can be found from eq.(23):

$$\tilde{s} = \{[(1-s^{m_o+1})^{\frac{m+1}{m_o+1}} - \frac{T}{T_k^*}]^{\frac{m_o+1}{m+1}} + s^{m_o+1}\}^{\frac{1}{m_o+1}} \qquad (28)$$

Stress-dependent exertion factor W_{sm} takes the value:

$$W_{sm} = s\{[(1-s^{m_o+1})^{\frac{m+1}{m_o+1}} - \frac{T}{T_k^*}]^{\frac{m_o+1}{m+1}} + s^{m_o+1}\}^{-\frac{1}{m_o+1}} \qquad (29)$$

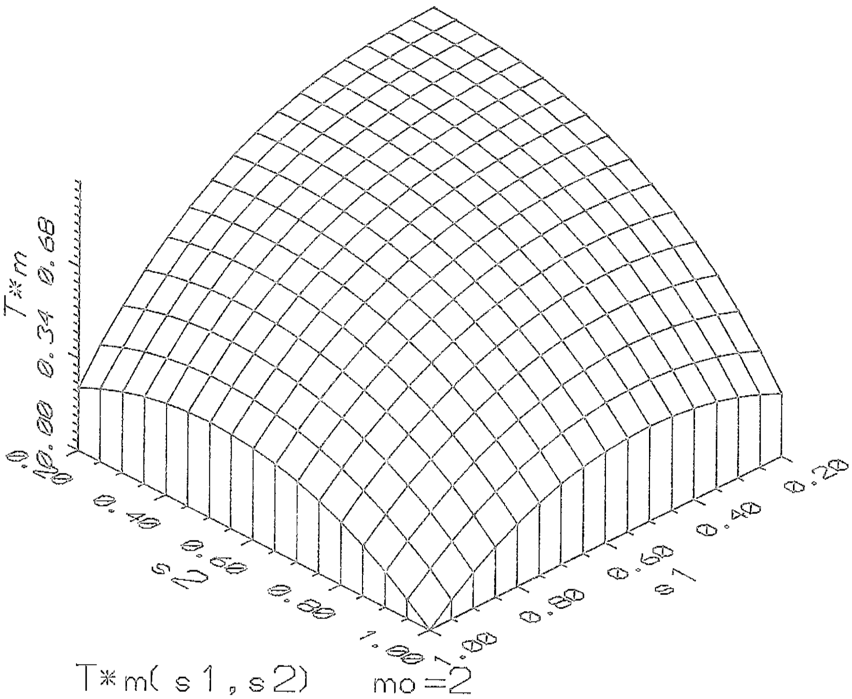

Fig.3. Time to failure due to modified theory

$$W_{tm} = \frac{T}{T_k^*(1-s^{m+1})} \qquad \text{and} \qquad W_{sm} = \frac{s}{\left(1-\frac{T}{T_k^*}\right)^{\frac{1}{m+1}}}$$

The ratio W_{tm} / W_{sm} can be less or greater than one depending on the relation of s to T_m:

For $s(1-s^{m+1}) \geq T_m(1-T_m)^{\frac{1}{m+1}}$ there is $W_{tm} \geq W_{sm}$.

It is seen from the above analysis that modified damage law gives more general approach to the evaluation of safety factor, which now depends on interrelation of applied stress and exploitation time.

3 SAFETY FACTOR FOR A STRUCTURE

3.1. Definitions and notations

The time at which damage variable reaches its critical value (normally set to 1) at least at one point of a body will be denoted by t_I. For the time $t>t_I$ the degradation process develops in a structure to come to the state when a surface on which damage is equal to 1 spans all the body at time t_{II}. This time is usually considered as an ultimate time of structure exploitation. One can consider that in the fractured cross-section a "super-hinge" is formed without any load bearing capacity. If the structure is statically indeterminate, the existence of such a hinge not necessarily means the total loss of loading capacity of a structure. The loading may be transferred to those parts of a structure which still can work, until it will become kinematically unstable as the result of consecutive appearance of "super-hinges". This time will be denoted as t_{III}. It is the main goal of the present paragraph to perform an analysis of a simple structure to evaluate all times t_i (i=I, II, III).

3.2. General assumptions and governing equations

The damage description was assumed to have the simplest form, i.e. scalar representation of damage variable ω ($0\geq\omega\geq1$), and stress governed damage rate law is:

$$\mathring{\omega}=A\langle\sigma_e/(1-\omega)\rangle^m , \quad \omega(0)=0 , \tag{30}$$

where A and m are material constants and σ_e is the equivalent stress according to Hayhurst et al.[3]:

$$\sigma_e = \alpha\sigma_1+(1-\alpha)\sigma_i \tag{31}$$

where σ_1 is maximum principal stress and σ_i is effective stress. The brackets $\langle\rangle$ in (30) denote that the damage accumulation occurs only for positive σ_e.

For such a formulation the stress redistribution in time plays an essential role in damage cumulation and therefore the general theory of nonstationary creep was adopted [4]:

$$\varepsilon_{ij}=\varepsilon^e_{ij} + \varepsilon^c_{ij} \tag{32}$$

$$\varepsilon^e_{ij}=D_{ijkl}\sigma_{kl} \tag{33}$$

$$\mathring{\varepsilon}^c_{ij}=\gamma\,[\sigma_i/(1-\omega)]^n s_{ij}, \quad \varepsilon^c_{ij}(0)=0 \tag{34}$$

where superscript e and c refer to elastic and creep deformation, respectively, s_{ij} is deviatoric stress, and γ and D are material functions.

As it can be seen from (34) that so called coupled theory of creep is used. Therefore the stress redistribution will be an effect of two processes: redistribution connected to transition from initial elastic solution at t=0 to the quasi stationary creep (for $t \to \infty$), and that associated with the damage growth. The latter will cause the stress to diminish at the points where damage develops, to drop finally to zero when $\omega = 1$ at time t_I.

For time $t>t_I$ all above equations will hold but the domain of integration will be reduced only to these points of the body at which $\omega<1$. This will imply the change in structure dimensions, which in turn will result in strain and damage localization. This effect will be shown by the results of example. The behavior of a structure for time $t > t_{II}$ depends first of all on the kinematic boundary conditions (statically determinate or indeterminate structures) and on the type of a structure (beams, plates, shells etc.). For statically determinate structures $t_{II}=t_{III}$ and the further analysis is impossible. In the following a simple statically indeterminate beam will be analyzed to show the effect of the damage development for $t>t_{II}$.

3.3. Example of a structure analysis

The beam of redundancy equal to 1 (Fig.4) with the rectangular cross-section 2h x b and loaded by a proportionate load q was chosen as the simplest example of a structure which behavior will cover all three stages of damage development.

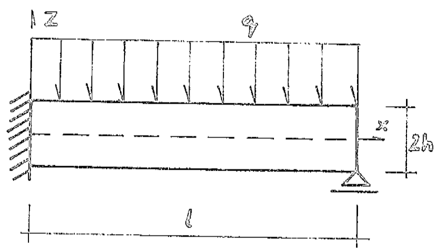

Fig.4. Analyzed beam

In the analysis the assumption of plane cross-section deformation was made and shear stresses were neglected. Numerical integration of governing equations was performed for the following boundary conditions:

$$w(x=0) = 0, \quad dw/dx(x=0) = 0, \quad w(x=l) = 0,$$

with time step limited by a criterion:

$$\Delta t = \{4(1+\nu)\} \ / \ \{3EKn \max_{x,z} [\sigma^{n-1}(x,z)]\}$$

proposed by Cormeau [2].

Distributions of stress, damage and creep strain are shown in Fig.5 (a, b, and c respectively) for time $t=t_{II}- \Delta t$. The fractured area of a cross-section at x=0 is marked as a dashed one. The essential damage and creep strain localization can be observed in the areas where first crack appears (x=0, z=h).

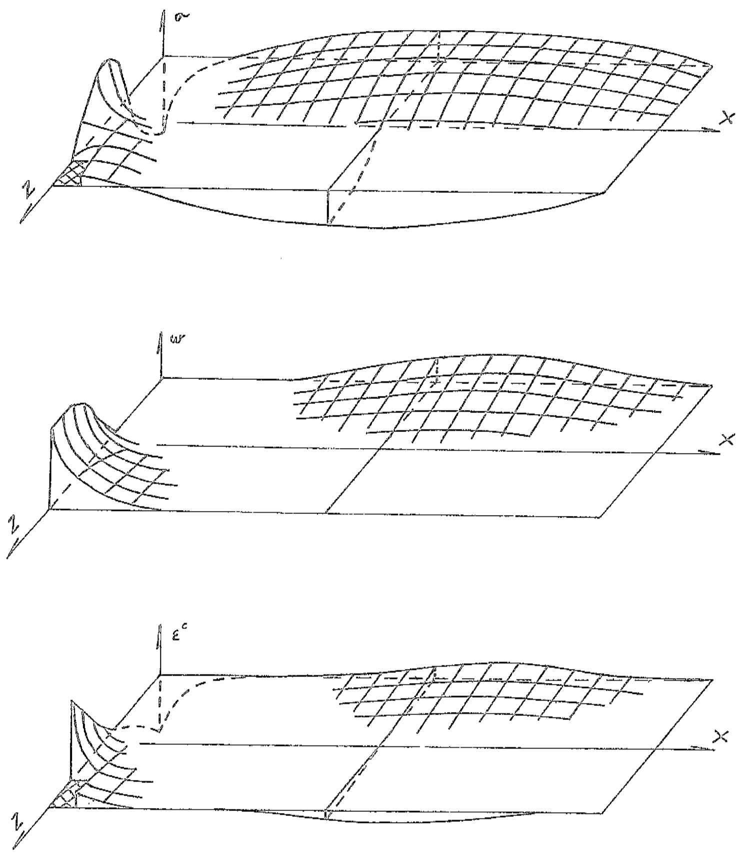

Fig.5. Distribution of stress (a) damage(b) and creep-strain (c) for $t = t \;\; - \Delta t$

3.4. Conclusions

The results of calculation give the following values of t_i: t_I = 115 [h], t_{II}= 128 [h], t_{III}= 178 [h], and show that the safety margin related to the time t_I is low for this example (t_{II}/t_I= 1.11 and t_{III}/t_I= 1.55). Undoubtedly, time t_{III} depends heavily on structure redundancy, and will grow with it. However, the most important factors influencing structure behavior are redistributions of bending moments and stress. These effects are connected with assumptions made. Table 1 summarize the comparison of t_I and t_{II} values for the following simplifying assumptions:

1. Structure was considered to be statically determinate but stress redistribution due to damage growth was taken into account,
2. Both effects of moment and stress redistribution were considered, as well as cross-section height reduction for $t > t_I$ (actual solution).
3. Steady-state creep theory for statically indeterminate structure was applied, without stress redistribution.

Table 1

	Redistribution of			t_I	t_{II}	t_{II}/t_I
	M	σ	h			
1	–	+	+	90.7	96.2	1.06
2	+	+	+	115	128	1.11
3	–	–	–	169	442	2.61

The general conclusion is that the evaluation of structure safety on the base of uncoupled steady-state creep theory is non-conservative (except the lower bound obtained for pure elastic moment distribution), and only full analysis with bending moments and stress redistribution taken into account should be used for a proper assessment.

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