The Concept of Equivalent Solid Plate in the Modal Analysis of Perforated Plates

S.N. Plyat, A.P. Villasor, Jr.
Westinghouse Electric Corporation, P.O. Box 355, Pittsburgh, Pennsylvania 15230, U.S.A.

The concept of equivalent solid plate elaborated for stress-strain problems of perforated plates is found sufficiently accurate for modal analysis of such plates. The equivalent solid plate should be taken as anisotropic with the effective elastic constants determined according to the theory by O'Donnell, et al.

By using finite elements, modal analyses of various perforated plates and their corresponding equivalent solid plates were performed and the first five natural frequencies compared. Both the triangular and the square penetration patterns were considered.

The replacement of the perforated plate by an equivalent anisotropic solid plate with any combination of simply supported/clamped boundaries gives a maximum error of ∿ 7% on the fundamental frequency and ∿ 12% on the first five natural frequencies.

1. Introduction

The modal analysis by the finite element method of perforated plates containing a large number of holes such as tubesheets, tube support plates and deck plates of steam generators becomes a herculean effort if all the individual penetrations of the plate are taken into account in the model. The concept of equivalent solid plate with modified elastic constants was originated by Mejers, O'Donnell, Slot, Porowski [1, 2, 3] and set forth in the American Society of Mechanical Engineers Boiler and Pressure Vessel Code. The objective of the present paper is to implement the concept of equivalent solid plate for the modal analysis of perforated plates.

2. Theory

Villasor and Plyat [4] have shown that there is a strong relationship between the fundamental frequency of a perforated plate and the maximum displacement due to its own weight. This relationship has the form:

$$f_1 = \frac{\alpha}{2\pi} \sqrt{g/W_{max}} \qquad (1)$$

where: f_1 = fundamental frequency; W_{max} = maximum static transverse displacement of the plate under its own weight; g = acceleration of gravity; α = constant

The constant "α" was found equal 1.257 for perforated plates with triangular penetration pattern [4] and 1.233 for perforated plates with square penetration pattern [5]. Jones [6] calculated "α" = 1.267 for solid plates. For transversely isotropic solid plates, "α" was computed to be 1.234.

Thus, for the solid isotropic and, at least, transversely isotropic plates as well as for perforated plates, eq. (1) is apparently true, using an average "α" of 1.248. In other words, all plates of this set which have the same displacement due to their own weights have approximately the same fundamental frequency.

The concept of equivalent solid plate has satisfactorily solved stress-strain problems of perforated plates. This is based on considering the perforated plate material as a solid material with modified or effective elastic constants. These constants are determined by requiring that the gross deformations of the perforated plate and the equivalent solid plate be the same under identical loading conditions. On this basis, the a priori assumption is made that the theory of equivalent solid plate elaborated previously for the solving of static stress-strain problems of perforated plates can be used as an approximation for modal analysis of perforated plates. Nevertheless, two questions arise: What is the accuracy of such an approximation? How many lower frequencies can be covered by this approximation?

3. Calculation Techniques

In order to evaluate the applicability of the concept of equivalent solid plate for calculating the transverse natural frequencies of perforated plates, modal analyses of actual perforated plates and the corresponding equivalent solid plates with effective elastic constants were compared. The modal analyses of both perforated and equivalent solid anisotropic/isotropic plates were performed with the finite element method, employing the

WECAN Computer Program [8] and its three-dimensional isoparametric element.

Considered were plates of different geometries, various sizes and different boundary conditions on the edges: simply supported, clamped, partially simply supported - partially clamped, partially supported - partially free.

The reduced modal analysis was used to determine the natural frequencies of the plates. The set of eigenvalues and eigenvectors were found by the Householder's method.

To determine the effective elastic constants of the equivalent solid plate, the theory advanced by Slot, Porowski and O'Donnell was used, considering the data for the state of generalized plane strain.

4. Modal Analysis of Perforated Plates with Triangular Penetration Pattern

The main results of the modal analysis of perforated plates with triangular penetration pattern were presented in [7], where the details are given. Figure 1 shows the finite element model.

The in-plane behavior of the perforated plate with triangular penetration pattern is isotropic while the anisotropy of the equivalent solid material is taken in the thickness direction only. Following Porowski and O'Donnell, the elasticity matrix of the equivalent solid material is:

$$D = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ D_{21} & D_{22} & D_{23} & 0 & 0 & 0 \\ D_{31} & D_{32} & D_{33} & 0 & 0 & 0 \\ D & 0 & 0 & D_{44} & 0 & 0 \\ D & 0 & 0 & 0 & D_{55} & 0 \\ 0 & D & 0 & 0 & 0 & D_{66} \end{bmatrix} \tag{2}$$

where:

$$\left.\begin{aligned}
D_{11} &= D_{22} = \frac{\bar{E}^*}{f\,(1+\bar{\nu}^*)}\left(1 - \frac{\bar{E}}{\bar{E}_Z^*}\,\nu^2\right); \\
D_{12} &= D_{21} = \frac{\bar{E}^*}{f\,(1+\bar{\nu}^*)}\left(\bar{\nu}^* + \nu^2\,\frac{\bar{E}^*}{\bar{E}_Z^*}\right); \\
D_{13} &= D_{31} = D_{23} = D_{32} = \frac{\bar{E}^*\,\nu}{f}\ ;\ D_{33} = \frac{\bar{E}_Z^*\,(1-\bar{\nu}^*)}{f}\ ; \\
D_{44} &= D_{55} = \frac{\bar{E}_Z^*}{2\,(1+\nu)}\ ;\ D_{66} = \frac{\bar{E}^*}{2\,(1+\bar{\nu}^*)}; \\
f &= 1 - \bar{\nu}^* - 2\,\frac{\bar{E}^*}{\bar{E}_Z^*}\,\nu^2
\end{aligned}\right\} \tag{3}$$

Here starred quantities relate to the equivalent solid plate; the bars, to generalized plane strain conditions. Unsubscripted quantities belong to the in-plane properties; the quantities with subscript Z, to properties in the thickness direction.

The resulting natural frequency calculations for both perforated and equivalent solid anisotropic plates are set forth in tables 1, 2, and 3. The natural frequencies of perforated plates in all cases were taken as 1.000 and only the relative natural frequencies of the equivalent solid plates were calculated. Note that the thickness of the plates were represented by the ratio "thickness/pitch", H/P.

The data in table 1 is the relative natural frequency of the plates with the following boundary conditions: simply supported (SS), clamped (CL), and partially simply supported and partially clamped, (SS + CL). Table 2 contains the relative natural frequencies of the plates with free (FR) and partially simply supported and partially free (SS + FR) boundaries. From table 1, the first five natural frequencies for both perforated and equivalent solid anisotropic plates are close. The absolute values of their differences were 0.1% (minimum) and 8.3% (maximum) for the first five frequencies. In table 2, it is seen that the differences between natural frequencies of the perforated and equivalent solid anisotropic plates are larger. A maximum discrepancy of 22% takes place for the first frequency; the deviation may reach 46% for the next four frequencies.

Table 3 shows the effect of the plate thickness on the natural frequency of perforated plate with simply supported edges. In this case, the frequency difference is less than 7.0% for the first five frequencies. Table 4 gives the results of the comparison of the natural frequencies of the perforated and equivalent solid isotropic plates based on the "isotropic" approach to the problem when the effective elastic constants in the in-thickness direction are supposed to be equal to the effective elastic constants in the in-plane direction. It is observed that the natural frequencies of the equivalent solid isotropic plate with simply supported edges might differ as much as ∿ 70% from the corresponding natural frequencies of the perforated plates.

5. Modal Analysis of Perforated Plates with Square Penetration Pattern

The finite element models for the perforated plate is shown in figure 2. The conditions of symmetry were imposed and the constraints on the other two edges were mixed: simply supported on one and clamped on the other. The hole diameter, the pitch, and the ligament were made the same as in the triangular penetration pattern case.

For the square penetration pattern, the equivalent material properties depend on the orientation of loading with respect to the symmetric axes of the pattern. The in-plane effective elastic constants of perforated plates with square penetration pattern are usually given in terms of the "pitch" and "diagonal" directions. The following are the non-zero entries in the elasticity matrix $D = [D_{ij}]$ for three recommendations concerning different kinds of averaging in-plane effective elastic constants of the equivalent plate:

Recommendation 1: Formulas are the same as in eq. (3), except

$$\bar{E}^* \to \bar{E}_p^*,\ \bar{\nu}^* \to \bar{\nu}_p^*;\ D_{66} = \frac{\bar{E}_d^*}{2\,(1 + \bar{\nu}_d^*)}$$

Recommendation 2: Formulas are the same as in eq. (3), except

$$\bar{E}^* \to \bar{E}_d^*,\ \bar{\nu}^* \to \bar{\nu}_d^*;\ D_{66} = \frac{\bar{E}_p^*}{2\,(1 + \bar{\nu}_p^*)}$$

Recommendation 3:

$$D_{11} = D_{22} = \frac{\bar{E}_p^*}{f\,(1+\bar{\nu}_p^*)}\left(1 - \frac{\bar{E}_p^*}{\bar{E}_Z^*}\nu^2\right) + 1/2\left(\frac{\bar{E}_d^*}{2\,(1+\bar{\nu}_d^*)} - \frac{\bar{E}_p^*}{2\,(1+\bar{\nu}_p^*)}\right)$$

$$D_{12} = D_{21} = \frac{\bar{E}_p^*}{f\,(1+\bar{\nu}_p^*)}\left(\bar{\nu}_p^* + \frac{\bar{E}_p^*}{\bar{E}_Z^*}\nu^2\right) - 1/2\left(\frac{\bar{E}_d^*}{2\,(1+\bar{\nu}_d^*)} - \frac{\bar{E}_p^*}{2\,(1+\bar{\nu}_p^*)}\right)$$

The remaining non-zero entries are defined by the Recommendation 1.

Table 5 contains the first five relative natural frequencies of perforated plates and the equivalent solid plates with different average non-zero entries in the elastic matrix D. From table 5, Recommendation 2 based on the "diagonal" averaging gives the best results. Therefore, the non-zero entries in $[D_{ij}]$ by Recommendation 2 should be used in the modal analysis of perforated plates with square penetration pattern.

The relative natural frequencies of perforated and equivalent solid rectangular plates are given in table 6 (rectangular plates with different aspect ratios) and in table 7 (square plates with different thicknesses). The absolute values of the differences between natural frequencies of the perforated and equivalent solid plates were 0.9% (minimum) and 5.1% (maximum) for the first frequency. For the first five frequencies, the maximum discrepancy is 12.4%.

6. Conclusion

It is established that the concept of the equivalent solid plate originally elaborated for solving of stress-strain problems of perforated plates can now be recommended as an approximation in the modal analysis of similar plates with no free or partially-free boundaries. To obtain sufficiently accurate results, the equivalent solid plate should be considered anisotropic with the effective elastic constants determined by the theory of O'Donnell, et al. However, for the square penetration pattern, the diagonal averaging method should be used to calculate the effective elastic constant.

MODE	BOUNDARY CONDITION					
	ss	cl	ss + cl	ss	cl	ss + cl
1	0.989	0.960	0.996	1.043	1.001	1.072
2	0.956	0.934	0.957	1.025	1.021	0.963
3	1.011	1.008	1.040	0.945	0.961	1.059
4	1.026	1.003	1.012	1.027	1.018	1.049
5	1.009	1.007	1.039	1.070	1.083	0.964
	RATIO H/P = 0.75			RATIO H/P = 0.10		

TABLE 1: Relative Natural Frequencies of Solid Anisotropic Plates Equivalent to Perforated Plates with the Triangular Penetration Pattern

MODE	BOUNDARY CONDITION					
	fr	ss + fr	ps	fr	ss + fr	ps
1	--	1.149	1.152	--	1.217	1.210
2	0.854	1.136	1.203	0.956	1.151	1.386
3	0.945	1.317	1.181	0.992	1.123	1.257
4	1.346	1.196	1.094	1.288	1.462	1.107
5	1.377	1.214	1.054	1.265	1.268	1.219
	RATIO H/P = 0.75			RATIO H/P = 1.10		

TABLE 2: Relative Natural Frequencies of Solid Anisotropic Plates Equivalent to Perforated Plates with the Triangular Penetration Pattern

MODE	RATIO H/P				
	1.50	0.75	0.40	0.20	0.10
1	0.975	0.989	0.997	1.017	1.043
2	0.926	0.956	0.995	0.965	1.025
3	1.025	1.001	0.998	0.976	0.945
4	0.979	1.026	1.033	1.023	1.027
5	0.981	1.009	1.008	1.010	1.070
	BOUNDARY CONDITION: SS = SIMPLY SUPPORTED EDGES				

TABLE 3: Relative Natural Frequencies of Solid Anisotropic Plates Equivalent to Perforated Plates with the Triangular Penetration Pattern

MODE	RATIO H/P				
	1.50	0.75	0.40	0.20	0.10
1	1.349	1.617	1.696	1.593	1.334
2	1.009	1.244	1.395	1.359	1.179
3	0.970	1.202	1.360	1.328	1.136
4	0.681	1.170	1.342	1.352	1.184
5	0.834	1.103	1.291	1.326	1.182
	BOUNDARY CONDITION: SS = SIMPLY SUPPORTED EDGES				

TABLE 4: Relative Natural Frequencies of Solid Isotropic Plates Equivalent to Perforated Plates with the Triangular Penetration Pattern

MOOE	ELASTICITY MATRIX RECOMMENDATION		
	I	II	III
1	0.859	0.953	0.842
2	0.841	0.934	0.829
3	0.847	0.918	0.801
4	0.891	1.076	0.875
5	0.680	0.916	0.671

TABLE 5: Comparative Natural Frequencies of Solid Anistropic Plates Equivalent to Perforated Plates with the Square Penetration Pattern

MOOE	ASPECT RATIO		
	4/4	3/4	2/4
1	0.953	0.973	0.949
2	0.934	0.950	1.005
3	0.918	0.939	1.023
4	1.076	1.083	1.030
5	0.916	0.983	1.114

TABLE 6: Relative Natural Frequencies of Solid Anisotropic Plates Equivalent to Perforated Plates with the Square Penetration Pattern

MOOE	RATIO H/P		
	0.5	1.0	4.0
1	0.949	0.953	0.991
2	0.932	0.934	1.031
3	0.881	0.918	1.075
4	1.042	1.076	0.993
5	0.924	0.916	1.126

TABLE 7: Relative Natural Frequencies of Solid Anisotropic Plates Equivalent to Perforated Plates with the Square Penetration Pattern

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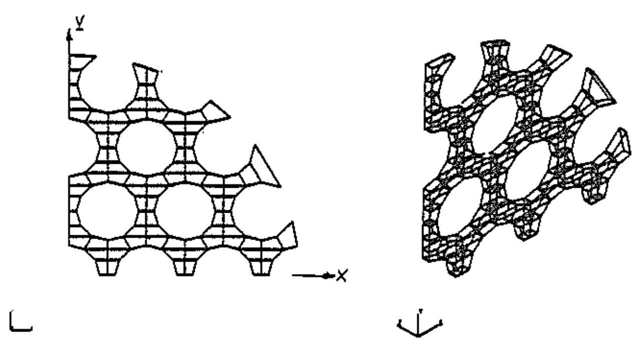

FIGURE 1: Finite Element Model of the Perforated Plate with the Triangular Penetration Pattern

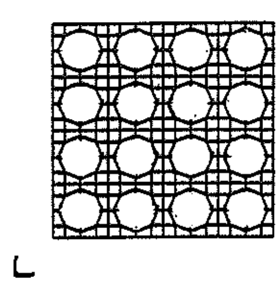

FIGURE 2: Finite Element Model of the Perforated Plate with the Square Penetration Pattern - shown for an Aspect Ratio of 4/4