Combined Error Correction and Multiuser Detection for Rayleigh Fading Synchronous CDMA Channels [1]

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Abstract *Code Division Multiple Access (CDMA) signaling is a promising candidate for the emerging mobile and wireless communication systems standards. Present-day CDMA systems, however, employ single-user (conventional) detection which is far from optimum. While multiuser detectors offer improved capacity, little work has been reported on employing them in conjunction with error correction coding. We evaluate the improvements achieved by the use of channel coding for several classes of multiuser detectors over the Rayleigh fading channel, and compare alternative implementations for two-stage detectors.*

1 Introduction

There has recently been much interest in Code Division Multiple Access (CDMA) techniques as an alternative to narrowband signaling methods in wireless communications systems, mainly due to potential capacity increases and the fact that capacity can be increased at the expense of some gradual degradation in performance for all users.

The two main problems in the multiuser environment are Multiple Access Interference (MAI) and multipath fading. In the reverse link (from mobile to base station), the detector faces a challenging problem since user signals are asynchronous and the link suffers from the "near-far" problem, where weaker users are poorly detected.

The simplest receiver structure for the multiuser detection problem is the conventional (or single-user) detector. However, its performance is generally poor for bandwidth efficient systems and weaker users suffer greatly from the near-far problem, necessitating the use of strict power control to ensure that signals from the different users arrive at the base station with close to equal power levels. The Maximum-likelihood detector, on the other hand, offers optimum performance [1] at the expense of high complexity which becomes prohibitive even for systems with a moderate number of users. Several suboptimum detectors that attempt to offer a better performance/complexity ratio have therefore been proposed, e.g. [2, 3, 4, 5]. However, most of the research effort reported so far has considered uncoded systems over the Additive White Gaussian Noise (AWGN) channel with a few exceptions (e.g. [6, 7, 8]).

In this work, we compare several low to medium complexity multiuser detectors using coding over the Rayleigh fading channel for different noise and MAI conditions; namely the conventional detector, the decorrelator, and the two-stage detectors with the conventional and the decorrelating first stage (TSCFS and TSDFS, respectively). We also compare two alternative implementations of the two-stage detectors with channel coding. We use a synchronous model and rely on computer simulations in our analysis. The multiuser detectors considered here can be generalized to the asynchronous case [3], and therefore the results of our comparative performance analysis can be carried over to the asynchronous channel found in practice.

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In Section 2, we describe the fading channel model and the CDMA system. In Section 3, we describe different CDMA receiver structures and in Section 4 our simulation model. Section 5 presents the results of our comparisons, and in Section 6 we compare two alternative schemes for two-stage detectors with coding. Finally, Section 7 presents conclusions and possibilities for future work in the area.

2 CDMA Fading Channel Model

In a CDMA system, each user is assigned a unique signature sequence and all users transmit simultaneously over the same channel. Let $b_k(j)$ denote the jth information bit of user k which is drawn from $\{-1, 1\}$ for BPSK modulation. We assume a frequency-nonselective channel which leads to a "multiplicative fading" expression for the received signal where the envelope for each user is multiplied by a realization, $c_k(t)$, of a zero-mean Gaussian complex random process. We consider a bit-synchronous system and since the information bits of each user are i.i.d., equiprobable and independent of other users', we may consider only one bit interval without loss of generality. We also assume slow fading in which case the amplitude and phase of $c_k(t)$ do not change significantly over one bit interval, and we may suppress the time-dependence of the fading coefficients for the duration of one bit. The received waveform is then given by

$$r(t) = \sum_{k=1}^{K} c_k b_k s_k(t) + n(t) \tag{1}$$

where $s_k(t)$ denotes the kth user's signature waveform, $n(t)$ is a realization of a zero-mean complex white Gaussian process with power spectral density σ^2, and c_k's are independent complex zero-mean Gaussian random variables. We consider normalized signature waveforms of duration T, i.e. $\int_0^T |s_i(t)|^2 dt = 1$.

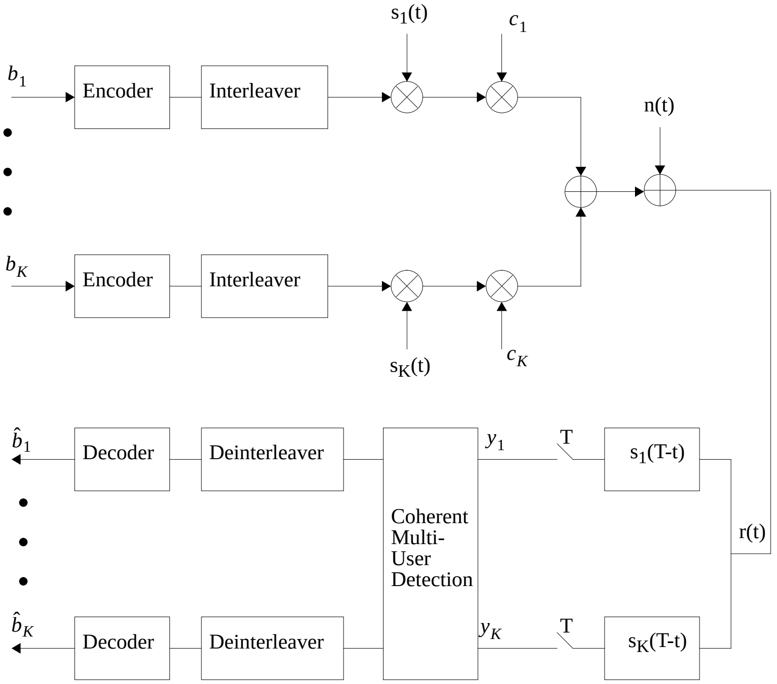

Figure 1: Uplink of a Coded CDMA system with fading

Figure 1 shows the uplink (from mobile to base station) of a coded CDMA system over a flat Rayleigh fading channel. The optimum filters at the receiver are a bank of filters matched to the signature waveforms of each user and the metrics at the output of the matched filters and samplers are given by

$$\mathbf{y} = \mathbf{RCb} + \mathbf{n} \tag{2}$$

where $\mathbf{C}=\mathrm{diag}\{c_1,...,c_K\}$ is the complex path gain matrix, and $\mathbf{n}$ is a zero-mean complex Gaussian $K-$vector with the covariance matrix $\sigma^2\mathbf{R}$ for both real and imaginary components. $\mathbf{R}$ is the matrix of cross-correlations between the normalized signature waveforms with elements :

$$R_{ij} = \int_0^T s_i(t)s_j(t)\ dt \tag{3}$$

In our comparative analysis, we consider the conventional, decorrelating, and two-stage detectors.

3 Multiuser Receiver Structures

The simplest detector is the conventional (or single-user) detector. It ignores Multiple Access Interference (MAI) and applies the decision rule

$$\hat{\mathbf{b}} = \mathbf{sgn}\ \grave{\mathbf{y}} \tag{4}$$

where

$$\grave{\mathbf{y}} = \mathbf{Re}[\mathbf{C}^*\mathbf{y}] = \mathbf{Re}[\mathbf{C}^*\mathbf{R}\mathbf{C}]\mathbf{b} + \mathbf{Re}[\mathbf{C}^*\mathbf{n}] \tag{5}$$

The decorrelator [2] is a linear detector which eliminates MAI at the expense of increased noise power. The decorrelating matrix, $\mathbf{R^{-1}}$, is applied to the output of the matched filter bank :

$$\tilde{\mathbf{y}} = \mathbf{R^{-1}y} = \mathbf{R^{-1}}[\mathbf{RCb} + \mathbf{n}] = \mathbf{Cb} + \tilde{\mathbf{n}} \tag{6}$$

where $\tilde{\mathbf{n}}$ is a zero-mean complex Gaussian $K-$vector with the covariance matrix $\sigma^2\mathbf{R^{-1}}$. The inputs to the threshold devices are the components of the vector

$$\grave{\tilde{\mathbf{y}}} = \mathbf{Re}[\mathbf{C}^*\tilde{\mathbf{y}}] = \mathbf{Re}[\mathbf{C}^*(\mathbf{Cb} + \tilde{\mathbf{n}})] = |\mathbf{C}|^\mathbf{2}\mathbf{b} + \mathbf{Re}[\mathbf{C}^*\tilde{\mathbf{n}}]\ , \tag{7}$$

and the decision rule is

$$\hat{\mathbf{b}} = \mathbf{sgn}\ \grave{\tilde{\mathbf{y}}} \tag{8}$$

Multistage detectors consist of an initial stage where tentative decisions are made for all users followed by one or more stages where more reliable decisions are obtained. While multi-stage detectors are non-linear detectors due to MAI cancelation that occurs in subsequent stages, they have linear complexity in the number of users. We consider the two-stage detector proposed in [3], with conventional and decorrelating first stages. The bit estimates of the first stage are denoted by $\hat{\mathbf{b}}(1) = [\hat{b}_1(1),...,\hat{b}_K(1)]$. Denoting the matrix $\{\mathrm{Re}[\mathbf{C}^*\mathbf{RC}]\}$ by $\mathbf{M}$, the second stage bit estimate for the kth user is given by :

$$\hat{b}_k(2) = \mathrm{sgn}\left[\grave{\mathrm{y}}_\mathrm{k} - \sum_{\mathrm{j}\neq\mathrm{k}} \hat{\mathrm{b}}_\mathrm{j}(1)\mathrm{M}_\mathrm{jk}\right] \tag{9}$$

where $\{\grave{y}\}_k$ are the elements of the vector $\grave{\mathbf{y}}$ given by Equation 5 and the first stage decisions are given by either of the conventional or decorrelating detectors.

Figure 2 shows a simplified diagram of a two-stage detector with arbitrary first stage.

4 The Simulation Model

Since closed form expressions for the probabilities of error in coded systems are very difficult to obtain (even some of the uncoded systems we study are hard to analyze, e.g. multistage detectors), we rely on computer simulations to compare the performance of the different detectors.

We study a 4-user bit-synchronous system with a set of signature waveforms derived from Gold sequences of length seven as used e.g. in [3, 4]. These signature waveforms have the property that the cross-correlation between the signals is as low as possible

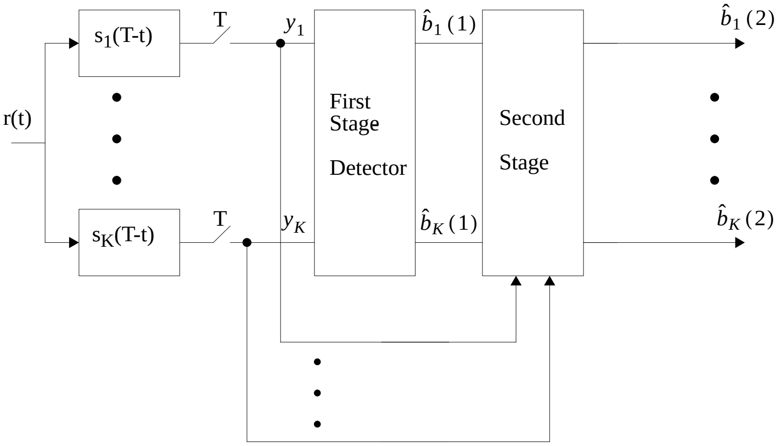

Figure 2: Two-stage Detector with Arbitrary First Stage

(given a certain allowable bandwidth) for all relative delays [3]. Since practical systems are asynchronous in nature, this is very important for drawing meaningful conclusions from our synchronous model. The resulting normalized cross-correlation matrix is [4]

$$\mathbf{R} = \frac{1}{7}\left[\begin{array}{cccc} 7 & -1 & 3 & 3 \\ -1 & 7 & -1 & 3 \\ 3 & -1 & 7 & -1 \\ 3 & 3 & -1 & 7 \end{array}\right] \tag{10}$$

We assume BPSK modulation with coherent detection. We make the usual assumption of perfect channel estimation, and perfect interleaving so that the fading coefficients, c_k, are uncorrelated from one bit interval to another. We perform our simulation runs between 10^6 and 10^7 times, depending on the estimated bit error rate. For a number of runs equal to $100/\hat{P}_e$, and for a confidence level of 95%, we obtain approximate confidence intervals equal to $(1.216\hat{P}_e, 0.822\hat{P}_e)$ calculated from Equation (9) of [9], where $\hat{P}_e$ is the estimated probability of error.

For the coded channel, we use the half-rate four-state convolutional encoder with generator sequences $\mathbf{g}_1 = [101]$ and $\mathbf{g}_2 = [111]$. It has a minimum free distance, d_{free}, of 5 and its encoder memory, m, is equal to 2. We chose this encoder since it has a small decoding delay and its minimum free distance ($d_{\text{free}} = 5$) is the largest possible for an $R = (1/2)$ code with $m = 2$ ([10], Table 11.1).

At the receiver, we implement a hard-decision truncated best-state Viterbi decoder with a path memory of 10 branches. The truncation depth was chosen to be five times the encoder memory, which results in a maximum ”truncation loss” of about 0.05 dB on the AWGN channel [11]. The above confidence intervals assume independent errors on the channel. We have error dependence due to encoding, however, which extends over 10 information bits. We obtain the same confidence intervals and confidence level, therefore, for a number of runs equal to $1000/\hat{P}_e$.

5 Comparison of Multiuser Detectors

Figures 3 and 4 show the BER plots versus average SNR for single-user and multiuser detectors for the uncoded and coded systems, respectively. The multiuser detectors generally outperform the conventional detector by several orders of magnitude of the BER. The TSDFS provides practically single-user performance over both the uncoded and coded channels. Table 1 gives the reduction in BER due to channel coding for the different detectors at different SNR points, and also the power gain in dB at a BER of 10^{-3}.

Table 1: Comparison of MU Detectors for Coded and Uncoded Systems on Fading Channel

	BER Reduction Factor (Orders of Magnitude)			Coding Gain @ 10^{-3} BER (dB)
	@13 dB	@18 dB	@23 dB	
TSCFS	1	≤ 2	2 to 3	NA
Decorrelator	0 to 1	1 to 2	2 to 3	11
TSDFS	1	≥ 2	3	12
SUB	1 to 2	≥ 2	3	11.5

We notice that the conventional detector's performance improves very little with coding and with increasing SNR. Figure 5 shows the performance of the conventional-based detectors for a system with the normalized cross-correlation matrix given by

$$\mathbf{R} = \begin{bmatrix} 1.0 & 0.2 & 0.2 & 0.2 \\ 0.2 & 1.0 & 0.2 & 0.2 \\ 0.2 & 0.2 & 1.0 & 0.2 \\ 0.2 & 0.2 & 0.2 & 1.0 \end{bmatrix} \tag{11}$$

We find that the conventional detector still has a relatively flat characteristic for SNR values beyond 10 and 15 dB for the uncoded and coded cases, respectively, resulting in a relatively high error floor.

On the other hand, returning to Figures 3 and 4, we find that the TSCFS detector's performance improves greatly with coding, and it's BER drops to about 10^{-6} before approaching the error floor. It also outperforms the decorrelator up to an average SNR value of 23 dB. The main advantage of the TSCFS detector (and non-decorrelating detectors generally) over the decorrelator is that no matrix inversion is required at the receiver. We also find that the TSCFS detector without coding does not approach its error floor over the SNR range of interest for the lower bandwidth-efficiency system of Figure 5.

We now study the important near-far situation where user 1's energy is, on average, equal to half the energies of the other three users. As expected, the conventional detector's BER is extremely high. User 1's performance is unaffected when the decorrelator is used since its performance is independent of other users' energies. Figure 6 shows the performance of two-stage detectors with conventional and decorrelating first stages for uncoded and coded systems. We observe that the TSCFS detector's performance is slightly worse than in the average equal SNR case. The change in performance is due to two opposing effects. Unlike in the 2-user case where the weaker user benefits greatly from MAI cancelation, the three stronger (on average) users here still interfere significantly with each other. Thus, they have only slightly better first stage estimates due to their higher energies and weaker interference from user 1. On the other hand, the metric at the output of the matched filter for user 1, i.e. y_1, now has higher MAI terms, which results in a slight overall increase in BER for user 1. We find that this is also the case for the AWGN channel for both the uncoded and coded systems.

6 Two-Stage Detector Implementations for Coded Systems

An alternative, and more complex, implementation (we refer to it as the complex scheme) than the one described in Figure 1 (referred to as the simple scheme) for two-stage detectors on a fading channel is shown in Figure 7. Here, error correction is applied to the decisions of the first stage detector. The decoding delay of this method is twice that of the simpler implementation of Figure 1.

While it may be expected that the complex scheme would outperform the simple scheme for all SNR values due to better first stage estimates (the only exception being when the BER is so high that the decoder introduces more errors), we find that it is not the case. For the additive white Gaussian noise channel, the superiority of a particular scheme depends on the average SNR and the cross-correlation between the systems users and the simple scheme is always at least as good as the complex scheme for the fading channel (Figure 8).

One explanation for this behavior is that the decoding applied to the first stage decisions introduces error dependence into the bit sequences entering the second bank of Viterbi decoders, and the channel ceases to be memoryless from that point onwards. Thus, the Viterbi decoders no longer provide optimum decisions. This effect is offset, however, by the improved decisions of the first stage and the complex scheme outperforms the simple scheme for low SNR values. The simple scheme always outperforms the complex scheme for higher SNR since the decoding of first stage decisions becomes less critical to subsequent stages, and the non-optimality of the second Viterbi decoder bank in the complex scheme results in a net degradation in performance.

The cross-over point between both schemes (if it exists) also depends on the cross-correlations between the system users. When we compare both schemes for a lower bandwidth-efficiency channel (**R** given by Equation 11), we find both schemes have equal performance, as shown in Figure 5. Moreover, for the AWGN channel, where MAI is less severe, the complex scheme outperforms the simple scheme for low SNR values when **R** is given by Equation 10.

The reason for this behavior is that for low cross-correlations between system users, the MAI interference cancelation capability of the second stage is very limited. Thus, the "changes" introduced by the second stage to the path chosen by the first decoder bank are very few and the non-optimality of the second decoder bank is insignificant. In addition, for low SNR values the advantage of improved first stage decisions being fed into the second stage results in better overall performance for the complex scheme. For the same level of cross-correlations between system users, the simple scheme does better than the complex scheme on the fading channel since it is inherently a "near-far" channel.

7 Conclusions

In this work, we evaluated the performance gap between single- and multi-user detectors for typical average SNR values on a flat fading channel. We find that with a simple half-rate code, two-stage detectors outperform the conventional detector by up to five orders of magnitude of the BER and that lower cross-correlations between system users do not significantly improve the conventional detector's performance. The TSCFS detector, on the other hand, improves greatly with lower user cross-correlations. For the coded system, it outperforms the decorrelator for up to a SNR value of 23 dB, and its BER for larger SNR values is almost 10^{-6}. Moreover, its performance in a simulated near-far situation is almost unaffected. The crucial advantage of the TSCFS detector over the decorrelator is that is does not require cross-correlation matrix inversion at the receiver.

We compare two alternative schemes (simple and complex) for employing error correction with two-stage detectors and analyze their performance. Under investigation is the use of concatenated coding to avoid the non-optimal decisions of the second Viterbi decoder bank in the complex scheme. We are also planning to study performance of simple multiuser detectors such as the TSCFS detector and interference cancelation method ([12]) for realistic Rayleigh fading channels.

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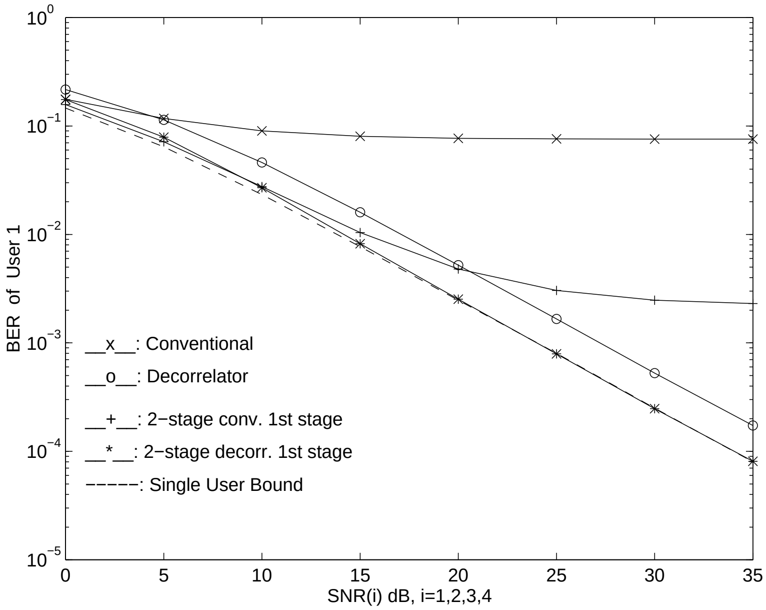

Figure 3: BER for Uncoded 4-user Systems on a Rayleigh Fading Channel

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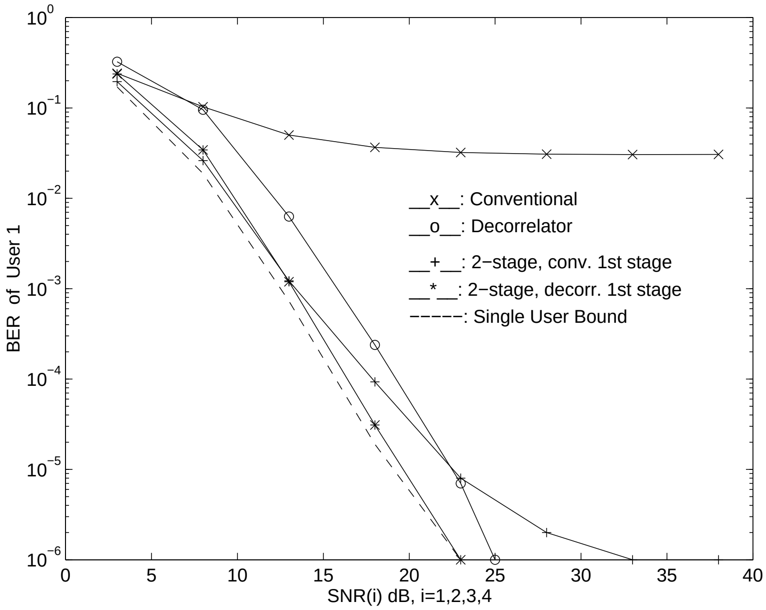

Figure 4: BER for Coded 4-user Systems on a Rayleigh Fading Channel

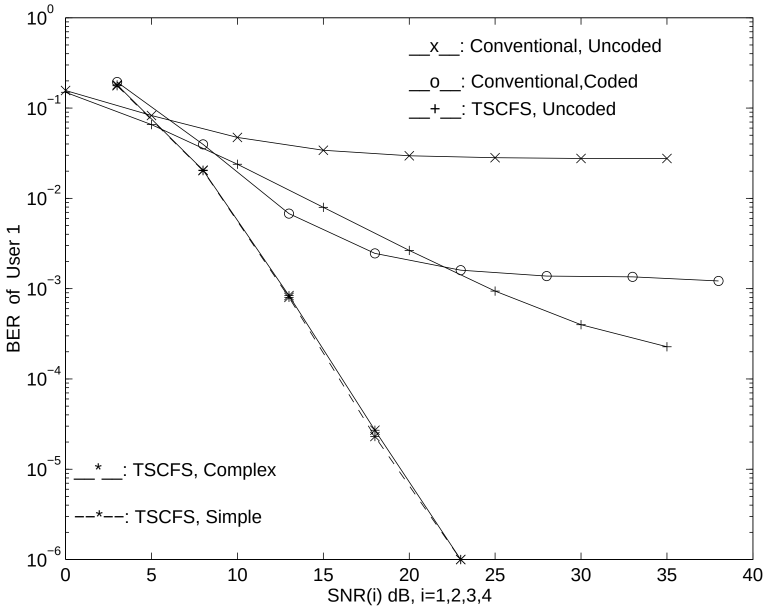

Figure 5: BER for Conventional-based Detection on a Lower Bandwidth-efficiency System

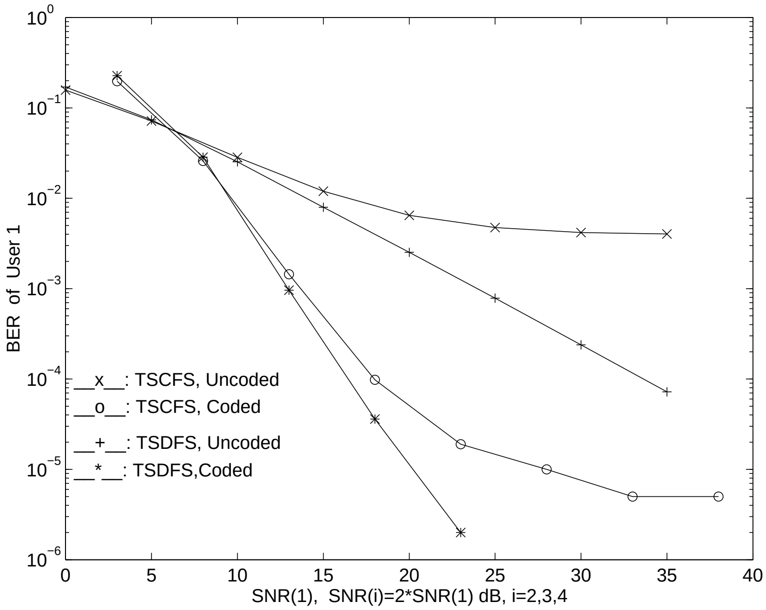

Figure 6: BER for Two-stage Detectors in a near-far environment

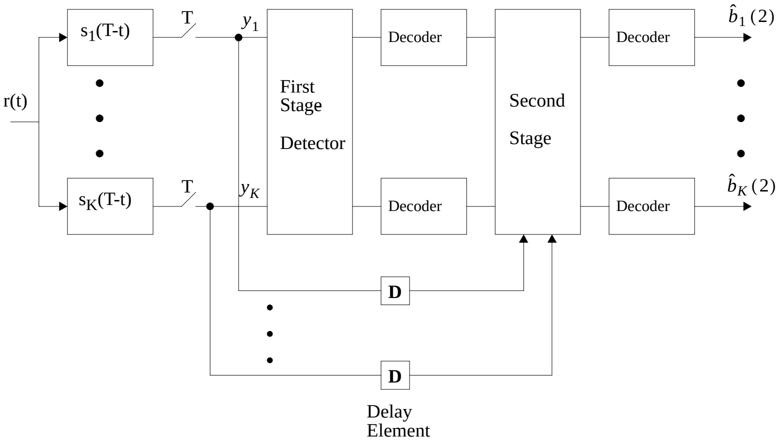

Figure 7: Complex Implementation of a Two-stage Detector with Coding

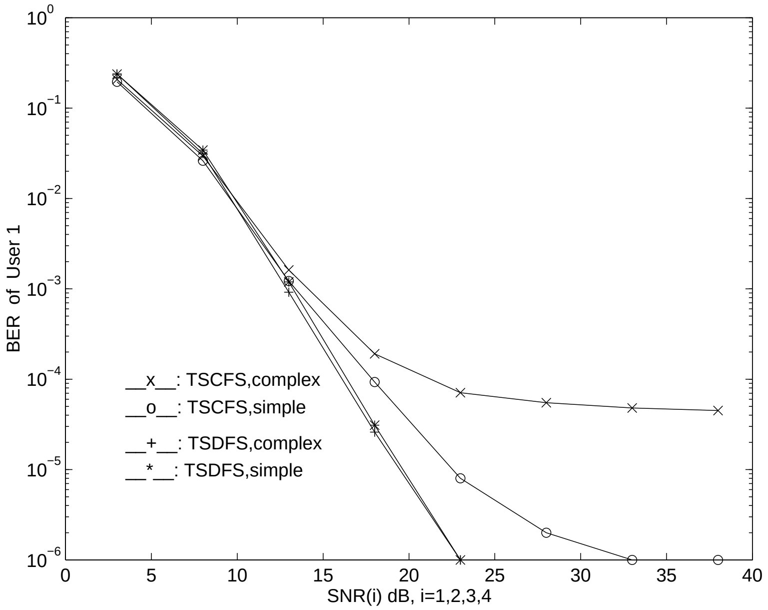

Figure 8: BER of "Simple" and "Complex" Schemes for Two-stage Detectors with Coding in a Fading Channel