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Fracture Energy and Size Effect Studies for Nuclear Concrete Structures

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1 ABSTRACT

The structural size effect law with influence of nonlinear fracture mechanics is revisited with the new experimental programme on notched / unnotched, plain and reinforced Three Point Bend (TPB) concrete beam specimens using conventional instrumentation, acoustic b-value analysis and high resolution image processing systems to evaluate the fracture energy of quasi-brittle concrete material. The target application areas of concrete nuclear structures are described, where in the nonlinear fracture mechanics principles can be used for the finite element inelastic analysis. This is explained with a few case studies using finite element cohesive crack and crack band fracture models and the issues of finite element mesh sensitivity as observed in the classical strength / strain based non-linear finite element theories are addressed. It is finally demonstrated that the limitations of the model and specimen testing for the prediction of prototype structural inelastic behaviour can be overcome through a rational approach by establishing the size effect law with the size independent fracture energy, which is being derived under the present experimental program.

2 INTRODUCTION

The design, analysis and testing of large size nuclear concrete structures pose problems due to varying sizes of the test specimens, models and prototype structures, which exhibit the structural size effect. Despite extensive research efforts and experimental testing programs undertaken by the nuclear industry the issue of structural size effect remains an enigma as the results obtained from such model and specimen tests cannot be directly used for predicting the inelastic and fracture behaviour of prototype concrete structures. In this paper the structural size effect law for such structures is revisited and is explained through nonlinear fracture mechanics models. The fracture energy evaluation on notched / unnotched, plain and reinforced Three Point Bend (TPB) concrete beam specimens using conventional instrumentation, acoustic b-value analysis and high resolution image processing systems is presented for Indian nuclear concrete structures. Further, a few case studies are analyzed with numerical finite element cohesive crack and crack band models to illustrate the issues of mesh sensitivity as observed in the classical strength / strain based non-linear finite element theories.

The structural size effect in plain and reinforced concrete structures have been earlier addressed by many investigators with an aim to quantify its effect on the structural strength, which determines the ultimate load carrying capacity based on stress/strain based theories. Many experimental and analytical simulation studies reported earlier though have shown the influence of the structural size effect on the strength of plain and reinforced concrete structures, still there remains many ambiguities in the interpretation of the observed results with regard to tensile, flexure and shear failures. With the evolution of non linear fracture mechanics theory for quasi-brittle material like concrete, it has been possible to quantify purely in a mechanistic manner the influence of the structural size effect with the help of brittleness number, characteristic length and fracture energy, which have been shown to be the typical material characteristics in a more rational manner. After defining the structural size effect laws based on the strength theories, further studies using the fracture energy of concrete have been reported by Bazant et al [1].

In this paper, we present the experimental and finite element simulation studies to address the structural size effect influence on the fracture energy of beam specimens. The three points bend plain and reinforced concrete beam specimens used for obtaining the fracture energy are indicated in the test-matrix of Table-1. The measurements during the tests are CMOD, load line displacement and acoustic data for evaluating the Fracture Process Zone (FPZ). The fracture energy obtained from the above tests can be directly used for inelastic finite element analysis of concrete structures. In order to obtain the mesh insensitive results with the numerical models, the adequacy of finite element cohesive crack and crack band fracture models is illustrated for identifying the critical structural and flaw size parameters that influence the maximum load carrying capacity, the softening behaviour and inelastic response up to the collapse load. A few numerical examples are presented and the peak load and inelastic softening response in agreement with the reported experimental results are obtained with the use of consistent fracture energy. Further the structural size effect is explained through non linear fracture mechanics and size independent fracture energy obtained from these tests are shown to be effective for predicting the collapse and inelastic behaviour of nuclear concrete structures.

TABLE 1

S.No	Depth (d), mm	Width (b), mm	Span (S= 3d), mm	Total length (L=4d), mm
1	94	47	282	376
2	188	94	564	752
3	750	375	2250	3000

3 THE STRUCTURAL SIZE EFFECT

The structural size effect is explained through nonlinear fracture mechanics principle with the equation

$$ _N = \quad _0 / (1+ \quad)^{1/2} \qquad (1)$$

With ratio = D/D_0, D= characteristic structural size, the above equation defines the failure stress $_N$ governed by conventional strength theory for <<1 and by the Linear Elastic Fracture Mechanics (LEFM) theory for >> 1 in the asymptotic limit. Normally for plain and reinforced concrete structures, the structural failure mode is governed in between the two limits due to the development of large Fracture Process Zone (FPZ) around cracks and both the theories are unconservative and hence the non-linear fracture mechanics treatment of the problem is essential.

During the experimental study the identification of the maximum load and the resulting displacement is required for studying the influence of the structural size effect. In addition, the detail investigation often is needed in the FPZ, which is obtained with the conventional optical techniques, the acoustic emission and the modern image processing techniques, all these methods help in validating the various size effect laws.

Acoustic Emission Technique

Acoustic emission (AE) method is one of the most widely used techniques to experimentally evaluate deformation, fracture and damage parameters in engineering practice. Stress waves generated by dynamic processes in materials are known as acoustic emission. The classical sources of AE are defect-related deformation processes such as crack growth and plastic deformation. AE is caused due to localized and rapid release of strain energy in a stressed material. AE energy causes stress waves to emanate which travels through the structures. These stress waves detected on the structure surface can be analyzed to deduce the magnitude and nature of damage present in the materials. AE is one of the oldest methods used in civil engineering practice and is well suited for monitoring fracture process as reported by Miller et. al., Perter et. al. and Ohtsu et. al. [3-5]. Among the various parameters, the most significant one is the b-value which is derived from the amplitude distribution data of AE following the methods use in seismology by Gutenberg and Richter [6] and Smith [7]. The b-value is defined as the 'log-linear slope of the frequency-magnitude distribution' of AE. It represents the scaling of magnitude distribution of AE and is measure of the relative numbers of small and large AE signals, which are signatures of localized failures in materials under stress as shown by Rao and Lakshmi [8], Colombo et. al. [9] and Shiotani et. al. [10]. The method used for the determination of b-value is important and the calculations are statistical dependent and improved b-values based on mean and standard deviations on brittle fracture has been reported.

b-value analysis

The b-value is generally calculated from Gutenberg-Richter empirical formula quantifying the magnitude - frequency of the earthquake seismology events, which is defined as:

$$\mathrm{Log}_{10}\, N = a - b\, M_L \qquad (2)$$

Where M_L is the Richter magnitude of the events, N is the incremental frequency, a and b are empirical constants. From equation (2) the b-value is the negative slope of the log-linear plot of earthquake frequency-magnitude and hence equation (2) represents the slope of the amplitude distribution. In term of AE technique, the Gutenberg-Richter formula is modified as

$$\mathrm{Log}_{10}\, N = a - b'\, A_{db} \qquad (3)$$

Where A_{db} = peak –amplitude of the AE events in decibels. b'-value obtained with AE technique should be multiplied by a factor of 20 to be comparable with one used in seismology. The b-value changes systematically with the different stages of fracture growth [2], so it can be used to estimate the development of fracture process.

4 The Present Experimental Programme

A detailed experimental study on plain and reinforced concrete Three Point Bend (TBP) specimens with different sizes as shown in Table 1 has been undertaken to address the influence of specimen size on the fracture energy of the concrete, wherein the quantitative analysis of fracture development using b-value for plain and reinforced concrete behaviour has been studied. Further in this investigation an attempt is made to obtain the influence of damage on *b-value* in plain concrete beam and reinforced concrete beams. Figure 1(a) shows the typical locations of sensors planned to be used for this study on TPB specimens with different sizes of plain and RCC specimens. The experimental setup presented in Fig 1(b) consists of loading frame with data acquisition system and an acoustic emission monitoring system, in which now image processing system has been augmented for displacement and strain characterization for notched / un-notched beams. Material testing system of capacity 1200 kN has been used as loading frame with data acquisition. The data acquisition records load, CMOD, midspan displacement, and time. The AE sensor diameter is 19mm and its height is 22.0mm and works in the temperature range of -65^0C to 177^0C. The AE sensor has peak sensitivity at 75 dB with reference 1V/(m/s) [1V/mbar]. The operating frequency is 35kHz – 100kHz. The appropriate coupling between the sensor face and the concrete test specimen surface is achieved by vacuum grease LR (high vacuum silicon grease) couplant in the present experimental study. The two sizes of specimens have been tested under CMOD control at the rate of 0.0004mm/sec using a displacement controlled loading frame. The midspan downward displacement is measured using Linear Variable Displacement Transducer (LVDT) placed at center of the specimen under bottom of the beam. A clip gauge is installed to measure the CMOD. AE signals over 40 dB is recorded as AE waves with classical AE parameters. The fracture energy of the two basic sizes of the beam specimens has been evaluated and b-value analysis methodology is employed to identify the fracture process zone, the micro and macro fractures and the peak load before the onset of softening. This measurement is to some extent qualitative and hence for evolving the parameters of the structural size effect law Eq (1), the high resolution image processing system backed up with strain measurement is planned to be used.

5 FRACTURE ENERGY EVALUATION BY FEM

This section on the numerical fracture energy evaluation for concrete beam specimens is based on the recently reported results by Singh et. al.[12]. Plain concrete though earlier regarded as a brittle material in the Griffith sense, with a sharp step drop in stress near the tensile strength, has shown residual load-carrying capacity after the crack initiation. The application of fracture mechanics to quasi-brittle plain and reinforced concrete has opened up a new field of research for modelling the cracking phenomena treated empirically in the past. Cohesive crack model proposed by Hillerborg [13] and crack band model by Bazant et. al. [14] with localization limiters are frequently used to study the tensile failure of concrete. Bazant et al. [15] recommended the use of the stress intensity factor or the critical strain energy rate as a cracking criterion. An alternative model developed by Bazant et al. [14] is based on the strain softening characteristics as a cracking parameter. The strain softening parameter is adjusted with respect to the element size, so that the total energy under the softening curve represents the fracture energy. Ouyang et. al. [16] used an R-curve approach to evaluate the fracture energy required for crack propagation for concrete treated as a quasi-brittle material. Kotsovos et al. [17] investigated the causes of size effects in structural-concrete members. Rabczuk, et al. [18] describes a two-dimensional approach to model fracture of reinforced concrete structures under static loading conditions. Carpinteri et. al. [19] proposed a theoretical model based on fracture mechanics concepts to analyze the mechanical damage of plain, reinforced and pre-stressed concrete beams. Only a very few publications can be cited on the simulation of crack in reinforced concrete using the fictitious crack model. Ouyong and Shah [20] proposed to estimate the fracture energy for reinforced concrete, where in the strain energy, debonding energy and sliding energy on the debonded interface of steel bars and concrete dissipated during cracking are included. The nonlinear fracture mechanics formulation accounts for the presence of fracture zone with a rising fracture resistance curve (R-curve).

A conventional constitutive model, based on the strength/strain theories with smeared crack analogy for cracking in tension zones and a plasticity algorithm for crushing in compression zones suffers from the mesh sensitivity. The fracture energy models for plain concrete with the fictitious crack model and the crack band model have been widely used for studying the fracture behaviour of concrete structures. In fictitious or Cohesive Crack Model (FCM), the fracture zone is replaced by the cohesive forces acting normal to both the crack surfaces. These surfaces are not fully cracked and enable the transfer of the tensile stress. The intensities of the cohesive forces are dependent on the crack opening displacement near the cracked surfaces, while outside the fracture zone behaves in linear elastic manner. The Crack Band Model (CBM) is the special case of the cohesive crack model. The constitutive relation with strain softening is associated with a reference width h_c of the crack band, which is treated as a material property in CBM. The crack band model can effectively handle the problem of mesh size sensitivity, provided it is localized within one element.

The concrete damage plasticity model uses concepts of isotropic damage elasticity in combination with isotropic tensile and compressive plasticity to represent the inelastic behavior of concrete. The advantage of this model over smeared crack model is that it can be used for concrete structure subjected to monotonic as well as cyclic loading under the low confining pressures. It consists of the combination of non-associated multi-hardening plasticity and scalar (isotropic) damage elasticity models to describe the irreversible damage that occurs during the fracturing process, which allows to control the stiffness recovery effects during cyclic load reversals. Further investigations are needed to include the dynamic fracture inertial effects using a combination of fracture and damage mechanics models for explaining the dynamic load induced failure in concrete and other quasi brittle materials.

Before the initiation of the present experimental programme, the strength/strain and fracture mechanics based various numerical models have been employed for the finite element analysis of a concrete notched beam specimen reported in Peterson's experiment [21] by Singh et al [12]. Further, an effort is made to simulate the experimental routine test based on recommendation of RILEM [22] for the determination of fracture energy of concrete beam specimen. The cohesive crack and crack band models, in addition to the conventional strength/strain based models are used for the numerical simulation of the above reported experimental test results. The mesh insensitive numerical results based on fracture energy models are evolved for inelastic analysis. The Peterson [21] notched beam example (Fig 2) subjected to three-point bending with the material properties shown in Table 3 is simulated by 8-noded plane stress elements. The typical beam geometry has depth (h) of 100.0 mm, the notch depth (a) of 10.0 mm and the width of the beam (b) is 150.0 mm with beam length (l) of 600.0 mm. The load and displacement control models are used for obtaining the complete failure curve (Fig. 3). The post softening curves shows the expected nonlinear behaviour of concrete with the fracture energy crack band model. As per RILEM [22] recommendation the fracture energy (G_f) can be calculated as

$$G_f = \frac{A_1 + mg\delta_0}{b(h-a)} \tag{4}$$

From Fig. 3, the area under load-deflection curve A_1 can be calculated using numerical integration. Here A_1 = 2.017 Nm; b = 150.0 mm; h = 100.0 mm; a = 10.0 mm; δ_0 = 0.325 mm; mg = 190.706 N and with all the above value in Eq. (4), fracture energy (G_f) = 154.0 N/m. Bazant et al.[14], introduced the crack band theory in the analysis of a plain concrete panel, which is the one of the simplest types of fictitious cracks models. The final equation derived for determining the strain at which tensile stress is zero ($\varepsilon_t^{\,f}$) can be expressed as,

$$\varepsilon_t^{\,f} = \frac{2G_f}{b\sigma_t} \tag{5}$$

$$G_f = (2.72 + 0.0214\sigma_t)\frac{\sigma_t^{\,2} D_a}{E_c} \quad (\sigma_t \text{ and } E_c \text{ are in psi, } D_a \text{ is in inch}) \tag{6}$$

The Eq. (5) can be successfully applied when the finite element mesh size is equal to or smaller than 75 mm. The value of fracture energy obtained using above Eq. (6) can be taken as the first good estimate. The actual value of the fracture energy may be slightly different because the above expression is the result of statistical analysis. For the present problem, the above equation gives a G_f of 147 N/m.

For the analysis of reinforced concrete structures, tension stiffening of concrete is taken into account due to the influence of reinforcement at the vicinity of cracks. The numerical examples presented are based on the three-point bending tested experimentally by Bosco et. al. [23]. The various values of G_f in the range of 90.0 to 150.0 N/m were employed in the present analysis. It was found that higher value of G_f gave results close to those obtained in the experiment by Bosco et al.[23]. Fig. 4 shows that load-deflection response for the value of G_f = 110.0 N/m, which seems to be in good agreement with the experimental results. The reinforcement area of 12.7 mm^2 was taken for the above analysis.

The superiority of fracture energy model in comparison with the conventional strength/strain based model is further investigated. Fig. 5-7 present the load-deflection curves obtained with strain based model and the two fracture energy models; namely the cohesive crack model and the crack band model respectively. In Fig. 6, strain based approach shows a sharp peak but it underestimates the ultimate peak load and post-softening response shows oscillations. The crack band fracture energy model traces the peak load (Fig. 7) in true manner in agreement with the experimental value of 10.3 KN and gives the true presentation of softening curve as observed in the reported experiment. In addition, the convergence for the fracture energy model with mesh refinement is monotonic as noticed in the load-deflection curve in case of fracture energy crack band model compared to the conventional strength/strain based approach. The cohesive crack fracture model computes the peak load in agreement with the experimental results but the post-softening behavior is inferior compared to the crack band model and minor mesh sensitivity is noticed for refined grids with 178, 196 and 214 elements. Fig. 8 presents the analytical results using elasticity model, stress-strain based plasticity model, cohesive crack fracture energy model and crack band fracture energy model. In this figure, the stress-strain based plasticity approach shows a sharp peak load of 7.95 KN and it underestimates the maximum load and post-softening response shows oscillations. The crack band fracture energy model traces the peak load of 10.04 kN accurately in agreement with the experimental value of 10.05 KN and gives the true representation of softening curve in agreement with the reported experiment. All the results are summarized in Table 4 for this example.

CONCLUSIONS

The present finite element analysis shows good agreement with the experimental results for the beam specimen fracture experiments. A numerical model (crack band model with localization limiters) was used in the present study for the estimation of fracture energy including the effect of body force. The fracture energy of concrete is most sensitive to the presence of reinforcement in the reinforced concrete structures. It was noted that with an increase in the area of reinforcement, the fracture energy value improves. Although all the numerical models show identical response before the softening with regard to the experimental results but in the post softening region, the responses differ. The study illustrated that both the fracture energy models trace the ultimate load carrying capacity in very good agreement with the experiment compared to the conventional strength/strain based numerical model, which shows lower peak load. The crack band model gives mesh insensitive results and convergence is achieved in a monotonic manner. Our elaborate experimental programme is directed towards evaluation the structural size effect law with supportive conventional instrumentation, AE b-value analysis and high resolution image processing measurements, which will be validated with the presently evaluated numerical fracture energy finite element models.

Table 2 Material properties of concrete beam

Material Properties of concrete	
Concrete grade	2
Comp. Strength, f_{ck} (MPa)	75.7
Young's Modulus, E_c(MPa)	34300.0
Fracture Energy, G_f (N/m)	90.0
Tensile Strength, f_t (MPa)	5.3

Table 3 Fracture energy for plain concrete

Loading condition	Fracture Energy (N/m)
Central point loading	154.0
Loading-unloading	**136.0**
Loading with body force	158.0
Experimental	90.0-140.0

Table 4 Comparison of results for concrete beam

	Crack Band Model	Cohesive Crack Model	Strength Based Model	EXPERIMENT
Ultimate load (KN)	10.04	11.51	7.95	10.05
Deflection at ultimate load (mm)	0.183	0.187	0.163	0.185

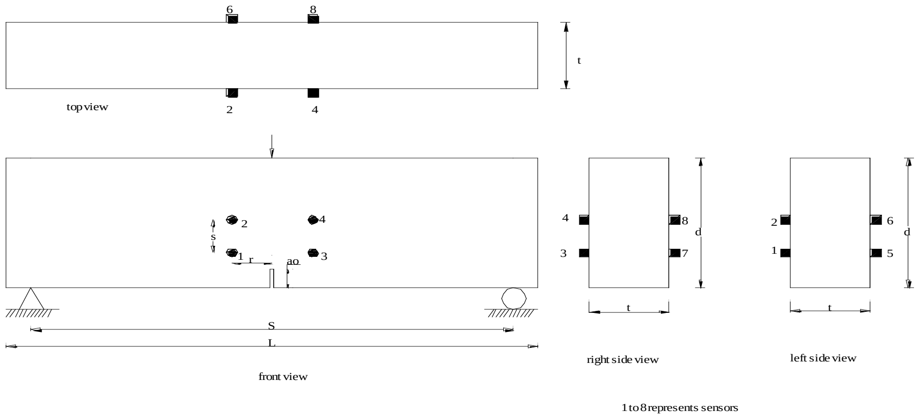


Fig 1 (a) Schematic of the Three Point Bend (TBP) Experiment with AE Sensors

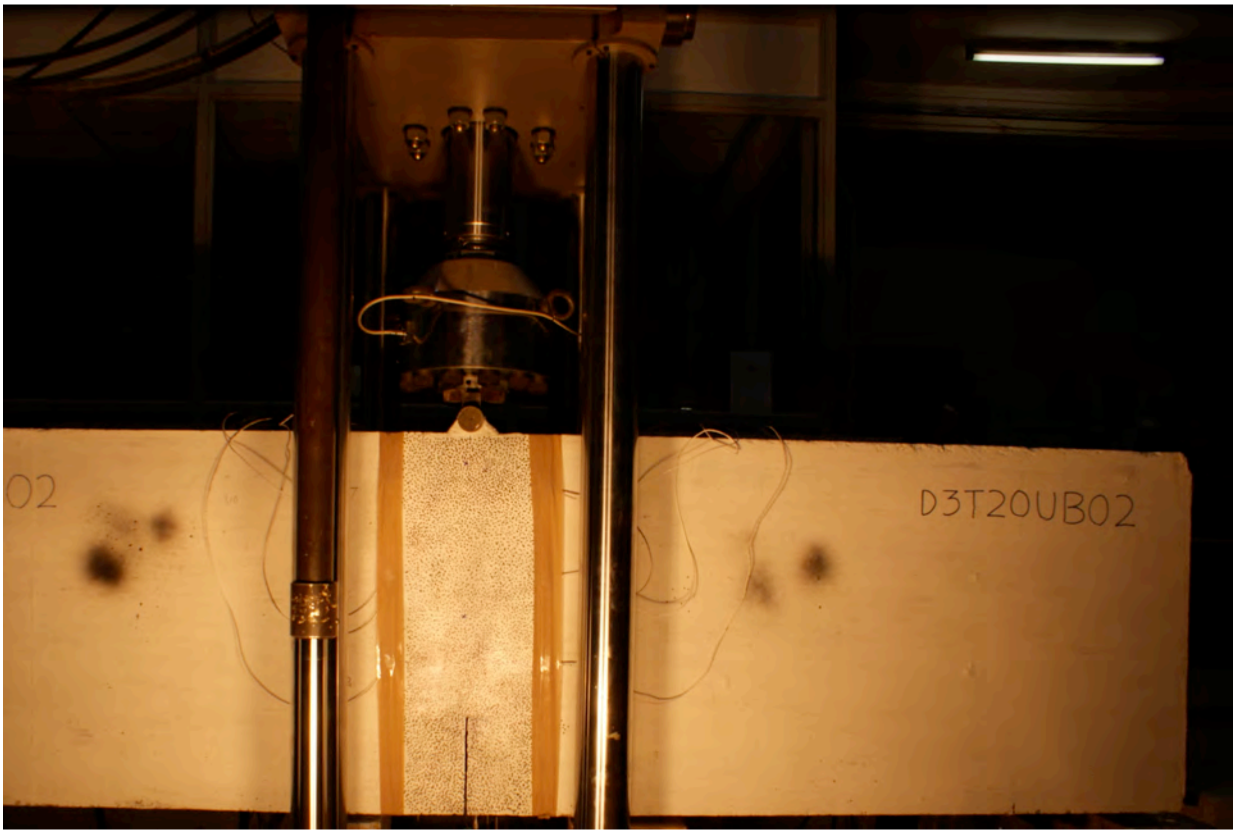


Fig 1 (b) Experimental Set-up with Image Processing System

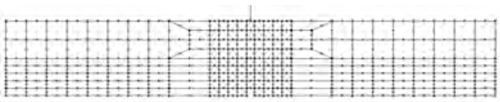


Fig. 2 FE Model of three points bend notched beam

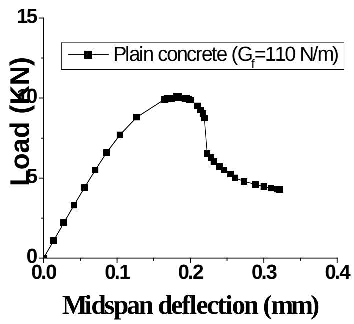


Fig. 3 - Load-deflection curve of notched beam with crack band model

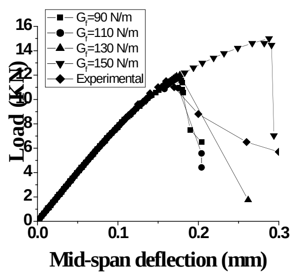
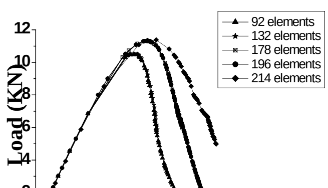
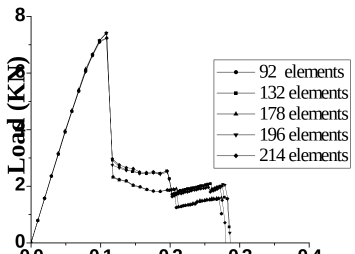


Fig. 4 Load-deflection curve for Different value of fracture energy





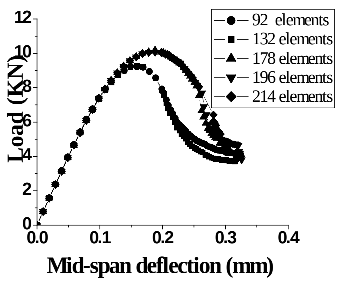


Fig. 7 Load deflection curve showing mesh insensitivity with crack band model

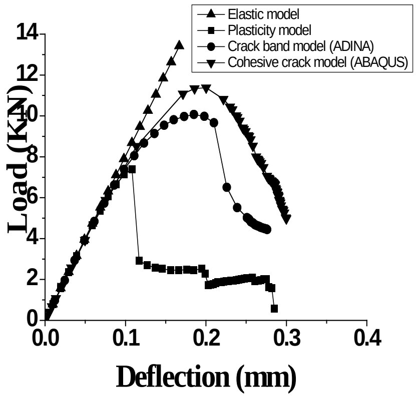


Fig. 8 Comparison of load-deflection curve with strain based and fracture energy models

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