Stochastic Finite Element Analysis of Uncertain Intrinsic Stresses Caused by Structural Misfits

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Abstract

The stresses caused by the external loads can be analyzed successfully by means of modern computational aids such as finite element method. The existence of uncertain intrinsic stresses, however, cannot be ignored in the process of elaborating the stress evaluation under the surveillance regarding structural safety and reliability. One of the well-known intrinsic stress is the residual stress generated locally along a weldment which has been investigated analytically or experimentally. On the other hand, suppose structural members with uncertain initial imperfections are assembled into an integrated form. When the misfits are cancelled in the assemblage, uncertain initial stresses whould be formed in a global sense. Owing to the difficulty to assess uncertain misfits, the latter case has not been analyzed systematically.

The present paper proposes a methodology to deal with the uncertain initial stresses of this sort. The formulation is based on the stochastic finite element method the authors have been developing. Uncertain initial imperfections are expressed by taking the representative parameters of the member as probabilistic. The element stiffness matrix is expanded in terms of those probability variables, and so are the initial nodal forces which reform the above imperfections. The members are assembled after the reformation, in which the context of conventional finite element method for initial strains is utilized. On the other hand, it is assumed that the unknown displacements are expanded with respect to the above probability variables. On the line of the second order perturbation method, the unknown coefficients of the displacements are determined. It is emphasized that the global stiffness equation is solved only once in this procedure. The expectation and variance of initial stress are subsequently evaluated for a given covariance matrix of the probability variables representing the uncertain imperfections. A numerical example is successfully given in the case of curved beams with uncertain radius and central angle integrated into a closed ring.

A use of the present stochastic finite element method in the field of structural safety and reliability is discussed especially from the aspect of reliability indeces. In the discussion, the present method is interpreted as a useful tool to estimate a quadratic-type failure surface as well as that to output the said statistics. It is found that the probability of failure is conveniently evaluated in the case of Gaussian random variables based on the recent development of quadratic limit state. The evaluation of 'generalized reliability index' is exemplified in the above case of a closed ring.

1. Introduction

Modern computational vehicles have enabled us to analyze various structural problems in good accuracy, while this fact does not necessarily guarantee the structural safety and reliability. This is because numerous uncertainties are involved in actual structural systems, and some of which are sensitive to the results. One of them which should be borne in mind in assembled structures is the misfits caused by geometrical uncertainties of members and the resultant initial stress. Suppose structural members are assembled into an integrated form. Although the members have misfits more or less in actual, the assemblage must be completed so that the misfits are cancelled in the structure. This causes initial stress distribution which is no longer local. Owing to the difficulty to assess misfits, the intrinsic stresses of this kind have been left out of scope of usual stress analyses. The objective of the present paper is to apply the stochastic finite element method the authors have been developing [1] to the above problem and to discuss the role of the proposed technique in the field of structural safety and reliability.

2. Stochastic FEM in assemblage of members with initial imperfections

There seems to have been at least two approaches to solve the initial stresses caused by the assemblage of imperfect members. A simple example is given in Fig. 1(a), in which two columns with deviations α_1 and α_2 in length are put between the walls. The initial axial stress may be evaluated by imposing a displacement $\alpha_1 + \alpha_2$ on a column of length $2\ell + \alpha_1 + \alpha_2$ as shown in Fig. 1(b). Figure 1(c) depicts the other approach, where two columns of length ℓ and with different initial strains are assembled. The latter approach, which is similar to that given by Martin [2], is superior to the former for complex structures. In the following formulation use is made of the concept of Fig. 1(c) in conjunction with the second order perturbation technique which allows a nonlinear relation between the imperfection and initial stress.

Firstly it is assumed that an element stiffness matrix $[k]$ is expanded in terms of probability variables $\alpha_r (r=1,2,\ldots,m)$ which represent uncertain initial imperfections in the element.

$$[k] = [k^0] + \sum_{r=1}^{m} [k_r^1]\alpha_r + \frac{1}{2}\sum_{r=1}^{m}\sum_{s=1}^{m} [k_{rs}^2]\alpha_r\alpha_s \qquad (1)$$

α_r is defined so that the ensemble average, $E[\alpha_r]$ equals zero. Secondly it is assumed that the initial imperfections of the element at the nodes are reformed by giving a small displacement $\{\tilde{u}\}$ to the nodes. $\{\tilde{u}\}$ is calculated simply by the geometrical relation and is approximated as eq.(2). This provides the corresponding nodal force $\{\tilde{f}\}$ in the form of eq.(3).

$$\{\tilde{u}\} = \sum_r\{\tilde{u}_r^1\}\alpha_r + \frac{1}{2}\sum_r\sum_s\{\tilde{u}_{rs}^2\}\alpha_r\alpha_s \qquad (2)$$

$$\{\tilde{f}\} = [k]\{\tilde{u}\} = \sum_r [k^0]\{\tilde{u}_r^1\}\alpha_r + \sum_r\sum_s\left(\frac{1}{2}[k^0]\{\tilde{u}_{rs}^2\} + [k_r^1]\{\tilde{u}_s^1\}\right)\alpha_r\alpha_s \equiv \sum_r\{\tilde{f}_r^1\}\alpha_r + \frac{1}{2}\sum_r\sum_s\{\tilde{f}_{rs}^2\}\alpha_r\alpha_s \qquad (3)$$

The reformation generates the initial strain and $\{\tilde{f}\}$ is nothing but the nodal force which restricts this initial strain. Therefore the element stiffness equation in the assemblage of initially imperfect members is given as follows according to the context of conventional FEM for initial strains [3].

$$[k]\{u\} + \{\tilde{f}\} = \{f\} \qquad (4)$$

Merging eq.(4) after proper transformation of co-ordinates whose matrix is given in the form of eq.(5), we have the global stiffness equation as eqs. (6) to (8).

$$[T] = [T^0] + \sum_r [T_r^1]\alpha_r + \frac{1}{2}\sum_r\sum_s [T_{rs}^2]\alpha_r\alpha_s \quad (5) \qquad [K]\{U\} + \{\tilde{F}\} = \{F\} \quad (6)$$

$$[K] = [K^0] + \sum_{k=1}^{n}[K_k^1]\alpha_k + \frac{1}{2}\sum_{k=1}^{n}\sum_{l=1}^{n}[K_{kl}^2]\alpha_k\alpha_l \quad (7) \qquad \{\tilde{F}\} = \sum_{k=1}^{n}\{\tilde{F}_k^1\}\alpha_k + \frac{1}{2}\sum_{k=1}^{n}\sum_{l=1}^{n}\{\tilde{F}_{kl}^2\}\alpha_k\alpha_l \quad (8)$$

Suffices k and l vary from one to n, the total number of probability variables involved in the structure. We re-define at this stage the mechanical boundary condition, corresponding unknown displacements, initial nodal forces and global stiffness matrix by $\{F\}$, $\{U\}$, $\{\tilde{F}\}$ and $[K]$ so that eq.(6) represents the reduced global stiffness equation. Substituting eqs.(7), (8) and the following (9) assumed for $\{u\}$ into eq.(6), we have eqs.(10) to (12) based on the principle of the second order perturbation method.

$$\{U\} = \{U^0\} + \sum_k\{U_k^1\}\alpha_k + \frac{1}{2}\sum_k\sum_l\{U_{kl}^2\}\alpha_k\alpha_l \quad (9) \qquad \{U_k^1\} = -[K^0]^{-1}(\{\tilde{F}_k^1\} + [K_k^1]\{U^0\}) \quad (11)$$

$$\{U^0\} = [K^0]^{-1}\{F\} \quad (10) \qquad \{U_{kl}^2\} = -[K^0]^{-1}(\{\tilde{F}_{kl}^2\} + [K_k^1]\{U_l^1\} + [K_l^1]\{U_k^1\} + [K_{kl}^2]\{U^0\}) \quad (12)$$

Equation (12) is obtained under the condition; $[K_{kl}^2] = [K_{lk}^2]$, $\{U_{kl}^2\} = \{U_{lk}^2\}$ and $\{\tilde{F}_{kl}^2\} = \{\tilde{F}_{lk}^2\}$. The unknown vectors $\{U^0\}$, $\{U_k^1\}$ and $\{U_{kl}^2\}$ can be evaluated based on the single computation of $[K^0]^{-1}$. Finally, a stress is evaluated in the form of eq.(14) through eq.(13) which is obtained by the substitution of $\{u\}$ of the part of eq.(9) into eq.(4).

$$\{f\} = \{f^0\} + \sum_k\{f_k^1\}\alpha_k + \frac{1}{2}\sum_k\sum_l\{f_{kl}^2\}\alpha_k\alpha_l \quad (13) \qquad \sigma = \sigma^0 + \sum_k\sigma_k^1\alpha_k + \frac{1}{2}\sum_k\sum_l\sigma_{kl}^2\alpha_k\alpha_l \quad (14)$$

The expectation and variance of a stress are subsequently given by eqs.(15) and (16) based on the second order approximation method and by eqs.(15-a) and (16-a) based on the first order one.

$$E[\sigma] = \sigma^0 + \frac{1}{2}\sum_k\sum_l\sigma_{kl}^2 E[\alpha_k\alpha_l] \quad (15)$$

$$Var[\sigma] = \sum_k\sum_l\sigma_k^1\sigma_l^1 E[\alpha_k\alpha_l] + \sum_k\sum_l\sum_p\sigma_k^1\sigma_{lp}^2 E[\alpha_k\alpha_l\alpha_p] + \frac{1}{4}\sum_k\sum_l\sum_p\sum_q\{\sigma_{kl}^2\sigma_{pq}^2(E[\alpha_k\alpha_l\alpha_p\alpha_q] - E[\alpha_k\alpha_l]E[\alpha_p\alpha_q])\} \quad (16)$$

$$E[\sigma] = \sigma^0 \quad (15\text{-a}) \qquad Var[\sigma] = \sum_k\sum_l\sigma_k^1\sigma_l^1 E[\alpha_k\alpha_l] \quad (16\text{-a})$$

3. Analysis of uncertain initial stress in assemblage of curved beams into ring

3.1 Stochastic stiffness matrix

According to the notation in Fig. 2, the stiffness matrix $[k]$ of a curved beam element with redius R, central angle β and flexural rigidity EI is given as follows provided the thickness t and height h are much smaller than R [2].

$$[k] = \begin{bmatrix} \mathbb{k}_{11} & \mathbb{k}_{12} \\ \mathbb{k}_{21} & \mathbb{k}_{22} \end{bmatrix} \quad (17)$$

$$\begin{array}{cccc} & u_1 & v_1 & \theta_1 \\ \mathbb{k}_{11} = \dfrac{EI}{R^3 G} & \left[\begin{array}{ccc} A & B & CR/\beta \\ & D & ER/\beta \\ SYM. & & FR^2/\beta \end{array}\right] & & \begin{array}{c} p_1 \\ q_1 \\ m_1 \end{array} \end{array} \quad (17\text{-a})$$

$$\mathbb{k}_{12}^T = \mathbb{k}_{21} = \mathbb{H}\cdot\mathbb{k}_{11} \quad (17\text{-c})$$

$$\mathbb{H} = \begin{bmatrix} -\cos\beta & -\sin\beta & 0 \\ \sin\beta & -\cos\beta & 0 \\ R(\cos\beta - 1) & R\sin\beta & -1 \end{bmatrix} \quad (18)$$

$\mathbb{k}_{22} = \{\mathbb{k}_{11}$ in which B and E are replaced by $-B$ and $-E\}$ (17-b)

$$\left.\begin{array}{l} A = e^2/\beta - d, \quad B = b - ae/\beta, \quad C = ad - be, \quad D = a^2/\beta - c, \quad E = ce - ab \\ F = b^2 - cd, \quad G = b(b - 2ae/\beta) + c(e^2/\beta - d) + ad^2/\beta = bB + cA + aC/\beta \end{array}\right\} \quad (19)$$

$$\left.\begin{array}{l} a = \beta - \sin\beta, \quad b = \cos\beta + 1/2\cdot\sin^2\beta - 1, \quad c = 3/2\cdot\beta - 2\sin\beta + 1/4\cdot\sin 2\beta \\ d = 1/2\cdot\beta - 1/4\cdot\sin 2\beta, \quad e = \cos\beta - 1 \end{array}\right\} \quad (20)$$

We deal with the imperfections of radius R and central angle β as defined in eq.(21), and expand the above equations with respect to ε and μ in order to evaluate $[k^0]$, $[k_r^1]$ and $[k_{rs}^2]$ appearing in eq.(1). ε is taken as equal to α_1 and μ to α_2.

$$R = \bar{R}(1 + \varepsilon) \quad , \quad \beta = \bar{\beta}(1 + \mu) \qquad (21)$$

3.2 Stochastic initial nodal force

In Fig. 2 a curved beam with initial imperfection is denoted by *1-2* and an as-designed one without imperfection by *1'-2'*. Following the context of the preceding chapter, nodes *1* and *2* are superposed on *1'* and *2'* respectively in the sense of tangent as well as co-ordinate. This reformation is equivalent to the displacements given in the followings.

$$u_1 = (R - \bar{R})\sin\beta/2 + \bar{R}\sin(\beta - \bar{\beta})/2 \,, \qquad v_1 = R - (R - \bar{R})\cos\beta/2 - \bar{R}\cos(\beta - \bar{\beta})/2$$

$$\theta_1 = (\beta - \bar{\beta})/2 \,, \qquad u_2 = -u_1, \quad v_2 = v_1, \quad \theta_2 = -\theta_1 \qquad (22)$$

Expanding eq.(22) in terms of $\varepsilon = \alpha_1$ and $\mu = \alpha_2$, we have $\{\tilde{u}\}$ in the form of eq.(2), and then, $\{\tilde{f}\}$ in the form of eq.(3). All the expansions are truncated properly to fit with the second order perturbation method.

3.3 Transformation matrix of co-ordinates and evaluation of stress

Curved beams are assembled into a closed ring, and nodes and elements are numbered up to eight in the example as shown in Fig. 3. It should be noted that the transformation matrix of co-ordinates, from local to global, must be formulated upon the proper expansion as regards ε and μ.

To evaluate the stress, eq.(13) is calculated firstly via eq.(4) to which $\{u\}$ on the basis of $\{U\}$ and $[T]$ is applied. Then substituting m and p, which are the components of $\{f\}$, into eqs.(23) and (24), we have the bending stress σ_b on the inner side of the ring and the hoop stress σ_h in the form of eq.(14).

$$\sigma_b = -6m\eta/th^2 \qquad (23) \qquad \text{with} \qquad \eta = (1 - h/6R)/(1 - h/2R) = (1 - h/6\bar{R})/(1 - h/2\bar{R})$$

$$- (h/3\bar{R})/(1 - h/2\bar{R})^2 \cdot \varepsilon - (h/3\bar{R})/(1 - h/2\bar{R})^3 \cdot \varepsilon^2 \qquad (23\text{-a}) \qquad\qquad \sigma_h = p/th \qquad (24)$$

3.4 Numerical example

A numerical example is given in the case of eight elements with rectangular section. Node *1* is fixed and the external force $\{F\}$ is taken as $\{0\}$. The second moments of ε and/or μ are assumed to have the form of

$$Cov[\varepsilon_i, \varepsilon_j] = Cov[\mu_i, \mu_j] = x^2(1 - 2d|j-i|/7), \quad Cov[\varepsilon_i, \mu_j] = yx^2(1 - 2d|j-i|/7) \qquad (25)$$

where i and j denote element number. When the variance of stress is evaluated based on the second order approximation, the third and fourth moments of ε and/or μ need to be given as seen in eq.(16). The following relations, which are known in the case of Gaussian random variables, are assumed herein.

$$E[\alpha_k\alpha_l\alpha_p] = 0 \,, \qquad E[\alpha_k\alpha_l\alpha_p\alpha_q] = E[\alpha_k\alpha_l]E[\alpha_p\alpha_q] + E[\alpha_k\alpha_p]E[\alpha_l\alpha_q] + E[\alpha_k\alpha_q]E[\alpha_l\alpha_p] \qquad (26)$$

Figure 5 shows a typical distirubion of the resultant standard deviation of σ_b along the ring, in which σ_b is normalized by $\xi\{(1 - \xi/6)/(1 - \xi/2)\}E/2$. ξ denotes the ratio of the height of the beam to the expected radius $h/\bar{R}$, admissible range of which may be $0 < \xi < 0.2$ when the accuracies of eqs.(17) to (20) and eq.(23-a) are taken into account. Through the computations, it is found that the hoop stress σ_h is much smaller than the bending stress σ_b.

4. A use of stochastic FEM in structural safety and reliability

As stated in the preceding chapter, stochastic structural systems are well analyzed by the present method, that is to say, the expectation $E[Y]$ and variance $Var[Y]$ of the structural behaviour Y are successfully evaluated. These statistics should be made much of from engineers' intuitive point of view. The well known principle of reliability index [4] may be adopted as its interpretation. It is desirable, however, that the solution of the present method in the form of

$$Y = Y^0 + \sum_k Y_k^1 \alpha_k + \sum_{kl}\sum Y_{kl}^2 \alpha_k \alpha_l \equiv Y^0 + \lfloor Y^1 \rfloor \{\alpha\} + \lfloor \alpha \rfloor [Y^2]\{\alpha\} \qquad (27)$$

such as eqs.(9), (13) and (14) serves something else besides to evaluate the said statistics. This will be made feasible because eq.(27) takes the guadratic form and is expected to be accurate for the wide range of probability variables. An idea is presented on the basis of the recent development in structural reliability.

In general α_k's in eq.(27) are correlated each other and $Var[\alpha_k]$ is not equal to $Var[\alpha_l]$ when $k \neq l$. As is done in the advanced first order second moment method, it is convenient to transform $\{\alpha\}$ into $\{\alpha'\}$ so that $\{C_\alpha'\}$, the covariance matrix of $\{\alpha'\}$ is equal to the unit matrix $[I]$. The transformation is given as [5]

$$\{\alpha'\} = ([\Phi]diag[\sqrt{\lambda i}][\Phi]^T)^{-1}\{\alpha\} \equiv [A]^{-1}\{\alpha\} \qquad (28)$$

where $[\Phi]$ is the modal matrix of $[C_\alpha]$ as the covariance matrix of $\{\alpha\}$ and λ_i's are the corresponding eigenvalues, ie.

$$[C_\alpha][\Phi] = [\Phi]diag[\lambda_i] \qquad (29)$$

$[\Phi]$ is normalized so as to satisfy $[\Phi]^{-1} = [\Phi]^T$. Substitution of the inverse form of eq.(28) into eq.(27) leads to

$$Y = Y^0 + \lfloor Y^1 \rfloor [A]\{\alpha'\} + \lfloor \alpha' \rfloor [A]^T [Y^2][A]\{\alpha'\} \equiv Y^0 + \lfloor Y^{1'} \rfloor \{\alpha'\} + \lfloor \alpha' \rfloor [Y^{2'}]\{\alpha'\} \qquad (30)$$

It should be noted that $E[\{\alpha'\}] = \{0\}$ holds. As shown later, eq.(30) gives a good approximation for Y over the wide range of $\{\alpha\}(\{\alpha'\})$ even when fairly strong nonlinearity is present. Therefore, in many cases, eq.(30) may be applied to the limit state equation. In case the approximation is not good enough, the expansion point may be moved from the means towards the failure surface by a suitable algorithm or even by an engineering judgement. As mentioned in the above, the expansion point needs not to be put very close to the failure surface by virtue of the guadratic approximation used here. Therefore, in general, a limit state may be written in the form of

$$Y_c - Y^0 - \lfloor Y^{1'} \rfloor \{\alpha'\} - \lfloor \alpha' \rfloor [Y^{2'}]\{\alpha'\} = 0 \qquad (31)$$

At this stage, one can find that the recent development of guadratic limit state is conveniently utilized. According to the work done by Fiessler et al. [6] , $\{\alpha'\}$ is again transformed as eq.(32) so that eq.(31) is rewritten as follows, $\{\alpha''\} = [\psi]^{-1}\{\alpha'\}$ (32)

$$Y_c - Y^0 - \lfloor Y^{1'} \rfloor [\psi] \{\alpha''\} - \lfloor \alpha'' \rfloor [\psi]^T [Y^{2'}][\psi]\{\alpha''\} \equiv Y_c - Y^0 - \lfloor Y^{1''} \rfloor \{\alpha''\} - \lfloor \alpha'' \rfloor [Y^{2''}]\{\alpha''\} = 0 \qquad (33)$$

where $[\Psi]$ is the modal matrix of $[Y^{2'}]$ which is normalized to satisfy $[\Psi]^{-1} = [\Psi]^T$, and thus $[Y^{2''}]$ is equal to the diagonal matrix whose components are the eigenvalues of $[Y^{2'}]$. After some calculations it is found that eq.(33) is reduced to the followings [6].

$$(\lfloor \alpha'' \rfloor - \lfloor \delta \rfloor)[Y^{2''}](\{\alpha''\} - \{\delta\}) = 1/4 \lfloor Y^{1''} \rfloor [Y^{2''}]^{-1}\{Y^{1''}\} + Y_c - Y^0 \qquad (34)$$

with

$$\{\delta\} = -1/2[Y^{2''}]^{-1}\{Y^{1''}\} \quad (35) \qquad \text{or} \qquad \sum_{i=1}^{n} Y_{ii}^{2''}(\alpha_i'' - \delta_i)^2 = 1/4\lfloor Y^{1''} \rfloor [Y^{2''}]^{-1}\{Y^{1''}\} + Y_c - Y^0 \equiv K_1 \quad (34\text{-a})$$

Again $E[\{\alpha''\}]$ equals $\{0\}$ and it should be noted that $[C''_\alpha]$, the covariance matrix of $\{\alpha''\}$, still remains as unit matrix. In case $\{\alpha\}$ are believed to be Gaussian random variables, then so are the linearly transformed $\{\alpha'\}$ and $\{\alpha''\}$, and it is found [6] that the probability of failure P_f can be calculated based on eq.(34-a) with a linear combination of noncentral chi-squared distribution formula [7], ie.

$$\left.\begin{aligned} &P_f = P\left(\sum_{i=1}^{n} Y_{ii}^{2''}(\alpha_i'' - \delta_i)^2 \geqq K_1\right) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \frac{\sin\theta(u)}{u\rho(u)}du \\ &\text{where} \\ &\theta(u) = \frac{1}{2}\sum_{j=1}^{n}[\tan^{-1}(Y_{jj}^{2''}u) + \delta_j^2 Y_{jj}u(1 + (Y_{jj}^{2''}u)^2)^{-1}] - \frac{1}{2}K_1 u \\ &\rho(u) = \prod_{j=1}^{n}(1 + (Y_{jj}^{2''}u)^2)^{1/4} \times \exp\{\sum_{j=1}^{n}(\delta_j Y_{jj}^{2''}u)^2/2(1 + (Y_{jj}^{2''}u)^2)\} \end{aligned}\right\} \quad (36)$$

In case $\{\alpha\}$ are not subjected to Gaussian distributions, the notion of generalized reliability index proposed by Ditlevsen [5] may be employed as a useful criterion, ie.

$$\beta_E = -\Phi^{-1}(P_f) \qquad (37)$$

where $\Phi^{-1}(\cdot)$ is the inverse form of standard normal distribution function, to which P_f as evaluated in eq.(36) is substituted.

A numerical example is given in the case of two probability variables, α_1 and α_2. α_1 represents ε_7 as well as μ_7 of the seventh element in Fig. 3. That is to say, full correlation exists between ε_7 and μ_7. Also α_2 denotes $\varepsilon_8 = \mu_8$ of the eighth element. It is assumed there are no other imperfections in the ring. The normalized bending moment $\hat{m} = m\cdot\bar{R}/EI$ at node 7 is evaluated for various combinations of α_1 and α_2 through the conventional (deterministic) FEM. The broken lines in Fig. 5 are the estimated contour lines of $\hat{m}$ in $\alpha_1 - \alpha_2$ plane. The solid lines of this figure denoted 'SOP' show the variation of the bending moment along the indicated sections as approximated by the present stochastic FEM, whose form is

$$\hat{m} = -0.100\alpha_1 - 0.204\alpha_1^2 + 0.0318\alpha_2 - 0.221\alpha_2^2 - 0.0235\alpha_1\alpha_2 \qquad (38)$$

When compared with the solid lines denoted 'EXACT' as obtained by the conventional FEM, the superiority of the second order perturbation method to the first order one (shown by the solid lines denoted 'FOP') is obvious. At least in this case, the mean centered second order perturbation method seems good for the evaluation of probability of failure or the reliability index for a proper magnitude of α_1 and α_2.

Suppose the covariance matrix of $\{\alpha\}$ takes the following value.

$$[C_\alpha] = 0.01^2\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \qquad (39)$$

Then the following relation is derived from the aforementioned context.

$$P_f = P[0.0000110(\alpha_1'' - 30.81)^2 + 0.0000336(\alpha_2'' + 8.48)^2 \geqq 0.0129 + \hat{m}_c] \qquad (40)$$

$\hat{m}_c$ denotes the critical bending moment, to which 0.001, 0.002 and 0.003 may be substituted for instance. P_f's are easily calculated by virtue of eq.(36) as 0.141, 0.0169 and 0.000895 respectively. The corresponding β_E's are 1.07, 2.12 and 3.12. Even if many random variables are involved in this procedure, CPU time is expected to be short, because only a single fold integral appears herein.

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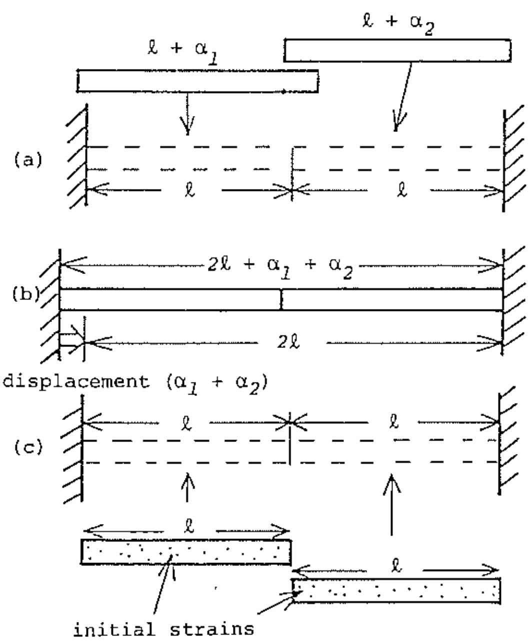


Fig. 1 Concept of analyses in assemblage of members with initial imperfection.

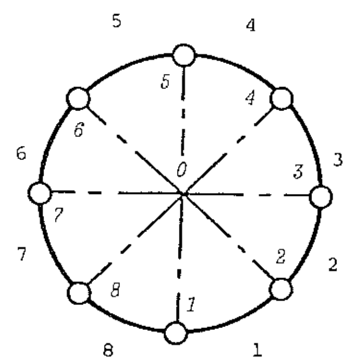


Fig. 3 Node and element numbers in closed ring model.

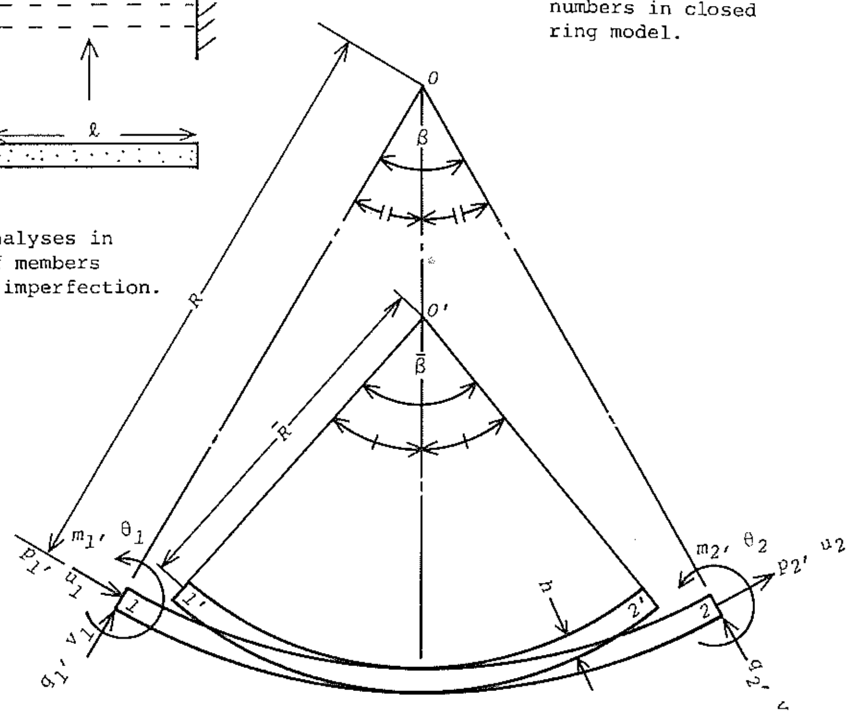
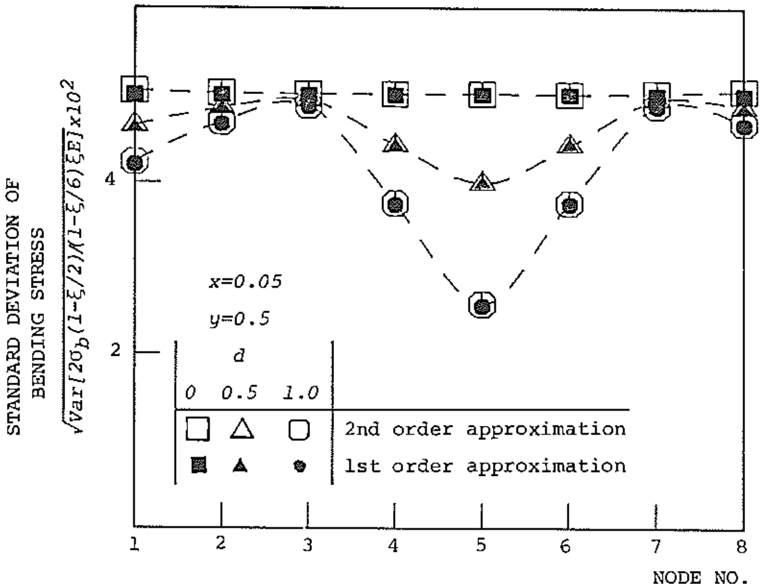


Fig. 2 Curved beam element with imperfect radius and central angle.



← Fig. 4 Standard deviation of initial bending stress at each node.

↓ Fig. 5 Distribution of normalized bending moment and its approximation by stochastic FEM.