Comparison of Methods for Seismic Risk Quantification

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INTRODUCTION

The focus of this paper is on the various methods that are currently in use for risk quantification in seismic Probabilistic Risk Assessment (PRA). In particular, algorithms for performing the Boolean operations are examined to determine their relative strengths and weaknesses. Ability to incorporate dependency between component failures is discussed and results of a sample test case are used to verify the equivalence of the approaches.

The seismic PRA methodology has been developed and applied to over twenty-five nuclear power plants. Briefly, a PRA requires the following three elements (PRA Procedures Guide, 1983):

(1) Seismic hazard - This relates to the seismicity of a particular region expressed as the annual frequency with which earthquakes equal to or greater than a given ground motion parameter (e.g. peak ground acceleration), occur at the site. To account for imperfect knowledge in estimating the various input parameters (e.g. seismic zonation, recurrence relation, maximum magnitude and ground motion model), the seismic hazard is represented by means of a family of subjectively weighted hazard curves.

(2) Component fragility evaluation - Component fragility expresses the seismic vulnerability of a structure or equipment as a probability of failure conditional on a ground motion parameter. A typical fragility function is represented by a family of fragility curves reflecting the uncertainty in estimating such quantities as material strength, seismic response etc. The component fragility is modeled in the form $A = A_m e_R e_U$, where A is the ground motion capacity, A_m is the median ground acceleration capacity, and e_R, e_U are random variables with unit median. They represent respectively, the inherent randomness about the median and the uncertainty in the median estimate. If it is assumed that both e_R and e_U are lognormally distributed with logarithmic standard deviations of β_R and β_U respectively, then the conditional frequency of failure f′ at any acceleration A and non-exceedance-probability level Q can be estimated by the following equation (PRA Procedures Guide, 1983):

$$f' = \Phi\left[\frac{\ln(A/A_m) + \beta_U\,\Phi^{-1}(Q)}{\beta_R}\right] \qquad (1)$$

where $\Phi^{-1}(.)$ is the standard Gaussian cumulative distribution function (CDF) and $Q = P[f < f' | _A]$

(3) Combination of components according to accident sequence - From event tree and fault tree analysis, Boolean expressions are developed that describe accident sequences in terms of basic component failures. Component fragilities are combined to yield for each sequence, a "plant level" fragility. Each plant level fragility is then convolved with the seismic hazard as in Equation 2, to give the seismic risk in terms of annual frequencies of failure.

$$f_S = \int_0^{\infty} - \frac{dH(A)}{dA} S(A)\, dA \tag{2}$$

where $-dH(A)/dA$ = the frequency with which earthquakes occur in the size range dA about A, and

S (A) = conditional probability of accident sequence

The uncertainty in the seismic hazard and the fragility functions is propagated through the convolution process to result in probability distributions on the frequency of seismically induced accident sequences (e.g. core melt, plant damage states and serious release).

NUMERICAL METHODS

The numerical schemes for risk quantification discussed herein fall into two broad, but by no means exclusive categories. The first category involves the discretization of analytical probability density functions (PDF) into discrete probability distributions (DPD) and is referred to as the DPD method. Figure 1 shows a discretization scheme where a continuous lognormal density function is approximated by a finite number of $\{<p_i, x_i>\}$ doublets. The second group, utilizing simulation techniques such as Latin Hypercube Sampling (LHS) and Monte Carlo Simulation (MCS), involves random sampling from a continuous PDF. The three methods (DPD, LHS, MCS) are described in the following sub-sections. For completeness, the existence of a fourth method should be mentioned. This is the Multiple Integration Method which formed the core of the systems analysis phase in the Seismic Safety Margins Research Program (Wells, et al 1981). This method does not strictly belong to either one or the other categories defined earlier; here probabilities of cut sets are represented by multinormal integrals and evaluated numerically using Gaussian quadrature.

The application example section used the large LOCA accident sequence, AE, taken from the Millstone 3 PRA (Ravindra et al 1984) together with site specific seismic hazard curves.

$$AE = (1 + 2 + 3 + 4 + 5 + 7 + 10 + R1) * (6 + 8 + 9) \tag{3}$$

where the numbers correspond to components listed in Table 1 and R1 corresponds to non-seismic failure of 2×10^{-4} per demand and with error factor of 6.0.

In this equation, "+" denotes probabilistic addition (or Union) and "*" denotes intersection between component failures.

Discrete Probability Distribution Method (DPD)

The DPD method applied to the "double lognormal" format in which component fragilities are cast, results in a family of fragility curves F_i as shown in

Figure 2. The steps are as follows. First, the PDF on the median capacity defined by A_m and β_U is discretized into a finite number of intervals as in Figure 1. A curve may be passed through the "mass centroid" X_i, of each interval to reflect the random variability defined by β_R associated with the median estimate. Each of the curves $F_i(a)$ is weighted; e.g. the first curve, $F_1(a)$, would have a subjective probability p_1, given by the area under the PDF within the first interval, and so forth. One may think of the family of curves as a set of doublets $\{<p_i,F_i>\}$, following the terminology of Kaplan and Lin (1987). An operation involving two components is then reduced to operation between two sets of doublets. For instance, an intersection operation between two independent sets, $x=\{<p_i,F_i>\}$ and $y=\{<q_j,G_j>\}$, yields a new set of doublet z defined by $z=\{<p_iq_j,F_iG_j>\}$. Rules of DPD arithmetic governs these Boolean combinations. At the end of one operation involving two independent components, the number of doublets in z is given by the product of the number of doublets in x and y. To prevent exponential escalation in storage requirements, a condensation procedure such as one proposed in Kaplan and Lin (1987) is performed after each operation.

The method can handle two extreme cases of dependency between component failures, i.e., either zero or full correlation in randomness and uncertainty.

Latin Hypercube Simulation (LHS)

The LHS method (Iman et al, 1980) differs from MCS in that a stratified sampling algorithm is used to efficiently span the probability space, thereby reducing the required number of trials. Stratified sampling of a lognormally distribution variable may be described in three steps:

1 Drawing stratified random uniform samples. The interval between 0 and 1 is discretized into N equal intervals where N is the number of simulations. A point is sampled at random within each interval. The N random uniform numbers are then permuted into random order to form a vector $\{X_i\}$, i=1,N.

2 Transform uniform samples to standard normal samples. The uniform random numbers are mapped onto a standard normal distribution by treating $\{X_i\}$ as standard Gaussian CDFs and computing $\Phi^{-1}(X_i)$. This step yields a vector of random standard normal variates $\{Y_i\}$, i=1,N.

3. Transform standard normal samples to lognormal samples. For a lognormal distribution defined by parameters A_m and β_U, this step is accomplished by the transformation $\{Z_i\} = \{A_m \exp(\beta_U Y_i)\}$.

At the end of step 3, one could construct a family of N fragility curves through the observation points $\{Z_i\}$ using the random variability parameter β_R. Each curve in the family would carry a subjective probability of 1/N. The above three steps are repeated for each of the M components appearing in a Boolean expression, resulting in vectors $\{Z1_i\}$, $\{Z2_i\}...\{ZM_i\}$. The Latin Hypercube Samples are then constructed by assembling the vectors into a (N x M) matrix. Each element of the matrix defines a weighted fragility curve and each row of the matrix represents one trial in the Latin Hypercube experiment.

LHS can handle partial correlation in uncertainty between components as discussed below. Correlation between M variables may be expressed in the form of a square, symmetric matrix [R] of size (M x M) with unity on the diagonal. If [R] is specified between non-normal samples, conversion to a correlation matrix $[R_0]$ between standard normal variables is required. Der Kiureghian and Liu (1986) describe procedures for accomplishing the conversion. $[R_0]$ is then decomposed into lower and upper triangular matrices via Cholesky's ($[R_0]=[L][L^T]$). The independent standard normal samples $\{Y1_i\},\{Y2_i\}$ ... $\{YM_i\}$, obtained in step 2 for the M variables, are assembled into a matrix [Y] of size NxM. Correlated standard normal samples are then obtained by the transformation $[Y^*]^T=[L][Y]^T$.

Monte Carlo Simulation (MCS)

The MCS method (Karimi, 1988) commences with selecting a random confidence level, i.e. Q in Equation (1), and evaluating the probability of failure at a given acceleration level. This is done at the same acceleration level for all seismic components appearing in an accident sequence. Non-seismic components are also sampled at random in the same manner from specified distribution on failure rate. Next, using the Boolean expression for the particular accident sequence, the plant level failure probability S(A) is computed. The trial is completed by selecting at random one of the seismic hazard curves and evaluating the integrand, $[H'(A)*S(A)]$, in Equation (2). By performing several such trials at the same acceleration level, a probability distribution on the integrand is obtained. The process is repeated by marching along the acceleration axis and storing selected percentiles of the distribution on the integrand. Finally, the probability distribution on the frequency of occurrence for the accident sequence is constructed by numerical integration of the stored percentiles.

In the above procedure, the plant-level fragility curves are not explicitly developed. An alternative is to terminate a trial after the sequence probability S(A) is calculated. Multiple trials at a given acceleration level lead to a distribution on sequence probability which may then be condensed into a DPD. The plant level fragility is constructed by marching along the acceleration axis. Convolution with the seismic hazard is performed later in a second stage, in the spirit of the DPD and LHS methods. This procedure was followed for the numerical example in Section 3.

Correlation in uncertainty is easily incorporated using MCS. For the case of full dependency in uncertainty, the randomly selected value for confidence level, Q is uniformly applied to all components during a trial. For partial correlation, the transformation described in Sub-section 2.2 for the LHS method is performed on the random standard normal variate (Q).

APPLICATION EXAMPLE AND CONCLUSIONS

Accident sequence frequencies estimated using the different approaches for the example large LOCA sequence are summarized in Table 2 and 3. Consistent results are predicted by all three methods. For postulated partial correlation in uncertainty between components, results by LHS and MCS are again comparable.

The numerical exercises presented herein are interim results of an on-going study to include correlation between components in seismic risk estimates. The preliminary conclusion is that where failures are partially correlated, LHS method appears to be better from the standpoint of versatility and computational efficiency. Areas needing further investigation include the treatment of partial correlation in randomness between components, assessment of correlation or lack thereof between seismic and non-seismic failures and its implication on the computational procedures. The more fundamental problem remains in the determination of correlation coefficients between components capacities for both uncertainty and randomness.

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TABLE 1
FRAGILITIES OF KEY STRUCTURES/COMPONENTS

COMPONENT #	DESCRIPTION	A_m(g)	β_R	β_U
1	Emergency diesel generator building	.88	.20	.46
2	Refueling water storage tank	.88	.30	.36
3	Emergency diesel generator anchorage	.91	.24	.43
4	Control building collapse	1.00	.24	.43
5	Service water pumphouse failure	1.30	.24	.49
6	Reactor coolant system piping (Large LOCA)	1.59	.48	.51
7	Engineered safety features building	1.70	.23	.43
8	Reactor vessel support	2.35	.48	.44
9	Reactor coolant pumps	2.68	.43	.47
10	Cable trays	2.70	.48	.42

TABLE 2
SUMMARY OF ACCIDENT SEQUENCE FREQUENCIES
(PER YEAR) WITH INDEPENDENT COMPONENTS

	DPD	LHS (1000)	MCS (1000)
Median	8.2E-8	7.0E-8	7.3E-8
Mean	7.1E-7	7.7E-7	7.5E-7
95% Conf.	3.3E-6	3.0E-6	2.9E-6

(Numbers in brackets indicate number of simulations)

TABLE 3
ACCIDENT SEQUENCE FREQUENCIES (PER YEAR) FOR PARTIAL CORRELATION IN UNCERTAINTY BETWEEN COMPONENTS

	LHS (1000)	MCS (1000)
Median	5.9E-8	5.9E-8
Mean	8.4E-7	9.2E-7
95% Conf.	3.8E-6	4.3E-6

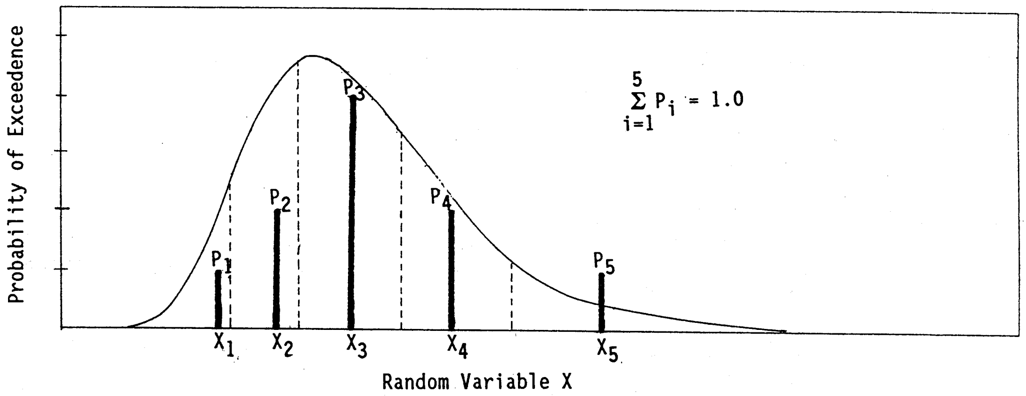

Figure 1: Discretized Probability Distribution

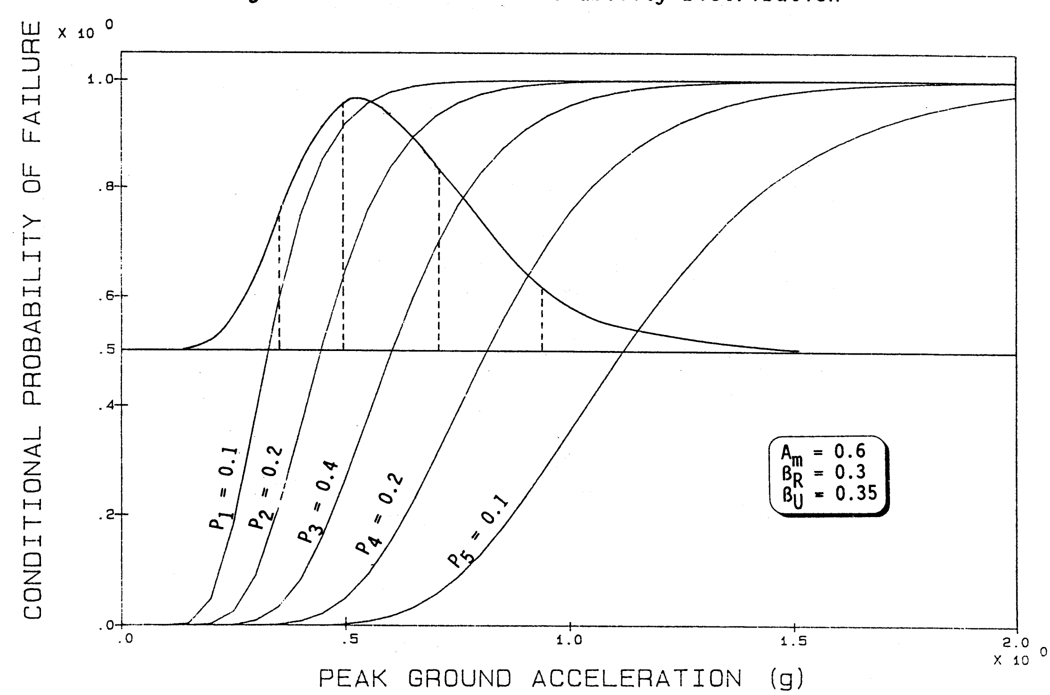

Figure 2: Family of Fragility Curves