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Application of a Modal Updating Technique to the FEM Schematization of Piping-pump Assembly: Two Case Studies

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ABSTRACT

A procedure for the updating of the physical parameter of a FE model is applied in two case studies. The considered cases refer to a pump-pipe assembly of two different chemical plants. Aim of the paper is to test the capability of the updating procedure in a real plant, i.e. outside laboratory controlled condition. The parameters that are updated in the case studies are, in the first case, the stiffness of the connection between the pump assembly and the base plate and, in the second one, the stiffness of the supports of the piping and the flexural stiffness of the reactor. The adopted method uses a comparison among the experimental and calculated eigenfrequencies, considering the uncertainties of the measurements, and taking into account the uncertainties of the parameters. The congruence of the identified parameters is discussed, considering different settings of the method.

KEYWORDS: structure vibration, parameter identification, FE model updating

INTRODUCTION

The updating of a finite element model of a real structure by means of the comparison with measured data is a topic that has received and still receives great attention: several methods have been proposed by different researches (among them [1], [2], [3], [5], [6]). All the methods are based on the comparison between the results of a Finite Element Model of the real structure and experimental data in the frequency domain (eigenfrequencies, mode shapes, frequency response functions) or in the time domain (response to impulsive or random excitation). There are two main categories of updating methods: those that modify the terms of the stiffness and mass matrices, the others that update the physical parameters of the model. The second category is preferable, when the main goal of the updating procedure is not only to match the experimental behaviour of the real structure in the tested condition, but also to properly reproduce its dynamic behaviour, in order to be able to predict the effect of structural modifications. Such structural changes could be related to modifications aiming to the dynamics optimisation of the structure, or alterations caused by damaging. In the second case, the localisation of the damaged area is useful for diagnostic activity.

This paper deals with a frequency domain method, developed by [1] which, considering modal parameters, enables to directly update the physical quantities of the finite element model, such as material properties or stiffness of lumped elastic elements. Applications to sample structures in laboratory have already been made with success ([5]): aim of this paper is to investigate the performance of the updating procedure when applied to a structurs in its operational environment. Two case studies, the piping and the pump assembly of the loop reactor of two different chemical plants, are considered. Although specific chemical plants are considered, the significance of the two cases is general, since it is not the case of a sample structure in the controlled environment of a laboratory, but a real structure in a real environment.

THE UPDATING PROCEDURE

The updating procedure is applied to a Finite Element (FE) schematisation of the system under investigation. The procedure compares the eigenfrequencies of the real structure, obtained in the considered cases from impact tests, and the ones obtained from the FE model. The first steps is the evaluation of the i-th eigenvalue λ_i derivative with respect to the generic j-th parameter θ_j :

$$\frac{\partial \lambda_i}{\partial \theta_j} = \Phi_i^T \left[\frac{\partial K}{\partial \theta_j} - \lambda_i \frac{\partial M}{\partial \theta_j} \right] \Phi_i \quad \text{with normalisation } 1 = \Phi_i^T M \Phi_i \tag{1}$$

being M and K the mass and stiffness matrices of the finite element model, Φ_i the eigenvector (mode shape) related to the i-th eigenvalue. A sensitivity matrix is then assembled by means of the derivatives (eq.1) of the eigenvalues. Several formulation of the problem are possible ([1]), according to the fact that the problem is overdetermined (number of parameters less than the number of measurements), or underdetermined (number of parameters greater than the number of measurements). The last case, in which the number of updating parameters is greater than the number of experimental data (i.e. the number of eigenfrequencies) has been considered, therefore the following penalty function $J(\delta\theta)$ has been used [1]:

$$J(\delta\theta) = \varepsilon^T W_{\varepsilon\varepsilon} \varepsilon + \delta\theta^T W_{\theta\theta} \delta\theta \qquad (2)$$

considering both the uncertainties assigned to the known parameters, and the uncertainties of the measured eigenvalues. $W_{\varepsilon\varepsilon}$ and $W_{\theta\theta}$ are weighting matrices, while ε is the vector of the error between measured and calculated eigenfrequencies, and $\delta\theta$ is the vector of parameters variation. The $W_{\varepsilon\varepsilon}$ matrix is related to the uncertainties of the measured frequencies, and is set as a diagonal matrix, being each element the inverse of the square of the standard deviation of the measured frequency (σ_m, in the following). On the other hand, matrix $W_{\theta\theta}$ reflects the degree of uncertainties with which the parameters are known or supposed, and is set as a diagonal matrix, being each element the inverse of the square of the standard deviation attributed to that parameter (σ, in the following). The function of the matrix $W_{\theta\theta}$, is also to limit the variation of the updating parameters at each iteration step: the influence of this term will be investigated further on. The problem stated by eq.2, is a problem of minimum, with the constraint related to the allowable parameter variation at each iteration step.

A least square iterative procedure is used for solving the problem (2). The iterations are neddend due to the non linearities, related to the dependence of the eigenvectors, necessary to calculate the eigenvalue derivatives (eq.1), on the parameters themselves. The updating of the parameters at the k-th step, is performed solving the following equation:

$$\theta_{k+1} = \theta_k + \left[[S]_k^T [W]_{\varepsilon\varepsilon} [S]_k + [W]_{\theta\theta} \right]^{-1} [S]_k^T [W]_{\varepsilon\varepsilon} \left(\underline{\lambda}_m - \underline{\lambda}_k \right) \qquad (3)$$

in which the $\underline{\lambda}_m$ vector collects the square of the measured frequencies (expressed as circular frequencies), and the vector $\underline{\lambda}_k$ represents the corresponding squared eigenfrequencies, calculated with the structure at the k-th step, i.e. with the current value of the $\underline{\theta}_k$ parameters. Eq.3 is moreover normalised in order to have the initial values of the parameters set to unity. This is done in order to avoid numerical problems when parameters with very different values are considered. Figure 1 resumes the iteration procedure: the natural frequencies and mode shapes are calculated from the FE model, and the sensitivity matrix is set-up, for the calculation of the updating of the parameters. The updated values of the parameters are then inserted into the FE model, and the procedure is iterated, until convergence is reached on the frequencies and the on parameters too. The recognition of the correspondence of the calculated mode shapes with the experimental ones is an essential aspect for the applicability of the method; this has been done comparing the related the mode shapes by means of the Model Assurance Criterion (MAC) [1]:

$$M.A.C. = \frac{\left| \Phi_{mj}^T \Phi_{kj}^T \right|^2}{\left(\Phi_{mj}^T \Phi_{mj}^T \right)\left(\Phi_{ak}^T \Phi_{ak}^T \right)} \qquad (4)$$

which is an indicator of the degree of the correlation between a measured mode Φ_{mj}, and a calculated one Φ_{ak}: values in the range 0.7÷1 indicate good correlation.

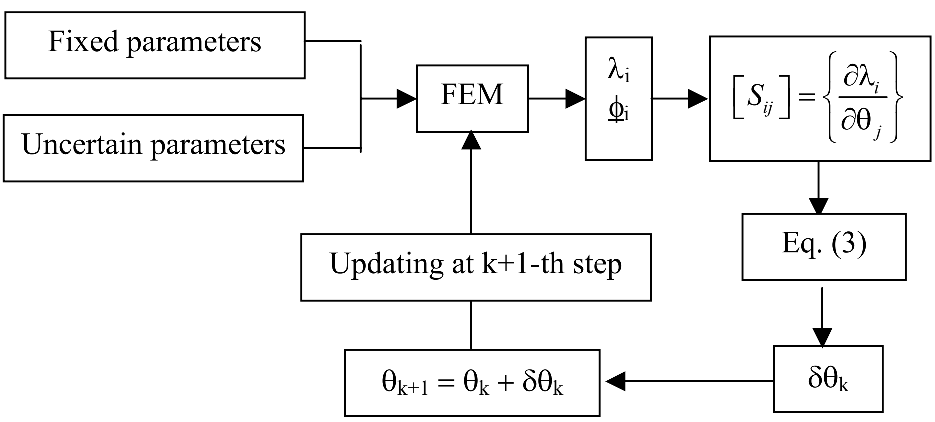

Fig. 1 Flow chart of the updating procedure.

As a matter of fact, it is not necessary to take into account all the modes, but only the ones that show a high value of the M.A.C. index . In fact, in the measurements, depending on the excitation procedure, and on the location of the applied excitation, not all the modes are always revealed. Another feature of the adopted method, since it refers to physical parameters, is that it enables also to take advantage from the knowledge of the location of the main uncertainties, limiting the updating only to some of the structural parameters.

CASE 1 STUDY

The first case which is considered is a part of a loop reactor of a chemical plant, only the lower portion of which is modelled, as shown in Fig.2. In this case the main technical problem was an excitation of a local mode (shown in Fig.3), because of a 4xrev component due to the interaction among the four propeller blades and the four fixed sectors in the pipe. There was a demand for shifting this local frequency. A possibility to increase the frequency was discarded, as there were not fully fixed points to which connect in order to stiffen the pump case, therefore it was decided to decrease this frequency. A possible intervention was the insertion of rubber elements in the connections among the pump case and the base-plate that supports the motor (Fig.4). Since a limitation to the flexibility is given because of the alignment of the pump shaft and the electric motor, a compromise had to be reached in order to significantly shift the critical frequency.

The excited local mode, found at 74.5 Hz, (see Fig.5), obtained by means of an impact-hammer test), after the insertion of the rubber elements, shifted its frequency to 69.5 Hz, getting out of the resonance condition. This modification is now used as test case for the method: the parameters to be updated are the stiffness of the rubber elements in the vertical and lateral direction. In this case we have the great advantage that it is known which are the changed parameters, therefore no other parameters are involved in the updating procedure.

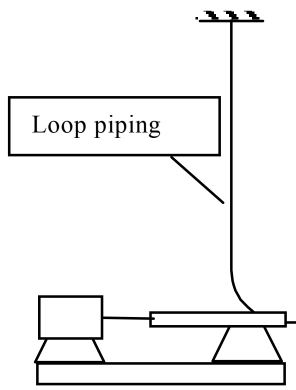

Fig. 2 Main sketch of the piping loop considered in the case 1.

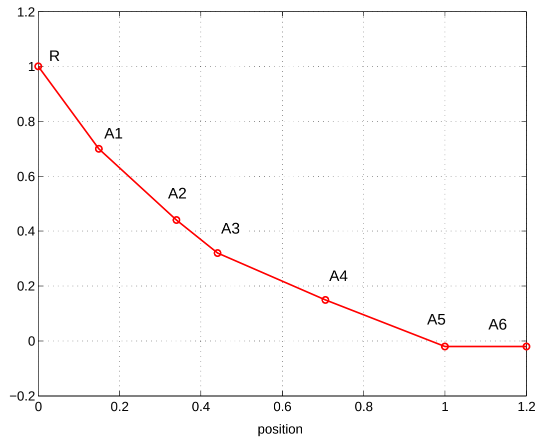

Fig. 3 Shape of the local mode measured at 74.5 Hz. The letters refer to the locations shown in Figure 4.

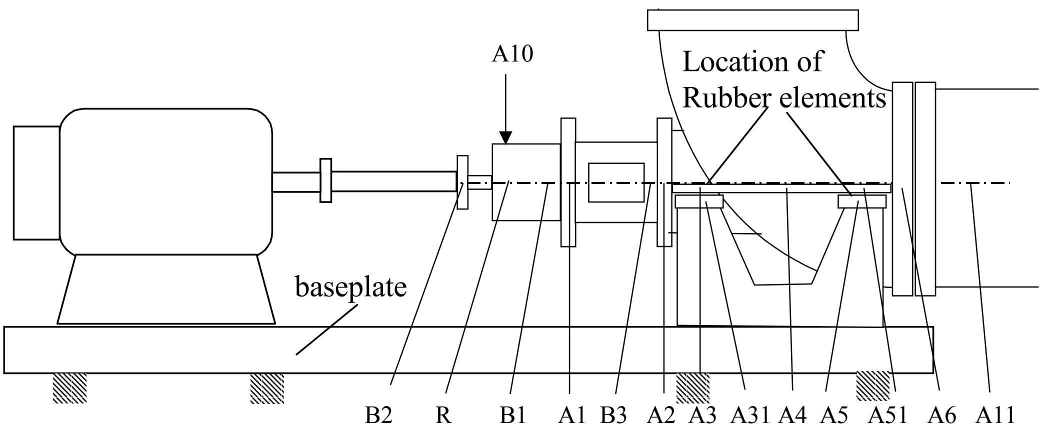

Fig. 4 Sketch of the pump assembly and base plate case involved in the investigated mode.

Results of the test case 1

The second step was to identify the value of the stiffness of the rubber element that corresponds to the effective frequency shifting. Figure 6 and Figure 7 show the convergence of the vertical stiffness (Kz) and the lateral stiffness (Ky) of the rubber elements, varying the starting value, and the iterative procedure set-up, in order to check the convergence of final values of the two parameters. The nominal value of the stiffness of the rubber was Kz=6MN/m for the vertical direction, while the lateral was set at 1/10 of the vertical one, i.e. Ky=0.6MN/m. It can be observed that the Kz value always reaches the same final value, while the Ky parameter does not reach always the same value, even if it remains in a bounded region. The reason for this different behaviour can be found in the sensitivity matrix: the most sensitive parameters, are the ones that are better identified, especially in the considered case, with more parameters than data. The modification of the settings of the method, affects the rate of convergence: decreasing the σ parameter /from 15% to 5%), which corresponds to limit the variation of each iteration step the convergence is slower, but in some cases, with this settings, a lower dispersion of the identified parameters is obtained (Fig.7, right).

During the iteration procedure some numerical problems can occour, these problems are related to the calculation of the eigenvalues/eigenvectors, which, in turn affect the sensitivity matrix. In such cases, regolarization tecnique coiuld be helpful [3]. The term of the sensitivity matrix corresponding to the vertical stiffness, for the two conditions σ=5%, and σ=15%, are reported, for the case study shown in Fig.7 on the right. When a smooth trend is obtained for the sensitivity matrix then a corresponding smooth trend is obtained for the identified parameter, and also for the frequency to be matched (Fig.8, left). In all the examined cases the target frequency is reached (Fig.8, left), with a trend depending on the history of the sensitivity matrix.

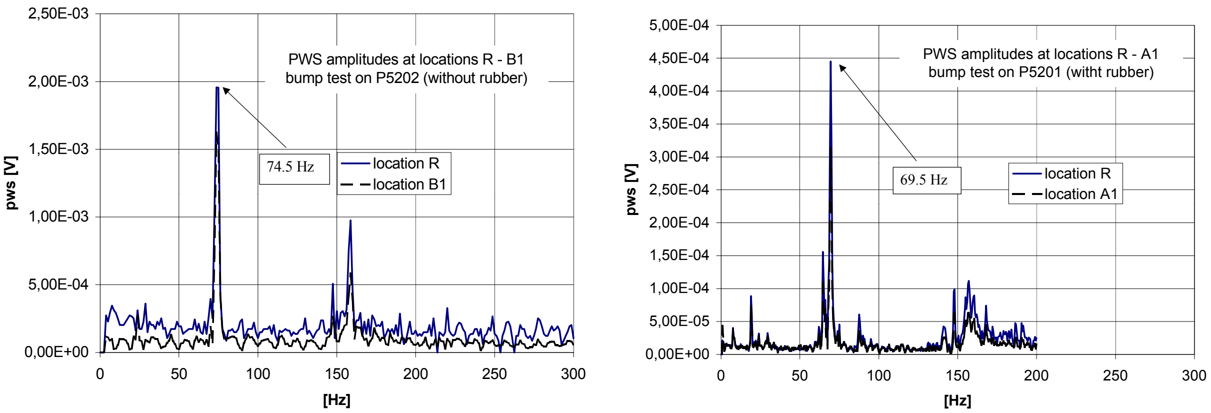

Fig. 5 Left: power spectrum of the experimental response of the pump case to impulse excitation in the original condition: the 74.5 Hz peak refers to the flexural mode of the assembly pump+case. Right: the frequency of the same mode, shifted to 69.5 Hz, measured after the installation of the rubber elements.

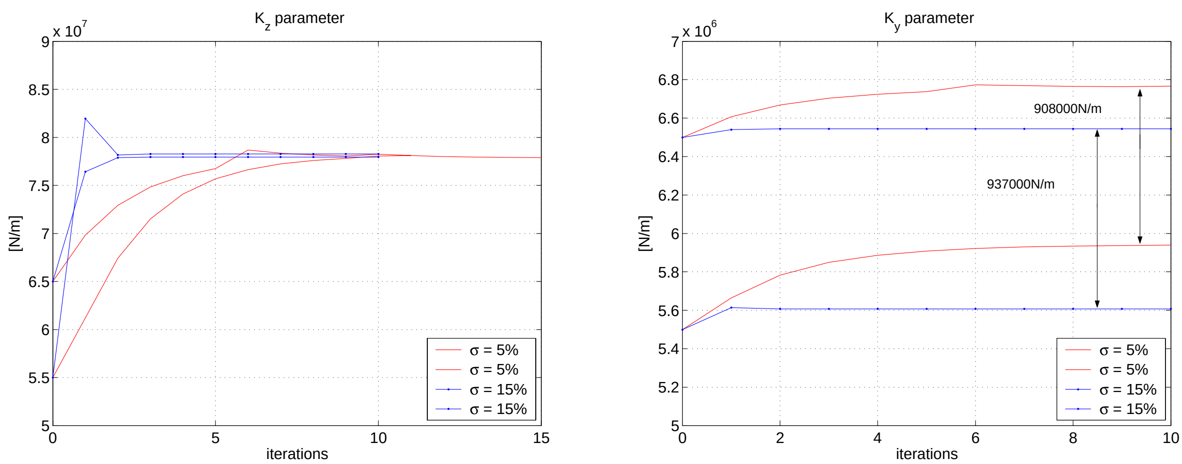

Fig. 6 Left: convergence, for the test case 1, of the Kz parameter (left) and for the Ky parameter (right), varying starting values and σ parameter.

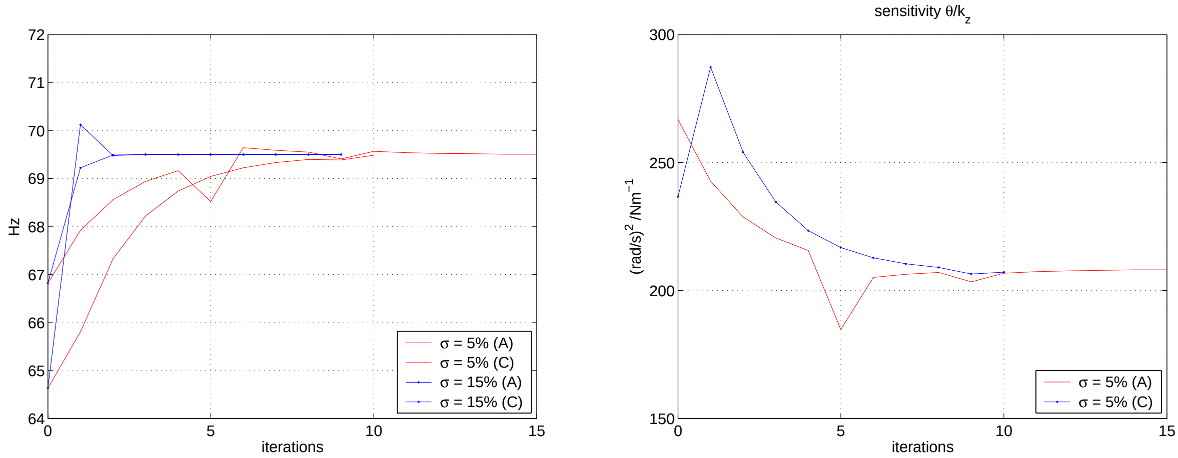

Fig. 7 Left: convergence, for the test case 1, of the frequency of the investigated mode. Right: sensitivity with repsect to the Kz parameters, as a function of the iteration step.

CASE 2 STUDY

The portion of the plant object of the second case study, which is involved in the identification process, consists (Fig.8) of a loop piping assembly plus a reactor, with a circulating pump located on the top. From the beginning some concerns raised about the occurence of high vibration induced by the pump. It is in fact clear that, due do its position on the top of the loop piping, the unbalance of the motor-pump system can easily excite those flexural modes that have significant in-plane and perpendicular to the plane components on the top of the loop piping, and fall inside the speed range of the pump, that does not work at a fixed speed, but can operate in the range 800-1800 rpm.

There are two kinds of connection between the loop piping and the supporting structure: supporting devices, (located at A and B position in Fig.8), and guiding devices, (located at 'G' positions in figure). The first ones give full restriction, while the second ones, constraint only the motion on the horizontal plane. The whole structure is schematised by means of beam element, whose characteristics are calculated according to the nominal dimensions. The supporting devices are modelled by means of a spring constant along all six directions (translation and rotations).

The main unknowns come from the stiffness of the supporting and guiding devices.Therefore, possible parameters to be identified are the stiffness of the supports and the guiding devices, the flexural stiffness of the heat exchanger, and the flexural stiffness of the connection of the motor, on the top of the piping. No kwnoledge at all was available for the stiffness of the supports, and the possible deviation from the nominal values of the flexural stiffness of the reactor and of the motor pump case connection. These parameters have been adjusted by tentative, looking for the correct mode shape.

Experimental data used for the identification procedure and FE model

Four modes have been identified from impact hammer tests, two belong to the x-z plane, two show prevalent motion perpendicular to the x-z plane. The top of the piping loop has the larger amplitude related to the first modes, which are expected to be general flexural motion of the loop piping itself, enhanced by the presence of the concentrated mass of the electric motor and pump assembly. Table 1 reports the values of the four frequencies identified from the impact tests, accompanied with the uncertainties of the obtained values. Fig.9 reports the calculated mod shapes, corresponding to the nominal values for the known parameters, and initially estimate for the unknown parameters.

One problem with the present investigation is that the modal shapes are measured only on a limited portion of it, even if they are singificant, since are the one that experience the higher amplitudes. As a consequence, the MAC indexes could be calculated limiting to only on a part of the structure. Fig.10 shows the graphical representation of the MAC indexes: it can be seen that the third mode has been not clearly identified as exact counterpart of a related mode of the FE model, while the values close to unit of the other modes assures that the numerical-experimental correspondence is good. Prelimiary investigations shown that the mode at 19.6Hz was weakly influenced by the considered parameters, if compared with the first two, therefore the updating procedure was applied considering the first two modes only.

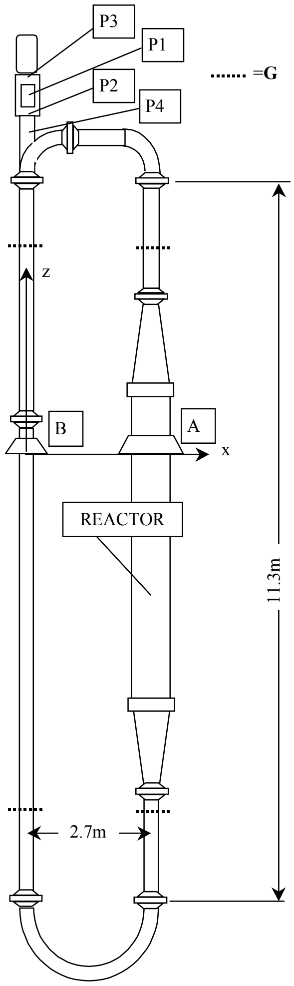

Fig. 8 Main sketch of the piping loop, with the location of the measurement points and the supports.

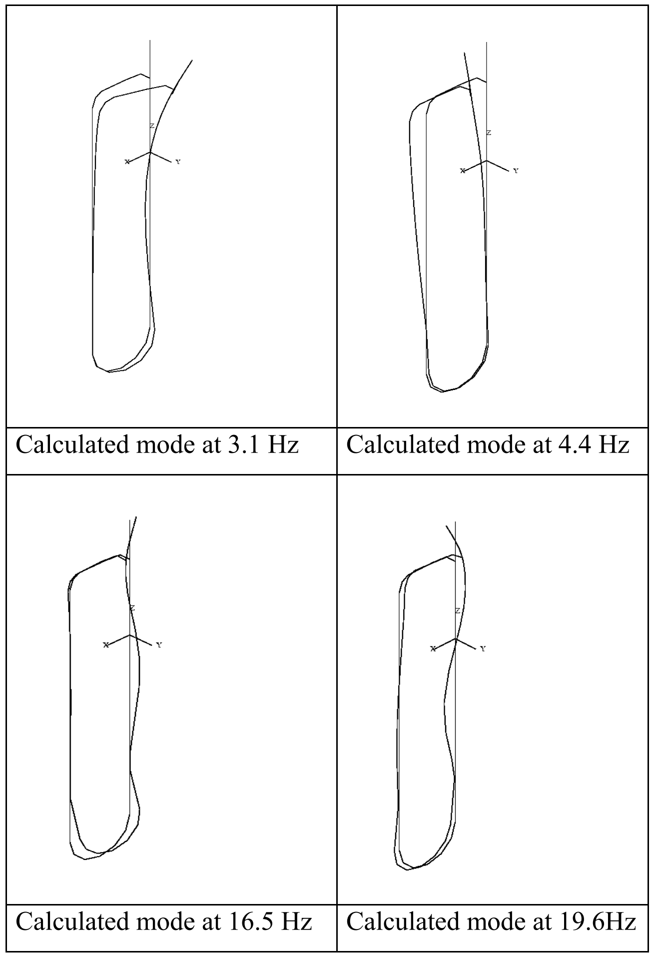

Fig. 9 Calculated mode shapes, with the nominal data, and with the starting values of the parameters.

Mode shape	Frequency [Hz] measured	Frequency [Hz] calculated
First flexural, perpendicular to x-z plane	3.75±0.25	3.16
First flexural, in x-z plane	5.5±0.25	4.4
Higher flexural mode, perpendicular to x-z plane	17±0.25	16.5
Higher flexural mode, in the x-z plane	320±0.25	19.6

Table 1.Comparison among measured and calculated natural frequencies, for the modes that could be paired.

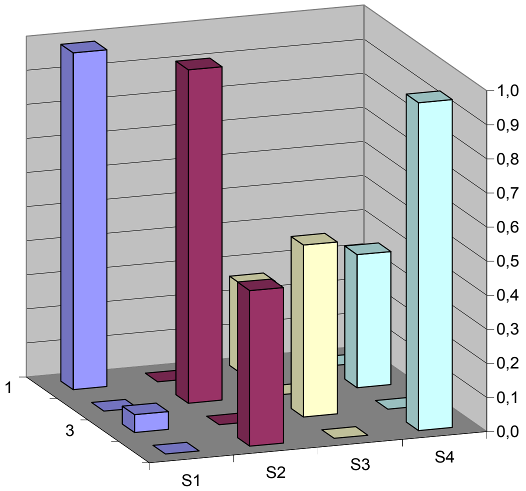

Fig. 10 Graphical representation of the MAC matrix, for the starting values of the parameters, for the test case 2.

Results of test case 2.

The aim of the application of the updating procedure is to tune the parameters reported in Table2I, in order to get closer to the first two target frequencies of the loop piping. The considered parameters are the longitudinal (K_x (A,B)) and (K_y (A,B)) lateral stiffness of A and B supports, their rotational stiffness ($K_{\theta x}$ and $K_{\theta y}$), different for the two supports because of their different dimension, while horizontal stiffness for the guides are considered differente accoridng to tob and bottom position. Flexural stiffness for the connection between the elecric motor and the piping (EI_C), and the flexural stiffness of the reactor (EI_C) are also considered among the variable parameters.
Table 2 shows the terms of the sensitivity matrix, in a non dimensional form, for the first two frequencies, for all the above mentioned parameters, calculated with their starting values (see Table 3).

Table 2. Non dimensional sensitivity of the parameters considered in the updating procedure.

	K_x (A,B)	K_y (A,B)	$K_{\theta x}$ (B)	$K_{\theta y}$ (B)	$K_{\theta x}$ (A)	$K_{\theta y}$ (A)	$K_x = K_y$ G(top)	$K_x = K_y$ G(bottom)	EI_C	EI_R
$\{\partial\lambda_1/\partial\theta_j\}\theta_j/\lambda_1 \times 100$	0.00	2.58	0.24	0.00	0.18	0.00	2.37	32.70	0.46	0.94
$\{\partial\lambda_2/\partial\theta_j\}\theta_j/\lambda_2 \times 100$	56.10	0.00	0.00	0.59	0.00	0.37	4.58	21.30	0.01	0.23

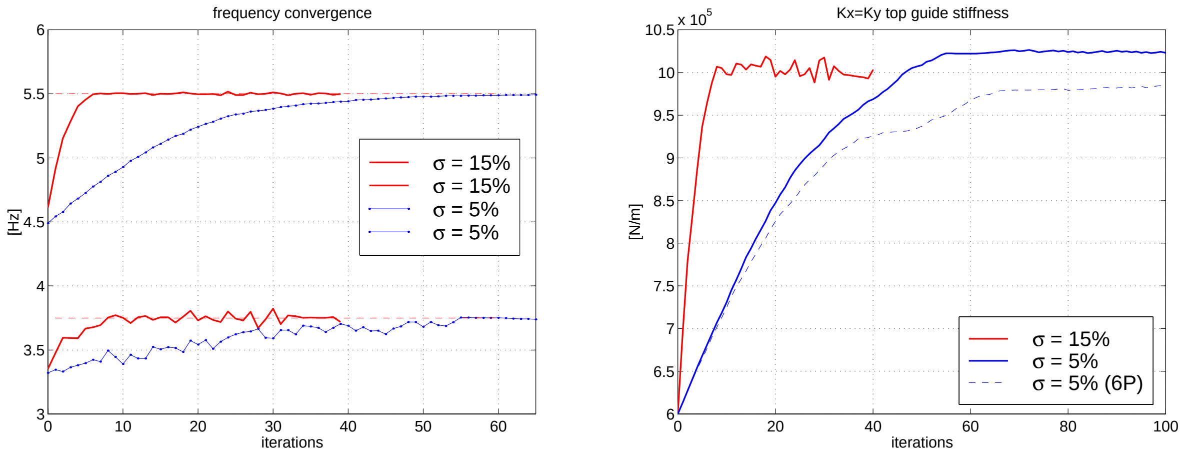

Fig. 11 Left: convergence, for the test case 2, of the frequency of the first and second modes. Right: convergence of the stiffness of the guiding devices at the top of the structure.

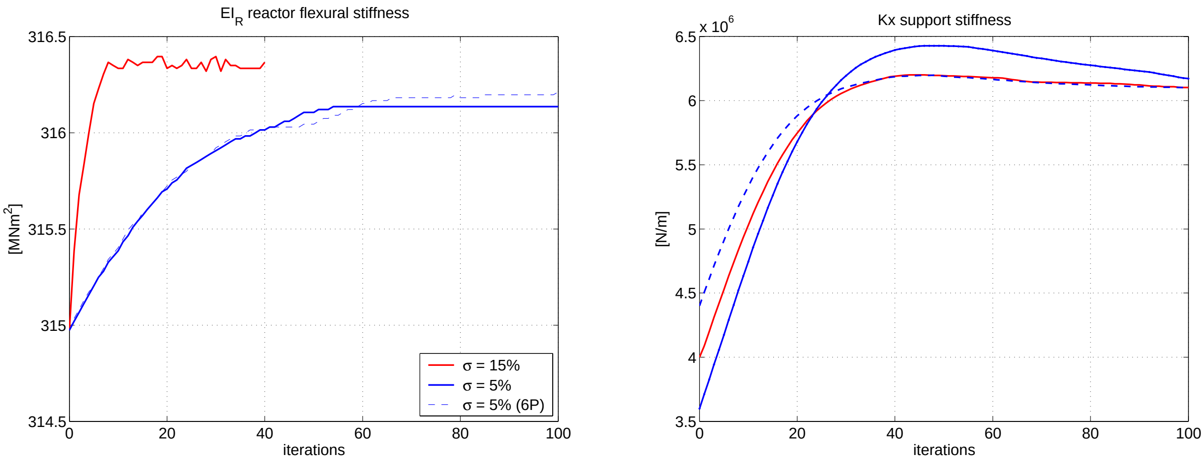

Fig. 12 Left: convergence of the flexural stiffness of the reactor flexural stiffness EI_R for different settings of the method. Right convergence of the Kx stiffness of the support A, (4 parameters), starting with different initial values.

Table 3. starting values and final values of the parameters, depending on the procedure settings.

	K_x MN/m (A,B)	K_y MN/m (A,B)	$K_{\theta x}$ MNm/rad (B)	$K_{\theta y}$ MNm/rad (B)	$K_{\theta x}$ MNm/rad (A)	$K_{\theta y}$ MNm/rad (A)	$K_x = K_y$ MN/m G(bottom)	$K_x = K_y$ MN/m G(top)	EI_R MNm^2	EI_C MNm^2
Initial values	4	8	1	1	3	3	6	0.6	314.97	32.672
σ = 15%, 10 par	6.104	8.163	1.022	1.004	3.005	3.007	6.366	1.003	316.36	32.959
σ = 5%, 10 par	6.162	8.135	1.019	1.004	3.004	3.006	6.341	0.973	316.17	32.890
σ = 5%, 6 par.	6.171	7.523	-	-	-	-	5.359	1.031	316.21	32.888
σ = 5%, 4 par.	6.172	7.415	-	-	-	-	5.639	1.028	-	-

Different settings of the methods (σ value), and different number of parameters, progressively reducing from ten to four (Table 3), have been adopted, to test the convergence of the remaining parameters. Some showed a good stability (Fig.11 and 12), while others showed a strong dependence on their initial values. Others, like the rotational stiffness, remained practically unchanged during the updating procedure (Table 3), and have been successively excluded from the procedure.

CONCLUDING REMARKS

In this paper the application of a updating procedure for FE models parameters, proposed by [1], has been applied to the pump-piping assembly of two chemical plants, in operating in-field condition. The procedure is based on the comparison among measured and calculated natural frequencies.

In the first test case study, it was known which were the parameters to be updated, and the procedure successfully converged to a realistic value of the updated parameter (the stiffness of inserted rubber elements).

In the second case study, the application of the procedure was made more difficult by the complete lack of knowledge on what parameters have to be updated, and on the initial values of most of them. Some reasonable hypothesis were formulated, and several combinations of parameters have been tested. It was found out that only some parameters could be identified with a sufficient degree of accuracy and stability, while for others, namely the ones less influencing the value of the considered frequencies, no stable convergence to the final values could be obtained, showing a dependence from the initial value. In all cases the frequencies converged at the target value.

Even if the measurements of the two considered cases were not specifically aimed to the application of the updating procedure, its applicability has been proven with partial satisfactory results, indicating also which are the requirements for its full application: completeness of the measurement points, adequate in number in order to ensure the correspondence of the calculated and measured modes, and the availability of a FE model with a proper level of details, in order to identify those parameters that effectively influences to values of the frequencies.

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