# The creep of multi-layered moderately thick shells of revolution under asymmetrical loading

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## 1 INTRODUCTION

As for the creep of shells of revolution many investigations have been made for not only simple geometries such as spherical shells, conical shells, cylindrical shells, but also general shells of revolution by Penny (1964), Murakami and Iwatsuki (1969), Takezono et al. (1973a, 1973b, 1975, 1980, 1981, 1983, 1984), and Yagawa et al. (1975). These investigations, however, are almost all concerned with the case of single layered shells and few studies on the layered shells composed of different materials are reported in spite of their importance in engineering.

In the present paper the authors study the creep deformation of the multi-layered thick shells of revolution under asymmetrical loads. The equations of equilibrium and the strain-displacement relations are derived from the Reissner-Naghdi theory (1941, 1957) for elastic shells where a consideration on the effect of shear deformation is given. In the theory of creep it is assumed that in a given increment of time the total strain increments are composed of an elastic part and a part due to creep. The elastic strains are proportional to the stresses by Hooke's law. For the constitutive equations in the creep range, McVetty's equation modified by Arrhenius' equation for thermal effect is employed.

The basic differential equations on the creep problems derived for the incremental values with respect to time are numerically solved by a finite difference method and the solutions at any time are obtained by summation of the incremental values. Resultant forces and resultant moments are given from numerical integration of the stresses by Simpson's 1/3 rules.

As a numerical example, the creep deformation of a simply supported two-layered cylindrical shell composed of mild steel SB42 and stainless steel SUS316 subjected to locally distributed loads is analyzed. The numerical computations have been carried out for three cases of the ratio of the thickness of the stainless steel layer to the shell thickness.

## 2 FUNDAMENTAL EQUATIONS

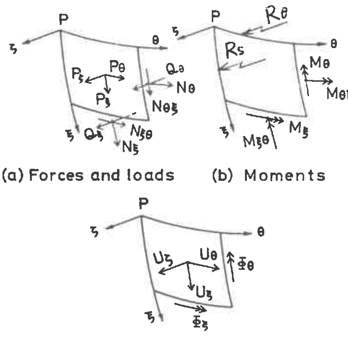
If the middle surface of the shells of revolution is given by $r=r(s)$ where $r$ is the distance from the axis and $s$ is the meridional distance measured from a boundary along the middle surface, the relations among the nondimensional curvatures $\omega_\xi(=a/R_s)$, $\omega_\theta(=a/R_\theta)$ and the nondimensional radius $\rho(=r/a)$ become:

$$\omega_\xi=-(\gamma'+\gamma^2)/\omega_\theta,\ \omega_\theta=\sqrt{1-(\rho')^2}/\rho \quad \omega'_\theta=\gamma(\omega_\xi-\omega_\theta),\ \rho''/\rho=-\omega_\xi\omega_\theta \quad \gamma=\rho'/\rho,\ \xi=s/a,\ (\ )'=d(\ )/d\xi \quad \cdots(1)$$

where $a$ is the reference length. An arbitrary point of the shell can be expressed in the orthogonal coordinate system $(\xi,\theta,\zeta)$.

Application of the Reissner shell theory to the shells of revolution and differentiation of the equilibrium equation with time yield:

$$\left.\begin{aligned}
&\frac{\partial\dot N_\xi}{\partial\xi}+\gamma(\dot N_\xi-\dot N_\theta)+\frac{1}{\rho}\frac{\partial\dot N_{\theta\xi}}{\partial\theta}+\omega_\xi\dot Q_\xi+a\dot P_\xi=0\\
&\frac{\partial\dot N_{\xi\theta}}{\partial\xi}+\gamma(\dot N_{\xi\theta}+\dot N_{\theta\xi})+\frac{1}{\rho}\frac{\partial\dot N_\theta}{\partial\theta}+\omega_\theta\dot Q_\theta+a\dot P_\theta=0\\
&\frac{\partial\dot Q_\xi}{\partial\xi}+\gamma\dot Q_\xi+\frac{1}{\rho}\frac{\partial\dot Q_\theta}{\partial\theta}-(\omega_\xi\dot N_\xi+\omega_\theta\dot N_\theta)+a\dot P_\zeta=0\\
&\dot Q_\xi-\frac{1}{a}\left[\frac{\partial\dot M_\xi}{\partial\xi}+\gamma(\dot M_\xi-\dot M_\theta)+\frac{1}{\rho}\frac{\partial\dot M_{\theta\xi}}{\partial\theta}\right]=0\\
&\dot Q_\theta-\frac{1}{a}\left[\frac{\partial\dot M_{\xi\theta}}{\partial\xi}+\gamma(\dot M_{\xi\theta}+\dot M_{\theta\xi})+\frac{1}{\rho}\frac{\partial\dot M_\theta}{\partial\theta}\right]=0
\end{aligned}\right\}\cdots\cdots\cdots\cdots(2)$$



(a) Forces and loads (b) Moments

(c) Displacements and rotations

Fig.1 Notations.

where the notations are shown in Fig.1.

The strain rates of the middle surface, $\dot\varepsilon_{\xi m}, \dot\varepsilon_{\theta m}$, $\dot\varepsilon_{\xi\theta m}$ are given by the displacements $U_\xi, U_\theta, U_\zeta$ in the next equations:

$$\dot\varepsilon_{\xi m}=\frac{1}{a}\left[\frac{\partial\dot U_\xi}{\partial\xi}+\omega_\xi\dot U_\zeta\right],\qquad \dot\varepsilon_{\theta m}=\frac{1}{a}\left[\frac{1}{\rho}\frac{\partial\dot U_\theta}{\partial\theta}+\gamma\dot U_\xi+\omega_\theta\dot U_\zeta\right],\qquad \dot\varepsilon_{\xi\theta m}=\frac{1}{2a}\left[\frac{1}{\rho}\frac{\partial\dot U_\xi}{\partial\theta}+\frac{\partial\dot U_\theta}{\partial\xi}-\gamma\dot U_\theta\right]\quad\cdots(3)$$

where $\dot\varepsilon_{\xi\theta m}$ is half the usual engineering shear strain rates.

The bending distortion rates $\dot\kappa_\xi, \dot\kappa_\theta, \dot\kappa_{\xi\theta}, \dot\kappa_{\theta\xi}$ are given as follows:

$$\dot\kappa_\xi=\frac{1}{a}\frac{\partial\dot\Phi_\xi}{\partial\xi},\ \dot\kappa_\theta=\frac{1}{a}\left(\frac{1}{\rho}\frac{\partial\dot\Phi_\theta}{\partial\theta}+\gamma\dot\Phi_\xi\right),\quad \dot\kappa_{\xi\theta}=\frac{1}{2a}\left(\frac{\partial\dot\Phi_\theta}{\partial\xi}-2\omega_\xi\dot\Phi_n\right),\quad \dot\kappa_{\theta\xi}=\frac{1}{2a}\left(\frac{1}{\rho}\frac{\partial\dot\Phi_\xi}{\partial\theta}-\gamma\dot\Phi_\theta+2\omega_\theta\dot\Phi_n\right)\quad\cdots(4)$$

where rotation rates $\dot\Phi_\xi,\dot\Phi_\theta,\dot\Phi_n$ are:

$$\dot\Phi_\xi=\frac{1}{a}\left(-\frac{\partial\dot U_\zeta}{\partial\xi}+\omega_\xi\dot U_\xi\right)+2\dot\varepsilon_{\xi\zeta m},\ \dot\Phi_\theta=\frac{1}{a}\left(-\frac{1}{\rho}\frac{\partial\dot U_\zeta}{\partial\theta}+\omega_\theta\dot U_\theta\right)+2\dot\varepsilon_{\theta\zeta m},\ \dot\Phi_n=\frac{1}{2a}\left(-\frac{1}{\rho}\frac{\partial\dot U_\xi}{\partial\theta}+\frac{\partial\dot U_\theta}{\partial\xi}+\gamma\dot U_\theta\right)\quad\cdots(5)$$

The strain rates at the distance $\zeta$ from the middle surface are:

$$\begin{aligned}
&\dot\varepsilon_\xi=(\dot\varepsilon_{\xi m}+\zeta\dot\kappa_\xi)/L_\xi,\ \dot\varepsilon_\theta=(\dot\varepsilon_{\theta m}+\zeta\dot\kappa_\theta)/L_\theta,\quad \dot\varepsilon_{\xi\zeta}=\dot\varepsilon_{\xi\zeta m}/L_\xi,\ \dot\varepsilon_{\theta\zeta}=\dot\varepsilon_{\theta\zeta m}/L_\theta,\ L_\xi=1+\zeta/R_s,\ L_\theta=1+\zeta/R_\theta\\
&\dot\varepsilon_{\xi\theta}=\left\{\frac{1}{2}(\dot\varepsilon_{\xi\theta m}+\dot\Phi_n)+\zeta(\dot\kappa_{\xi\theta}+\dot\Phi_n/R_s)\right\}/L_\xi+\left\{\frac{1}{2}(\dot\varepsilon_{\xi\theta m}-\dot\Phi_n)+\zeta(\dot\kappa_{\theta\xi}-\dot\Phi_n/R_\theta)\right\}/L_\theta
\end{aligned}\quad\cdots(6)$$

In the theory of creep it is assumed that in a given increment of time the total strain increments are composed of an elastic part and a part due to creep. Then the total strain rates may be expressed as follows:

$$\{\dot\varepsilon\}=[D]^{-1}\{\dot\sigma\}+\{\dot\varepsilon^T\}+\{\dot\varepsilon^c\}\cdots\cdots\cdots\cdots(7)$$

where

$$\{\dot\varepsilon\}=\{\dot\varepsilon_\xi,\ \dot\varepsilon_\theta,\ \dot\varepsilon_{\xi\theta},\ \dot\varepsilon_{\xi\zeta},\ \dot\varepsilon_{\theta\zeta}\}^T\quad \{\dot\varepsilon^T\}=\{\alpha\dot T_e,\ \alpha\dot T_e,\ 0,\ 0,\ 0\}^T$$
$$\{\dot\sigma\}=\{\dot\sigma_\xi,\ \dot\sigma_\theta,\ \dot\sigma_{\xi\theta},\ \dot\sigma_{\xi\zeta},\ \dot\sigma_{\theta\zeta}\}^T\quad \{\dot\varepsilon^c\}=\{\dot\varepsilon^c_\xi,\ \dot\varepsilon^c_\theta,\ \dot\varepsilon^c_{\xi\theta},\ \dot\varepsilon^c_{\xi\zeta},\ \dot\varepsilon^c_{\theta\zeta}\}^T$$

$$[D]=\frac{E}{1-\nu^2}\begin{bmatrix}1&\nu&0&0&0\\ \nu&1&0&0&0\\ 0&0&1-\nu&0&0\\ 0&0&0&1-\nu&0\\ 0&0&0&0&1-\nu\end{bmatrix}\quad\cdots(8)$$

where, $\alpha$ is thermal expansion coefficient and $T_e$ is a temperature rise from the original temperature $T_0$ to the present temperature $T$, namely,

$$T_e(\xi,\theta,\zeta,t)=T(\xi,\theta,\zeta,t)-T_0\cdots\cdots\cdots(9)$$

Now, if it is assumed that the creep strain in the uniaxial state for constant stress may be given by McVetty's equation modified by Arrhenius' equation for thermal effect:

$$\varepsilon^c=Ae^{-\beta/T}\sigma^m(1-e^{-qt}+ct)\ \cdots\cdots\cdots\cdots\cdots(10)$$

Then the creep strain rates $\{\dot\varepsilon^c\}$ in the multi-axial state assumed in the shell theory may be written as follows for the time hardening theory:

$$\{\dot{\varepsilon}^c\}=Ae^{-\beta/T}\sigma_e^{m-1}(qe^{-qt}+c)[B]\{\sigma\} \cdots\cdots\cdots (11)$$

$$[B]=\begin{bmatrix} 1 & -1/2 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 3/2 & 0 & 0 \\ 0 & 0 & 0 & 3/2 & 0 \\ 0 & 0 & 0 & 0 & 3/2 \end{bmatrix} \cdots\cdots (12)$$

where $A, m, q, c,$ and $\beta$ are material constants to be obtained from a fit of uniaxial test data, and $T$ is the absolute temperature, and $\sigma_e$ is effective stress given by:

$$\sigma_e=(\sigma_\xi^2+\sigma_\theta^2-\sigma_\xi\sigma_\theta+3\sigma_{\xi\theta}^2+3\sigma_{\xi\zeta}^2+3\sigma_{\theta\zeta}^2)^{1/2} \cdots\cdots\cdots\cdots\cdots (13)$$

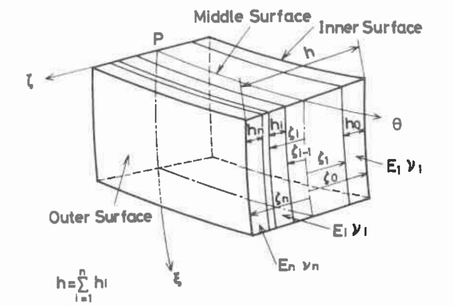



Fig.2 Multi-layered shell element.

Substituting Eqs.(6) into Eqs.(7) and solving them about stress rates,the stress rates are written as:

$$\left.\begin{aligned}
&\dot{\sigma}_\xi=\frac{E}{1-\nu^2}\{(\dot{\varepsilon}_{\xi m}+\zeta\dot{\kappa}_\xi)/L_\xi+\nu(\dot{\varepsilon}_{\theta m}+\zeta\dot{\kappa}_\theta)/L_\theta\}-\dot{\sigma}_\xi^c-\dot{\sigma}^T \qquad \dot{\sigma}_\theta=\frac{E}{1-\nu^2}\{(\dot{\varepsilon}_{\theta m}+\zeta\dot{\kappa}_\theta)/L_\theta+\nu(\dot{\varepsilon}_{\xi m}+\zeta\dot{\kappa}_\xi)/L_\xi\}-\dot{\sigma}_\theta^c-\dot{\sigma}^T \\
&\dot{\sigma}_{\xi\theta}=\frac{E}{1+\nu}\left[\left\{\frac{1}{2}(\dot{\varepsilon}_{\xi\theta m}+\dot{\Phi}_n)+\zeta\left(\dot{\kappa}_{\xi\theta}+\frac{\dot{\Phi}_n}{R_\xi}\right)\right\}/L_\xi+\left\{\frac{1}{2}(\dot{\varepsilon}_{\xi\theta m}-\dot{\Phi}_n)+\zeta(\dot{\kappa}_{\theta\xi}-\frac{\dot{\Phi}_n}{R_\theta}\right\}\Big/L_\theta\right]-\dot{\sigma}_{\xi\theta}^c \\
&\dot{\sigma}_{\xi\zeta}=\frac{E}{1+\nu}\dot{\varepsilon}_{\xi\zeta m}/L_\xi-\dot{\sigma}_{\xi\zeta}^c \qquad \dot{\sigma}_{\theta\zeta}=\frac{E}{1+\nu}\dot{\varepsilon}_{\theta\zeta m}/L_\theta-\dot{\sigma}_{\theta\zeta}^c \qquad \{\dot{\sigma}^c\}=[D]\{\dot{\varepsilon}^c\},\ \{\dot{\sigma}^T]=[D]\{\dot{\varepsilon}^T\}
\end{aligned}\right\} \cdots\cdots (14)$$

By the use of Eqs.(14) with the next approximation, the rates of change of the resultant forces and the resultant moments for the multi-layered shell (Fig.2) may be obtained as Eqs.(16).

$$\frac{L_\theta}{L_\xi}\simeq 1-\left(\frac{1}{R_\xi}-\frac{1}{R_\theta}\right)\zeta+\frac{1}{R_\xi}\left(\frac{1}{R_\xi}-\frac{1}{R_\theta}\right)\zeta^2 \cdots\cdots\cdots\cdots\cdots (15)$$

$$\begin{Bmatrix}\dot{N}\\ \dot{M}\end{Bmatrix}=\int_{-h/2}^{h/2}[L]\begin{Bmatrix}\dot{\sigma}\\ \dot{\sigma}^*\zeta\end{Bmatrix}d\zeta=[A]\begin{Bmatrix}\dot{\varepsilon}_m\\ \dot{\kappa}\\ \dot{\Phi}_n\end{Bmatrix}-\begin{Bmatrix}\dot{N}^c\\ \dot{M}^c\end{Bmatrix}-\begin{Bmatrix}\dot{N}^T\\ \dot{M}^T\end{Bmatrix} \cdots\cdots\cdots (16)$$

where

$$\{\dot{N}\}=\{\dot{N}_\xi, \dot{N}_\theta, \dot{N}_{\xi\theta}, \dot{N}_{\theta\xi}, \dot{Q}_\xi, \dot{Q}_\theta\}^T$$

$$\{\dot{M}\}=\{\dot{M}_\xi, \dot{M}_\theta, \dot{M}_{\xi\theta}, \dot{M}_{\theta\xi}\}^T$$

$$[L]=\begin{bmatrix}L_1 & 0\\ 0 & L_2\end{bmatrix},\quad [L_1]=\begin{bmatrix}L_\theta & & & & & 0\\ & L_\xi & & & & \\ & & L_\theta & & & \\ & & & L_\xi & & \\ & & & & L_\theta & \\ 0 & & & & & L_\xi\end{bmatrix},\quad [L_2]=\begin{bmatrix}L_\theta & & & 0\\ & L_\xi & & \\ & & L_\theta & \\ 0 & & & L_\xi\end{bmatrix}$$

$$\{\dot{\varepsilon}_m\}=\{\dot{\varepsilon}_{\xi m}, \dot{\varepsilon}_{\theta m}, \dot{\varepsilon}_{\xi\theta m}, \dot{\varepsilon}_{\xi\zeta m}, \dot{\varepsilon}_{\theta\zeta m}\},\ \{\dot{\kappa}\}=\{\dot{\kappa}_\xi, \dot{\kappa}_\theta, \dot{\kappa}_{\xi\theta}, \dot{\kappa}_{\theta\xi}\}$$

$$\{\dot{N}^c\}=\{\dot{N}_\xi^c, \dot{N}_\theta^c, \dot{N}_{\xi\theta}^c, \dot{N}_{\theta\xi}^c, \dot{Q}_\xi^c, \dot{Q}_\theta^c\}^T=\int_{-h/2}^{h/2}[L_1]\{\dot{\sigma}^c\}d\zeta=\sum_{i=1}^{n}\int_{\zeta_{i-1}}^{\zeta_i}[L_1][D_i]\{\dot{\varepsilon}^c\}d\zeta$$

$$\{\dot{M}^c\}=\{\dot{M}_\xi^c, \dot{M}_\theta^c, \dot{M}_{\xi\theta}^c, \dot{M}_{\theta\xi}^c\}^T=\int_{-h/2}^{h/2}[L_2]\{\dot{\sigma}^c\}^*\zeta d\zeta=\sum_{i=1}^{n}\int_{\zeta_{i-1}}^{\zeta_i}[L_2][\bar{D}_i]\{\dot{\varepsilon}^c\}^*\zeta d\zeta \qquad [\bar{D}_i]=\frac{E_i}{1-\nu_i^2}\begin{bmatrix}1 & \nu_i & 0\\ \nu_i & 1 & 0\\ 0 & 0 & 1-\nu_i\end{bmatrix}$$

$$\{\dot{N}^T\}=\{\dot{N}_\xi^T, \dot{N}_\theta^T, \dot{N}_{\xi\theta}^T, \dot{N}_{\theta\xi}^T, \dot{Q}_\xi^T, \dot{Q}_\theta^T\}^T=\int_{-h/2}^{h/2}[L_1]\{\dot{\sigma}^T\}d\zeta=\sum_{i=1}^{n}\int_{\zeta_{i-1}}^{\zeta_i}[L_1][D_i]\{\dot{\varepsilon}^T\}d\zeta$$

$$\{\dot{M}^T\}=\{\dot{M}_\xi^T, \dot{M}_\theta^T, \dot{M}_{\xi\theta}^T, \dot{M}_{\theta\xi}^T\}^T=\int_{-h/2}^{h/2}[L_2]\{\dot{\sigma}^T\}^*\zeta d\zeta=\sum_{i=1}^{n}\int_{\zeta_{i-1}}^{\zeta_i}[L_2][\bar{D}_i]\{\dot{\varepsilon}^T\}^*\zeta d\zeta \quad (\{a_1, a_2, a_3, a_4, a_5\}^T)^*=\{a_1, a_2, a_3\}^T$$

$$[A]=\sum_{i=1}^{n}[E_i][A_i] \qquad [E_i]=\frac{E_i}{1-\nu_i^2}[1, 1, 1-\nu_i, 1-\nu_i, 1-\nu_i, 1-\nu_i, 1, 1, 1-\nu_i, 1-\nu_i]$$

$$[A_i]=\begin{bmatrix}
\zeta_{1,i}^*, & \nu_i\zeta_{1,i}, & 0, & 0, & 0, & \zeta_{2,i}^*, & \nu_i\zeta_{2,i}, & 0, & 0, & 0\\
\nu_i\zeta_{1,i}, & \zeta_{1,i}^{**}, & 0, & 0, & 0, & \nu_i\zeta_{2,i}, & \zeta_{2,i}^{**}, & 0, & 0, & 0\\
0, & 0, & \frac{1}{2}(\zeta_{1,i}^*+\zeta_{1,i}), & 0, & 0, & 0, & 0, & \zeta_{2,i}^*, & \zeta_{2,i}, & (a)\\
0, & 0, & \frac{1}{2}(\zeta_{1,i}^{**}+\zeta_{1,\zeta}), & 0, & 0, & 0, & 0, & \zeta_{2,i}, & \zeta_{2,i}^{**}, & (b)\\
0, & 0, & 0, & c\,\zeta_{1,i}^*, & 0, & 0, & 0, & 0, & 0, & 0\\
0, & 0, & 0, & 0, & \bar{c}\,\zeta_{1,i}^{**}, & 0, & 0, & 0, & 0, & 0\\
\zeta_{2,i}^*, & \nu_i\zeta_{2,i}, & 0, & 0, & 0, & \zeta_{3,i}^*, & \nu_i\zeta_{3,i}, & 0, & 0, & 0\\
\nu i\zeta_{2,i}, & \zeta_{2,i}^{**}, & 0, & 0, & 0, & \nu_i\zeta_{3,i}, & \zeta_{3,i}^{**}, & 0, & 0, & 0\\
0, & 0, & \frac{1}{2}(\zeta_{2,i}^*+\zeta_{2,i}), & 0, & 0, & 0, & 0, & \zeta_{3,i}^*, & \zeta_{3,i}, & (c)\\
0, & 0, & \frac{1}{2}(\zeta_{2,i}^{*}{}^*+\zeta_{2.i}), & 0, & 0, & 0, & 0, & \zeta_{3,i}, & \zeta_{3,i}^{**}, & (d)
\end{bmatrix} \qquad (17)$$

where

$$(\mathrm{a}) = \frac{1}{2}(\zeta^*_{1,i} - \zeta_{1,i}) + \frac{\omega_\xi}{a}\zeta^*_{2,i} - \frac{\omega_\theta}{a}\zeta_{2,i},\ (\mathrm{b}) = \frac{1}{2}(\zeta_{1,i} - \zeta^{**}_{1,i}) + \frac{\omega_\xi}{a}\zeta_{2,i} - \frac{\omega_\theta}{a}\zeta^{**}_{2,i}$$

$$(\mathrm{c}) = \frac{1}{2}(\zeta^*_{2,i} - \zeta_{2,i}) + \frac{\omega_\xi}{a}\zeta^*_{3,i} - \frac{\omega_\theta}{a}\zeta_{3,i},\ (\mathrm{d}) = \frac{1}{2}(\zeta_{2,i} - \zeta^{**}_{2,i}) + \frac{\omega_\xi}{a}\zeta_{3,i} - \frac{\omega_\theta}{a}\zeta^{**}_{3,i}$$

$$\zeta_{n,i} = \frac{1}{n}(\zeta_i^n - \zeta_{i-1}^n) \quad (n=1, 2, \cdots, 5)$$

$$(n=1,\ 2,\ 3)$$

$$\zeta^*_{n,i} = \zeta_{n,1} - \frac{1}{a}(\omega_\xi - \omega_\theta)\zeta_{n+1,i} + \frac{\omega_\xi}{a^2}(\omega_\xi - \omega_\theta)\zeta_{n+2,i},\quad \zeta^{**}_{n,i} = \zeta_{n,i} - \frac{1}{a}(\omega_\theta - \omega_\xi)\zeta_{n+1,i} + \frac{\omega_\theta}{a^2}(\omega_\theta - \omega_\xi)\zeta_{n+2,i}$$

and $c$ and $\bar{c}$ are the shear correction factors.

If $n = 1$ is selected in the Eqs.(16),(17), they coincide with the equations for the ordinary single layered shells by Takezono and Uchida (1984).

A complete set of field equations for 55 independent variables: $\dot{N}_\xi, \dot{N}_\theta, \dot{N}_{\xi\theta}, \dot{N}_{\theta\xi}, \dot{M}_\xi, \dot{M}_\theta, \dot{M}_{\xi\theta}, \dot{M}_{\theta\xi}, \dot{Q}_\xi, \dot{Q}_\theta, \dot{N}^c_\xi, \dot{N}^c_\theta, \dot{N}^c_{\xi\theta}, \dot{N}^c_{\theta\xi}, \dot{M}^c_\xi, \dot{M}^c_\theta, \dot{M}^c_{\xi\theta}, \dot{M}^c_{\theta\xi}, \dot{Q}^c_\xi, Q^c_\theta, \dot{N}^T_\xi, \dot{N}^T_\theta, \dot{M}^T_\xi, \dot{M}^T_\theta, \dot{\sigma}_\xi, \dot{\sigma}_\theta, \dot{\sigma}_{\xi\theta}, \dot{\sigma}_{\xi\zeta}, \dot{\sigma}_{\theta\zeta}, \dot{\sigma}^c_\xi, \dot{\sigma}^c_\theta, \dot{\sigma}^c_{\xi\theta}, \dot{\sigma}^c_{\xi\zeta}, \dot{\sigma}^c_{\theta\zeta}, \dot{\sigma}^T, \dot{\varepsilon}^c_\xi, \dot{\varepsilon}^c_\theta, \dot{\varepsilon}^c_{\xi\theta}, \dot{\varepsilon}^c_{\xi\zeta}, \dot{\varepsilon}^c_{\theta\zeta}, \dot{U}_\xi, \dot{U}_\theta, \dot{U}_\zeta, \dot{\Phi}_\xi, \dot{\Phi}_\theta, \dot{\Phi}_n, \dot{\varepsilon}_{\xi m}, \dot{\varepsilon}_{\theta m}, \dot{\varepsilon}_{\xi\theta m}, \dot{\varepsilon}_{\xi\zeta m}, \dot{\varepsilon}_{\theta\zeta m}, \dot{\kappa}_\xi, \dot{\kappa}_\theta, \dot{\kappa}_{\xi\theta}, \dot{\kappa}_{\theta\xi}$ is now given by 55 equations, (2),(3)-(5),(11),(14),(16),(17).

## 3 NONDIMENSIONAL EQUATIONS

In order to analyze the problem of the shells under arbitrary unsymmetrical loads, the loads $\dot{P}_\xi, \dot{P}_\theta, \dot{P}_\zeta, \dot{T}_e$ and the 50 independent variables except the creep strain rates $\dot{\varepsilon}^c_\xi, \dot{\varepsilon}^c_\theta, \dot{\varepsilon}^c_{\xi\theta}, \dot{\varepsilon}^c_{\xi\zeta}, \dot{\varepsilon}^c_{\theta\zeta}$ are expanded into Fourier series.

Substituting these into the above fundamental equations, the equations among the Fourier coefficients relating to the variables are obtained. Eliminating appropriately the coefficients, the resultant set for the displacement rates $\dot{u}_\xi, \dot{u}_\theta, \dot{u}_\zeta$ and the rotation rates $\dot{\varphi}_\xi, \dot{\varphi}_\theta$ then can be derived as follows:

$$\left.\begin{aligned}
&a_1\dot{u}''_\xi + a_2\dot{u}'_\xi + a_3\dot{u}_\xi + a_4\dot{u}'_\theta + a_5\dot{u}_\theta + a_6\dot{u}'_\zeta + a_7\dot{u}_\zeta + a_8\dot{\varphi}''_\xi + a_9\dot{\varphi}'_\xi + a_{10}\dot{\varphi}_\xi + a_{11}\dot{\varphi}'_\theta + a_{12}\dot{\varphi}_\theta = c_1\\
&a_{13}\dot{u}'_\xi + a_{14}\dot{u}_\xi + a_{15}\dot{u}''_\theta + a_{16}\dot{u}'_\theta + a_{17}\dot{u}_\theta + a_{18}\dot{u}_\zeta + a_{19}\dot{\varphi}'_\xi + a_{20}\dot{\varphi}_\xi + a_{21}\dot{\varphi}''_\theta + a_{22}\dot{\varphi}'_\theta + a_{23}\dot{\varphi}_\theta = c_2\\
&a_{24}\dot{u}'_\xi + a_{25}\dot{u}_\xi + a_{26}\dot{u}_\theta + a_{27}\dot{u}''_\zeta + a_{28}\dot{u}'_\zeta + a_{29}\dot{u}_\zeta + a_{30}\dot{\varphi}'_\xi + a_{31}\dot{\varphi}_\xi + a_{32}\dot{\varphi}_\theta = c_3\\
&a_{33}\dot{u}''_\xi + a_{34}\dot{u}'_\xi + a_{35}\dot{u}_\xi + a_{36}\dot{u}'_\theta + a_{37}\dot{u}_\theta + a_{38}\dot{u}'_\zeta + a_{39}\dot{u}_\zeta + a_{40}\dot{\varphi}''_\xi + a_{41}\dot{\varphi}'_\xi + a_{42}\dot{\varphi}_\xi + a_{43}\dot{\varphi}'_\theta + a_{44}\dot{\varphi}_\theta = c_4\\
&a_{45}\dot{u}'_\xi + a_{46}\dot{u}_\xi + a_{47}\dot{u}''_\theta + a_{48}\dot{u}'_\theta + a_{49}\dot{u}_\theta + a_{50}\dot{u}_\zeta + a_{51}\dot{\varphi}'_\xi + a_{52}\dot{\varphi}_\xi + a_{53}\dot{\varphi}''_\theta + a_{54}\dot{\varphi}'_\theta + a_{55}\dot{\varphi}_\theta = c_5
\end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (18)$$

where the superscript (n) on Fourier coefficients will be omitted for convenience. $a_1 \sim a_{55}$ are the coefficients determined from the shell geometries and the elastic constants, $E_i, \nu_i$. $c_1 \sim c_5$ are constants determined from the distributed loads and the internal forces due to thermal load and creep in addition to the shell geometries.

The rates of the internal forces related to the creep and the temperature rise are given as follows:

$$\left.\begin{aligned}
&\sigma_0 h_0 \sum_{n=0}^{\infty}[A_n]\{\dot{n}^{c(n)}_\xi, \dot{n}^{c(n)}_\theta, \dot{n}^{c(n)}_{\xi\theta}, \dot{n}^{c(n)}_{\theta\xi}, q^{c(n)}_\xi, \dot{q}^{c(n)}_\theta\}^T = \sum_{i=1}^{n}\int_{\zeta_{i-1}}^{\zeta_i}[L_1][D_i]\{\dot{\varepsilon}^c\}d\zeta\\
&\frac{\sigma_0 h_0^3}{a}\sum_{n=0}^{\infty}[B_n]\{\dot{m}^{c(n)}_\xi, \dot{m}^{c(n)}_\theta, \dot{m}^{c(n)}_{\xi\theta}, \dot{m}^{c(n)}_{\theta\xi}\}^T = \sum_{i=1}^{n}\int_{\zeta_{i-1}}^{\zeta_i}[L_2][\bar{D}_i]\{\dot{\varepsilon}^c\}^*\zeta d\zeta\\
&[A_n] = [\cos n\theta, \cos n\theta, \sin n\theta, \sin n\theta, \cos n\theta, \sin n\theta] \qquad [B_n] = [\cos n\theta, \cos n\theta, \sin n\theta, \sin n\theta]
\end{aligned}\right\} \quad \cdots\cdots (19)$$

The creep strain rates in right hand sides of Eqs.(19) are related to the stresses by Eqs.(11). The integrations are carried out numerically by the use of Sympson's 1/3 rule.

## 4 NUMERICAL METHOD

The incremental solutions at any calculation stage are obtained from Eqs.(18) with appropriate boundary conditions. The solutions at any time are given by summation of the incremental values. The equations (18) are numerically solved by the finite difference method as described in the paper by Takezono and Uchida (1984).

## 5 NUMERICAL EXAMPLE

As a numerical example of creep deformations of multi-layered shells of revolution, a simply supported two layered cylindrical shell composed of stainless steel SUS316 and mild steel SB42 subjected to locally distributed loads is treated (Fig.3).

The material constants of stainless steel SUS316 and mild steel SB42 have been determined from experiments at 600°C as follows (Fig.4):

SUS 316 : $E_1=164$ GPa, $\nu_1=0.3$ SB 42 : $E_2=155$ GPa, $\nu_2=0.3$

$m_1=6.69$, $A'_1=A_1e^{-\beta_1/T}$ $m_2=5.53$, $A'_2=A_2e^{-\beta_2/T}$

$=2.8\times10^{-19}$ $\mathrm{MPa}^{-m_1}$ $=2.9\times10^{-13}$ $\mathrm{MPa}^{-m_2}$

$c_1=0.51\ \mathrm{h}^{-1}$, $q=0.9\ \mathrm{h}^{-1}$ $c_2=0.5\ \mathrm{h}^{-1}$, $q_2=0.39\ \mathrm{h}^{-1}$

The calculations for the shell have been carried out under assumption of stress free at 600°C.

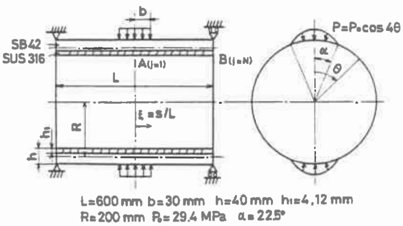


Fig.3 cylindrical shell.

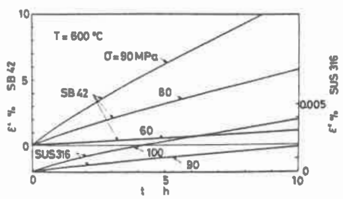
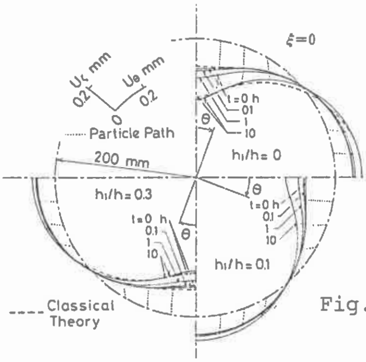


Fig.4 Creep curves.



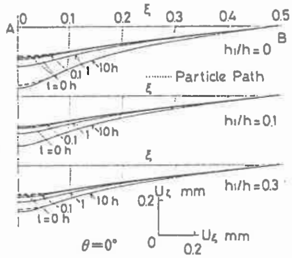


Fig.5 Deformations.

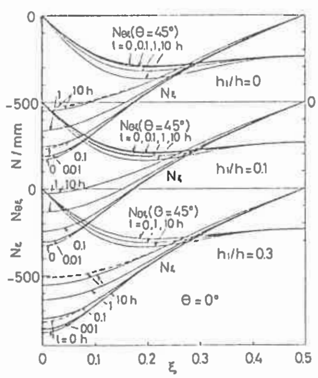


Fig.6 $N_\xi$, $N_{\theta\xi}$

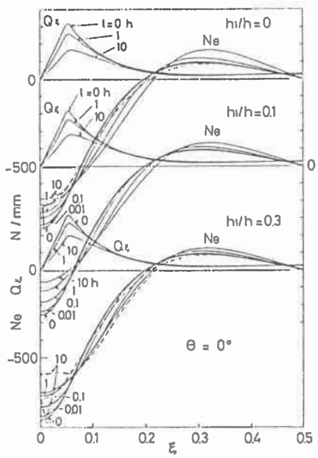


Fig.7 $N_\theta$, $Q_\xi$

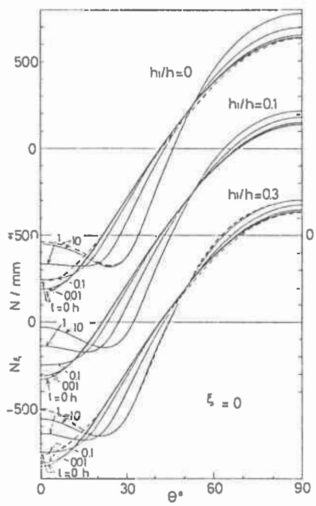


Fig.8 $N_\xi$

Some of the essential features of the solutions from the time hardening law are shown in Fig.5-13. The numerical computations have been carried out for three cases of ratio of thickness of stainless steel layer to shell thickness. It is found from the computations that the deformations and the stress distributions are significantly varied depending on the ratio.

The numerical calculations for these examples need about 10-15 minutes in FACOM M-382S+VP100.

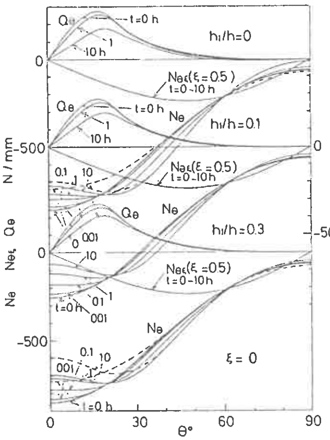



Fig.9 $N_\theta$, $N_{\theta\xi}$, $Q_\theta$

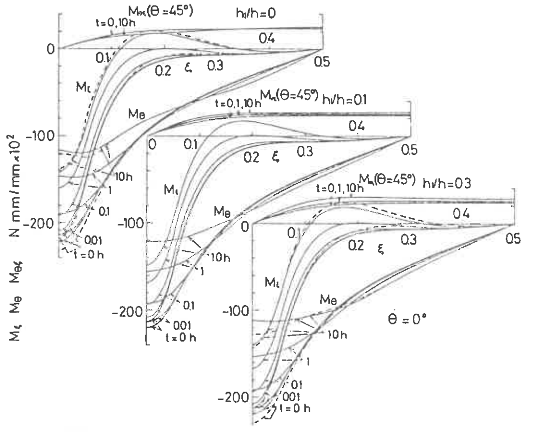



Fig.10 $M_\xi$, $M_\theta$, $M_{\theta\xi}$

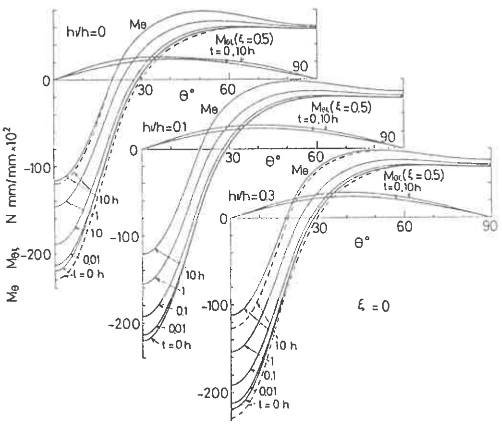



Fig.11 $M_\theta$, $M_{\theta\xi}$

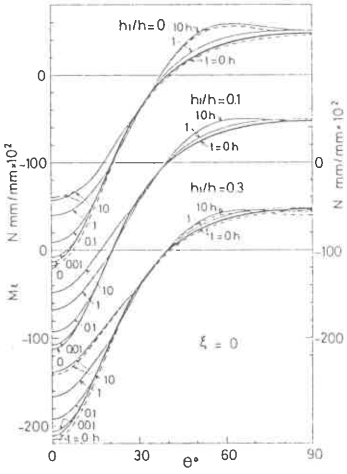



Fig.12 $M_\xi$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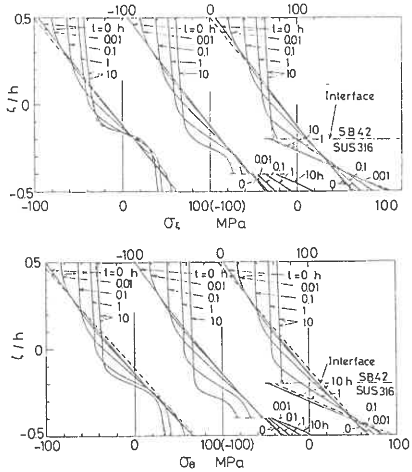


Fig.13 $\dot{\sigma}_\xi$, $\dot{\sigma}_\theta$ $(\xi=0, \theta=0^\circ)$