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Nonlinear Dynamic Responses of Nuclear Reactor Building Under Earthquake Excitations

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Abstract

Linear-elastic model is primary adopted for the dynamic analysis of nuclear engineering under earthquake excitations. And the responses of the structure under SL-1 earthquake are usually apparently smaller than that under SL-2 earthquake. If nonlinear model is used, more reasonable results can be obtained. In this condition, the responses of the structure under SL-1 earthquake will be close to and even larger than that under SL-2 earthquake. Using nuclear reactor building as an example, this paper introduces the features of linear and nonlinear dynamic calculations, and the responses under linear-elastic and elastic-plastic models are calculated and compared.

KEY WORD : Reactor building, Earthquake, Nonlinear Dynamic reponse

INTRODUCTION

In nuclear engineering, linear-elastic analysis methods are primary adopted for the dynamic calculations of the structures under earthquake excitations. But under dynamic loadings, the structure material often shows apparent nonlinear characteristics, which make the behavior of the structure quite different from that under static loadings. Therefore, nuclear codes in many countries stipulate that the dynamic calculations under strong earthquake must consider such nonlinear characteristics. Chinese nuclear codes also regulate that when the structures behave apparent non-linearity, this non-linearity must be include in the calculations.[1,2] However, in the real engineering project, it is usually very difficult to obtain the nonlinear characteristics of some structures (such as the reactor building) under strong earthquakes. And their seismic analysis can only be carried out approximately by linear models.

The example used in this paper is a nuclear reactor building. Its producer carries out a lot of experiments under strong earthquakes and obtains the nonlinear restoring force characteristics of the material. Therefore, elastic-plastic models can be used in the dynamic calculations. This paper will introduce the seismic analysis of this building, the different features of linear and nonlinear dynamic calculations are emphasized, and the responses of the structure are compared.

SEISMIC ANALYSIS OF REACTOR BUILDING

Analysis model and earthquake excitations

The building has different structure in East-West cross-section and North-South cross-section. Therefore, the analysis model is different in the two cross-sections. Only the East-West cross-section analysis model is introduced in this paper.

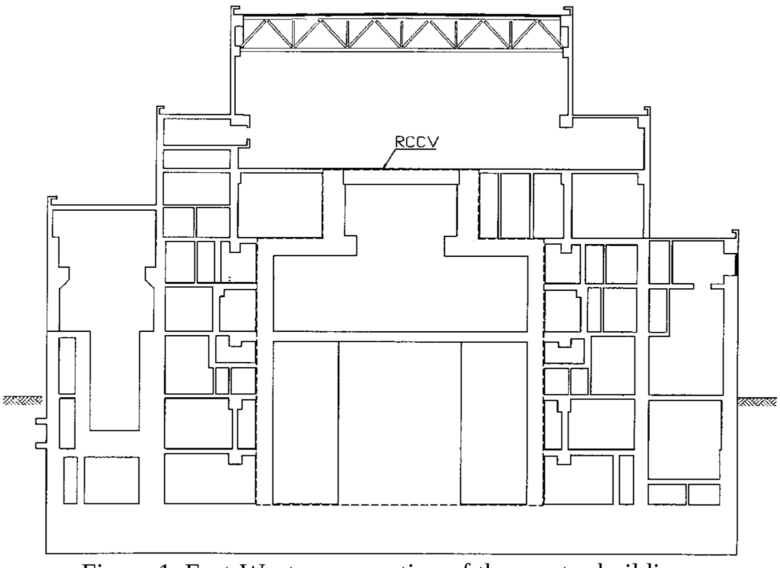

Figure 1. East-West cross-section of the reactor building

The structure of the building in East-West cross-section is shown in Figure 1. The figure shows that this is a Soil-Structure Interaction (SSI) problem. Strictly saying, the filtering effects of the soil to the earthquake waves and the nonlinear effects of the soil to the dynamic characteristics of the structure must be taken into consideration. On the other hand, the feedback effects of the structure to the soil properties must also be considered. However, base on the regulations of Chinese nuclear code, the SSI effects can be neglected if the mean shear wave velocity of the foundation soil exceeds 1100m/s.[1] In this condition, the foundation soil can be treated as rock, which is a perfect elastic body. Transmitting in such a body, the earthquake waves dissipate very little and the SSI effects is very weak. In this example, the shear wave velocities of the foundation soil all exceed 1100m/s. Therefore SSI effect can be neglected and the structure can be analyzed without considering the foundation soil.

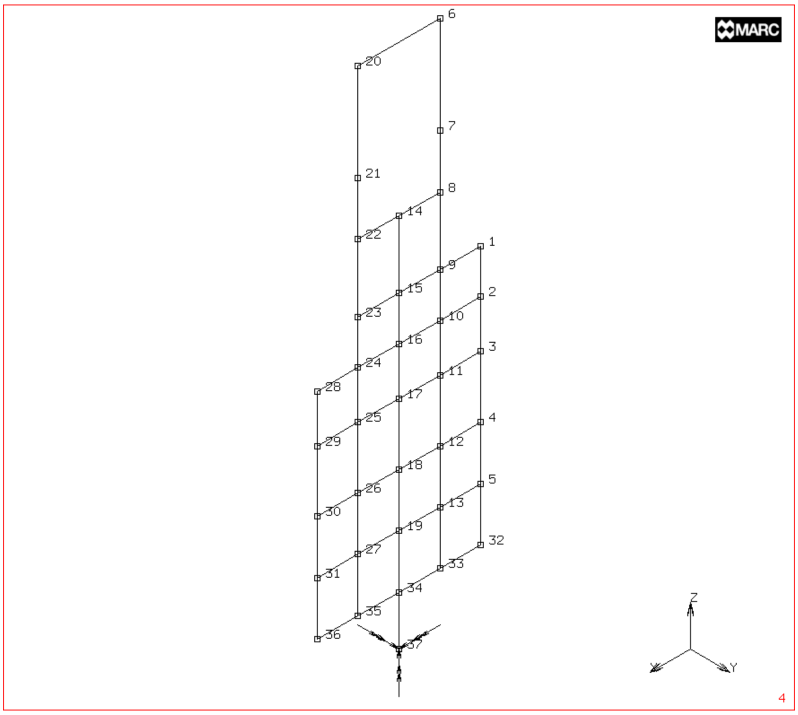

Figure 2. FEM model of the building in East-West cross-section

The FEM model of the structure in East-West cross-section is shown in Figure 2. In this figure, X axis represents East-West direction, Y axis represents North-South direction, and Z axis represents vertical direction. The model is established by 2-node beam elements, where the vertical bundles represent various part of the building and the horizontal ones are the rigid connection elements. Because there are no requirements in Chinese nuclear codes to make difference between horizontal and vertical analysis model, both the horizontal and vertical analysis is carried out based on the model in Figure 2. [1,2]

The FEM program used in this example is MARC/MENTAT.[3]

The earthquake waves are the artificial acceleration time-histories defined on the rock of the building. Based on the regulations of Chinese nuclear codes, two levels of earthquake motions — Operational safety ground motion (SL-1) and Ultimate safety ground motion (SL-2) — must be considered and the structure must be analyzed in three orthogonal directions. [1,2] Therefore, there are totally six input waves in this example, representing the earthquake motions of two levels in X, Y, and Z directions. These six waves satisfy the statistically independent requirement.[4] For the SL-1 earthquake, the maximum horizontal acceleration is $3.91 m/s^2$, while the vertical is $2.61 m/s^2$. For the SL-2 earthquake, the maximum horizontal acceleration is $4.73 m/s^2$, while the vertical is $3.16 m/s^2$. Each wave has the time interval of 0.01s and lasts for 35.9s.

Eigenvalue Analysis

The basic component of dynamic analysis is eigenvalue analysis. Determination of the fundamental frequencies and eigenvectors of the structure can be used to characterize its dynamic behavior.

Considering a linear un-damped structure without dynamic load, the equation of motion is: [5]

$$[M]\{\ddot{u}\}+[K]\{u\}=0 \tag{1}$$

Its eigenvalue equation is:

$$\left([K] - \omega^2 [M] \right) \{u\} = 0 \tag{2}$$

The nonzero solution of the above equation will result in pairs of ω_i and $\{u\}_i$, which are the angular frequency and the corresponding eigenvector respectively.

According to the nuclear code, the highest interested frequency of a structure is 33Hz under earthquake motions. [4] Therefore eigenvalue analysis is carried out on the condition that the bottom of the model is totally fixed and the cut-off natural frequency is 33Hz. Analysis results show that there are thirteen natural frequencies within 33Hz. The results are listed in Table 1.

Table 1. Eigenvalues and Eigenvectors of the Model

Num	Freq. (Hz)	Description of the eigenvectors
1	6.40	Sway in Y-Z plan, first order of beam eigenvector
2	6.42	Sway in X-Z plan, first order of beam eigenvector
3	10.55	Expand and shrink along Z-axis, first order of beam eigenvector
4	12.52	Sway in Y-Z plan, second order of beam eigenvector
5	12.53	Sway in X-Z plan, second order of beam eigenvector
6	19.08	Sway in Y-Z plan, third order of beam eigenvector
7	19.10	Sway in X-Z plan, third order of beam eigenvector
8	20.43	Expand and shrink along Z-axis, second order of beam eigenvector
9	25.54	Sway in Y-Z plan, third order of beam eigenvector. Local vibration at the top.
10	25.65	Sway in X-Z plan, third order of beam eigenvector. Local vibration at the top.
11	27.29	Sway in Y-Z plan, third order of beam eigenvector. Local vibration at the top.
12	27.88	Sway in X-Z plan, third order of beam eigenvector. Local vibration at the top.
13	29.88	Expand and shrink along Z-axis, third order of beam eigenvector

The results show that the numbers of the lumped mass of the model are greater than twice the number of the natural frequency, which satisfy the requirement of the nuclear code.[1] The eigenvectors shows that the vertical bunches of the beam element will vibrate as the cantilever beam, while the top part will superimpose some local vibration in the high natural frequencies.

After the eigenvalue analysis, time-history analysis of the model subjected to earthquake excitations is performed.

Seismic Analysis under SL-1 Earthquake

For the SL-1 earthquake motions, the structure material can be treated as elastic and the linear-elastic model can be adopted for the calculations. The analysis method used here is modal superposition method.

1) Theoretical Background

Considering a multi-degree system under ground motion, its dynamic equation is: [5]

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -[M]\{\ddot{u}_g\} \tag{3}$$

where $\{\ddot{u}_g\}$ is the acceleration vector of the ground motion.

Assuming that the system is linear elastic and the damping matrix $[C]$ is orthogonal, the eigenvalue matrix of the system can be used as the transition matrix to transfer the above equation into a set of non-coupled differential equations. After solving each of the differential equations, the results are superimposed and the dynamic response of the system can be obtained.

This analysis procedure, which is called modal superposition method, is more effective than the time integration method because equation (3) is a large scale coupled differential equations. After transferring, the equations are decoupled and their scale is reduced as the number of natural frequencies within the cut-off frequency.

If the error in spatial discretization is not considered, the only error of the method is due to the error in solving the differential equations base on a given load history. If the load change is linear within an increment, the solving will be accurate.

This method can only be applied successfully to the structure excited in the lower frequency range because the determination of higher order frequencies is usually less accurate. As the earthquake vibration consists mainly of lower frequency motions, this method is just suitable for seismic analysis.

Because the damping matrix $[C]$ is assumed to be orthogonal, each natural vibration mode can be given a specific viscous damping factor.

2) Dynamic calculations

The aforementioned modal superposition method is adopted for the model in Figure 2 based the results in Table 1. The excitation is input at the bottom of the model. The viscous damping factor of 5% is chosen for each natural vibration mode of the model.[1] The calculation time step is 0.01s, which is the same with the time step of the change of the excitation. Totally 35.9 seconds are computed, and the total increment is 3590 steps. The results are shown in Table 2.

Seismic Analysis under SL-2 Earthquake

SL-2 Earthquake belongs to strong earthquake, and the structure material could go into the damage stage. Therefore, nonlinear model must be used for the calculation. The analysis method used here is one of the direct integration methods, which is called Newmark method.

1) Theoretical Background

The Equation (3) can be integrated directly in time domain. This is called direct integration method. Many approaches have been derived, and the Newmark method is the most commonly used. It assumes that within a specific time interval ($\Delta t = t^{n+1} - t^{n}$), the variations of displacements, velocities, and accelerations are all linear. Therefore, Equation (3) can be written in an incremental form: [5]

$$[M]\{\Delta \ddot{u}\}^{n} + [C]\{\Delta \dot{u}\}^{n} + [K]\{\Delta u\}^{n} = -[M]\{\Delta \ddot{u}_{g}\}^{n} \tag{4}$$

The following assumptions are used:

$$\{u\}^{n+1} = \{u\}^{n} + \Delta t\{\dot{u}\}^{n} + \left(\frac{1}{2} - b\right)\Delta t^{2}\{\ddot{u}\}^{n} + b\Delta t^{2}\{\ddot{u}\}^{n+1} \tag{5}$$

$$\{\dot{u}\}^{n+1} = \{\dot{u}\}^{n} + (1-c)\Delta t\{\ddot{u}\}^{n} + c\Delta t\{\ddot{u}\}^{n+1} \tag{6}$$

where b and c are the parameters which determine the integration accuracy and stability of the algorithm.

Table 2. The maximum responses of the structure under SL-1 earthquake

Vertical bundles	Elevation, [mm]	Shear force, [$\times 10^4$ t]	Moment of bend, [$\times 10^5$ t·m]	Horizontal displacement, [mm]	Horizontal acceleration, [m/s^2]
OW-J	27500			7.10	12.11
	22100	1.951	2.406	6.00	10.44
	16300	2.712	2.922	4.70	8.75
	8800	3.221	3.606	2.90	6.68
	2300	4.212	5.882	1.40	4.35
	–4200	4.107	7.709	0.10	3.37
IW-H	54100			13.20	26.95
	42200	0.688	0.391	10.10	17.90
	35700	1.352	1.218	8.60	14.19
	27500	1.817	2.681	7.10	12.11
	22100	2.306	2.594	6.00	10.44
	16300	2.466	3.149	4.70	8.75
	8800	3.324	7.275	2.90	6.68
	2300	4.095	10.556	1.40	4.35
	–4200	4.731	14.208	0.10	3.37
RCCV	35700			8.60	14.19
	27500	1.769	0.032	7.10	12.11
	22100	1.905	0.626	6.00	10.44
	16300	2.050	0.699	4.70	8.75
	8800	2.324	0.866	2.90	6.68
	2300	2.136	1.129	1.40	4.35
	–4200	2.311	1.451	0.10	3.37

IW-B	54100			13.20	26.95
	42200	0.668	0.379	10.20	18.25
	35700	1.483	1.216	8.60	14.19
	27500	1.836	2.660	7.10	12.11
	22100	2.755	4.286	6.00	10.44
	16300	2.033	5.090	4.70	8.75
	8800	3.247	7.364	2.90	6.68
	2300	4.061	10.970	1.40	4.35
	–4200	4.665	14.644	0.10	3.37
OW-A	22100			6.00	10.44
	16300	2.692	3.294	4.70	8.75
	8800	3.868	4.041	2.90	6.68
	2300	4.063	5.556	1.40	4.35
	–4200	4.072	7.737	0.10	3.37
	-10200	19.887	60.759	0.00	3.37

Substitute the incremental form of Equation (5) and (6) into Equation (4), the incremental form of $\{\Delta u\}^n$, $\{\Delta \dot{u}\}^n$, and $\{\Delta \ddot{u}\}^n$ can then be calculated. Therefore, Equation (4) can be integrated from the beginning of the time and the response of the structure can be calculated at every time intervals.

If b=0.25 and c=0.5 is adopted, the Newmark method is unconditionally stable. If T is the period associated with the highest frequency of interested, choosing a time step of 0.1T will result in less than three percent error.

For the nonlinear problems, the dynamic equation can be written in the general form: [5]

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+\{F\}=-[M]\{\ddot{u}_g\} \tag{7}$$

Compared with Equation (3), the only difference is that the elastic force vector $[K]\{u\}$ is substituted by resuming force vector $\{F\}$.

If the linear assumption within Δt is held effective and a suitable Δt is chosen, the accumulation of the increments can give approximately correct results of the non-linear procedure. That means:

$$\{F\}^{n+1}=\{F\}^n+[\overline{K}]\{\Delta u\}^n \tag{8}$$

where $[\overline{K}]$ is the incremental stiffness matrix that changes with the deformation of the structure. Therefore the incremental form of Equation (7) can be written as:

$$[M]\{\Delta\ddot{u}\}^n+[C]\{\Delta\dot{u}\}^n+[\overline{K}]\{\Delta u\}^n=-[M]\{\Delta\ddot{u}_g\}^n \tag{9}$$

Equation (9) has the same form with Equation (4), therefore it can be calculated by Newmark method. The primary difference compared with the linear calculation is that the stiffness matrix is a non-linear function of the displacement. In order to improve the estimation of $\{u\}^{n+1}$, an iterative approach can be derived using the standard techniques for nonlinear equations. The most commonly used technique is modified Newton-Raphson scheme.

Most nonlinear problems have little effect on the higher frequency, hence, the stable time step suitable for the linear transient analysis does not need appreciable change. Also, if the time step is small enough, the accuracy can be guaranteed.

In the direct integration approach, damping is usually specified as Rayleigh damping, which has the following form: [5]

$$[C]=\alpha[M]+\beta[K] \tag{10}$$

namely, the damping matrix is given as a linear combination of the mass and stiffness matrix. Therefore the damping of the system can be determined by two factors α and β. These two factors can be determined by the damping factors of two natural frequencies: [5]

$$\alpha=\frac{2\omega_r\omega_s(\xi_s\omega_r-\xi_r\omega_s)}{(\omega_r^2-\omega_s^2)} \tag{11}$$

$$\beta = \frac{2(\xi_r\omega_r - \xi_s\omega_s)}{(\omega_r^2 - \omega_s^2)} \quad (12)$$

where ω_r , ω_s are the angular frequencies and ξ_r , ξ_s are the corresponding viscous damping factors.

2) Nonlinear Properties of the Structure

Based on the experiments under strong earthquakes, the nonlinear restoring force characteristics of the structure material, namely the $M-\phi$ relationship and $\tau-\gamma$ relationship, are obtained. The skeleton $M-\phi$ is a bilinear model, and the skeleton $\tau-\gamma$ is a trilinear model. In different places of the structure, the nonlinear characteristics are different.

The nonlinear properties of beam element in the MARC program has the following expression:[3]

$$\{DF\} = [D] \times \{DE\} \quad (13)$$

where $\{DF\} = \{\Delta F, \Delta Q_{xy}, \Delta Q_{yz}, \Delta M_x, \Delta M_y, \Delta T\}^T$ is the matrix of the increments of the generalized stress; $\{DE\} = \{\Delta\varepsilon, \Delta\gamma_{xy}, \Delta\gamma_{yz}, \Delta\phi_x, \Delta\phi_y, \Delta\psi\}^T$ is the matrix of the increments of the generalized strain; and $[D]$ is the matrix of cross-section stiffness properties which has the following expression:

$$[D] = \begin{bmatrix} E\cdot A & & & & & \\ & G\cdot A & & & & \\ & & G\cdot A & & & \\ & & & E\cdot I_x & & \\ & & & & E\cdot I_y & \\ & & & & & G\cdot J \end{bmatrix}$$

Based on the experimental data, the matrix $[D]$ can be expressed as the function of $\Delta\phi$ and $\Delta\gamma$.

The user subroutine UBEAM of the MARC program is used to carry out the calculation. After the increments of the generalized strain $\{DE\}$ are read in from the MARC program, the matrix $[D]$ is determined and the increments of the generalized stress $\{DF\}$ can then be calculated from Equation (13). Therefore the total generalized stress at time t^{n+1} can be calculated as:

$$\{F\}^{n+1} = \{F\}^n + \{DF\} \quad (14)$$

3) Dynamic calculations

The acceleration time-history of SL-2 earthquake is input at the bottom of the model and the Newmark method is used to carry out the nonlinear dynamic calculation. The parameters of b and c in Equation (5) and (6) are chosen as 0.25 and 0.5.

The viscous damping factor is chosen as 7%.[1] Based on Table 1 and Equation (11) and (12), the first two natural vibration modes are used to calculate the two factors in Rayleigh damping. The results are $\alpha = 3.503$ and $\beta = 0.001315$.

Table 3. The maximum responses of the structure under SL-2 earthquake

Vertical bundles	Elevation, [mm]	Shear force, [×104 t]	Moment of bend,[×105t·m]	Horizontal displacement, [mm]	Horizontal acceleration, [m/s^2]
OW-J	27500			7.30	11.87
	22100	1.929	2.317	6.10	10.40
	16300	2.595	2.885	4.90	9.02
	8800	2.916	3.675	3.10	7.35
	2300	3.936	5.886	1.50	5.45
	–4200	4.029	7.714	0.10	4.61
IW-H	54100			14.50	24.54
	42200	0.604	0.350	10.50	19.96
	35700	1.211	1.083	8.90	14.78
	27500	1.771	2.501	7.30	11.87

	22100	2.353	2.492	6.10	10.40
	16300	2.528	3.106	4.90	9.02
	8800	3.470	7.402	3.10	7.35
	2300	4.080	10.574	1.50	5.45
	–4200	4.929	14.230	0.10	4.61
RCCV	35700			8.90	14.78
	27500	1.729	0.029	7.30	11.87
	22100	1.893	0.603	6.10	10.40
	16300	2.061	0.690	4.90	9.02
	8800	2.504	0.882	3.10	7.35
	2300	2.259	1.130	1.50	5.45
	–4200	2.512	1.452	0.10	4.61
IW-B	54100			14.50	24.54
	42200	0.592	0.341	10.90	17.61
	35700	1.281	1.075	8.90	14.78
	27500	1.814	2.480	7.30	11.87
	22100	2.788	4.118	6.10	10.40
	16300	2.096	5.016	4.90	9.02
	8800	3.384	7.494	3.10	7.35
	2300	4.061	10.984	1.50	5.45
	–4200	4.764	14.680	0.10	4.61
OW-A	22100			6.10	10.40
	16300	2.716	3.254	4.90	9.02
	8800	3.422	4.120	3.10	7.35
	2300	3.774	5.562	1.50	5.45
	–4200	4.010	7.741	0.10	4.61
	-10200	22.456	61.046	0.00	4.61

Because the highest frequency of interested in seismic analysis is 33Hz, the time step that can guarantee a good accuracy is:

$$\Delta t = \frac{0.1}{33Hz} = 0.003\ s$$

A time step of 0.002s is chosen in the calculations. The Total computational time is still 35.9 seconds, so the total increment is 17950 steps. The results are shown in Table 3.

DISCUSSION

Comparing the results listed in Table 2 and 3, it can be found that the seismic forces of the structure under the SL-1 earthquake are no less than, in some part, even larger than that under the SL-2 earthquake. The same conclusion can be obtained for the accelerations of the structure. But the displacements of the structure under the SL-1 earthquake are a little bit smaller than that under the SL-2 earthquake. These are due to the nonlinear characteristics of the structure material under the strong earthquake. After the material going into the nonlinear stage under the external excitation, the structure stiffness will decrease. Therefore, even though the structure displacements increase, the internal forces of the structure will not increase with the displacements proportionally. And the accelerations of the structure also do not increase proportionally when the external excitation becomes stronger.

If linear-elastic model is adopted both for the SL-1 and SL-2 seismic analysis, the responses of the structure under the SL-1 earthquake are usually apparently smaller than that under the SL-2 earthquake. Compared with the linear-elastic model, the nonlinear model can obtain more actual results and can make the structural design more reasonable.

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