Nonlinear Analysis of Reinforced Concrete Members in Nuclear Power Plants

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Nonlinear analysis of reinforced concrete systems is of particular importance when such systems are sensitive to concrete cracking, such as a nuclear plant's response to different cases of loading. Analysis of such a system requires the consideration of several unique factors such as the non-homogeneity of the member cross sections, nonlinearity of material relationships, inelastic behavior of concrete requiring adequate yield and failure criteria, and the influence of progressive material cracking. Many methods are presently available for the nonlinear analysis of different reinforced concrete systems. However, most of these methods are available only on special programs or on programs with limited distribution, which restricts their use on a wide basis by practicing engineers.

The method is based on the concept of grid framework, and is applicable to the inelastic analysis of plane stress problems. However, it could easily be expanded to handle plane strain and three-dimensional problems.

The force deformation relation of the concrete elements can simulate cracking of concrete zones under tensile stresses, as well as the nonlinear behavior of those zones under compressive stresses. The steel reinforcement is simulated by one dimensional elements, which have a bilinear stress-strain relation. The solution of the total nonlinear system is performed using the well-known tangent stiffness approach. Comparisons are made between the results of this method and test results with a good agreement. Additionally, results of the method will be compared with the results of a finite element approach.

1. Introduction

Nonlinear analysis of reinforced concrete members is an important aspect of the design of nuclear power plants, especially when these members are located in a sensitive area from a safety point of view. Numerous nonlinear reinforced concrete models have been investigated. Some of these models use finite element methods of analysis [1, 2, 3 and 4]. Most of these methods require special analytical structural elements, as well as special treatment of the material constitutive equations, which limits the use of them by practicing Engineers. This paper presents a simple, accurate and economical method of analyzing reinforced concrete members. The method is based on the extension of the plane framework method [5, 6 and 7] to reinforced concrete members.

2. Theoretical Background

2.1 Mathematical Model

The mathematical model of this paper is shown in Figure 1. The concrete body is represented by a set of truss elements. The aspect ratio of the panels of this model were recommended to be close to unity [5]. The areas of these truss members are [5 and 7];

$$A_1 = \text{Area of diagonal members} = \frac{0.75}{\sqrt{2}}\, ah \qquad (1)$$

$$A_2 = \text{Area of horizontal and vertical members} = 0.75\ ah \qquad (2)$$

where

a = Length of the square panel

h = Thickness of the reinforced concrete member.

The steel reinforcement can be modeled as a truss element in parallel to the concrete elements at any required location, as shown in Figure 1. This method makes it very easy to model any reinforcement, especially stirrups, without any additional cost or effort.

2.2 Force-Deformation Relationships of Concrete

There are many well-known nonlinear stress-strain relationships of concrete members [1, 2 and 4]. However, the uniaxial stress-strain relationship of concrete suggested by CEB-FIP International Recommendations [8] will be used in this work due to its simplicity. The relationship is given by;

$$\sigma = kf_c' \left[2\left(\frac{\epsilon}{0.002}\right) - \left(\frac{\epsilon}{0.002}\right)^2\right] \text{ for } \epsilon \leqslant 0.002 \qquad (3)$$

$$= kf_c' \quad \text{for } 0.002 \leqslant \epsilon \leqslant 0.0035 \qquad (4)$$

$$= 0 \quad \text{for } \epsilon \geqslant 0.0035 \qquad (5)$$

where

σ = Normal stress in concrete

ϵ = Normal strain in concrete.

f_c' = Strength of a standard cylindrical specimen in uniaxial compression.

k = 1.0, for the purpose of this study.

Equations (3) to (5) can be used to evaluate the force, F, in any member;

$$F = \sigma A \tag{6}$$

The strain, ϵ, of any member can be evaluated by;

$$\epsilon = (U_1 - U_2)/L \tag{7}$$

U_1, U_2 = Displacements at the two ends of the member.

L = Member length.

Equations (3) through (7) can be expressed for each concrete member in an incremental form as;

$$\Delta F = K\,(\Delta U_1 - \Delta U_2) \tag{8}$$

where

ΔF = Incremental force.

$\Delta U_1, \Delta U_2$ = Incremental displacements.

K = Material and geometrical constant.

The tensile stress-strain equations of this model are;

$$\sigma = k f_c' \left[2 \left(\frac{\epsilon}{0.002}\right) - \left(\frac{\epsilon}{0.002}\right)^2 \right] \quad \text{for } \sigma \leq f_c' \alpha \tag{9}$$

$$= 0 \qquad \text{for} \qquad \sigma > f_c' \alpha \tag{10}$$

α = Ratio of axial tensile strength to axial compressive strength.

The tangent or secant stiffness matrix of the concrete member can then be evaluated using the method of reference [7].

2.3 Force-Deformation Relationship of Steel

The nonlinear force-deformation relationship of the steel element in this model is shown in Figure 2. This constitutive model is assumed to be bilinear, for simplicity. The member tangent and secant stiffness matrices can be formed using the method of reference [7].

2.4 Methods of Analysis

Two different methods of analyzing this type of problem were discussed in reference [7]. They are the tangent method and the accelerated method. However, the accelerated method experienced numerical difficulties, and did not converge. The tangent method of analysis yielded satisfactory results when the incremental length was sufficiently small. It was decided that the tangent method of analysis be used for this work.

2.5 State of Stresses

Reference [7] presented a method of evaluating the state of stresses at any point inside a plain strain continuum. This method is applicable to the evaluation of stresses in the concrete continuum.

Stresses in steel can be evaluated using the computed displacements and Figure 2.

3. Example Problem

The mathematical model of this work was used tosolve the reinforced concrete beam of Figure 3. The properties of this beam are

$f_c' = 4000.0$ PSI

$\alpha = .075$

F_y = Yield stress of steel

$= 40000.0$ PSI

$E_1 = 29 \times 10^6$ PSI

$E_2 = 2.0 \times 10^6$ PSI

A_s = Area of steel reinforcement

$= 4.0 \text{ in}^2$

The same beam problem was solved using finite element techniques, reference 2 . The load-displacement relation at midspan resulting from the two methods, as well as the test results, are shown in Figure 4. The agreement between the three curves is satisfactory.

4. Conclusions

A simple, accurate and economical method of the analysis of reinforced concrete members is presented. Most computer programs that have a truss element can be modified, with a small effort and cost, to implement this method. The comparison between the results of this method, a finite element method and test results show satisfactory agreement.

5. Acknowledgment

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6. References

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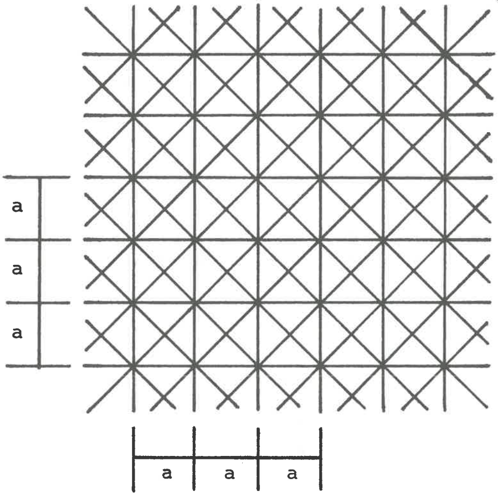
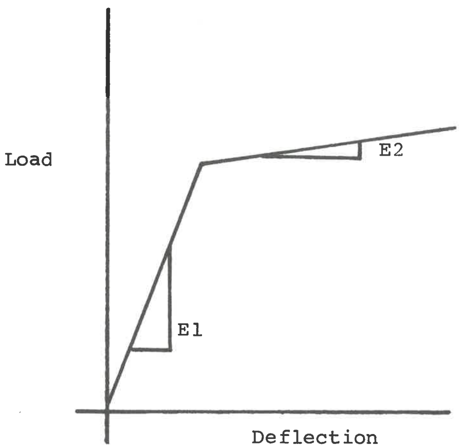
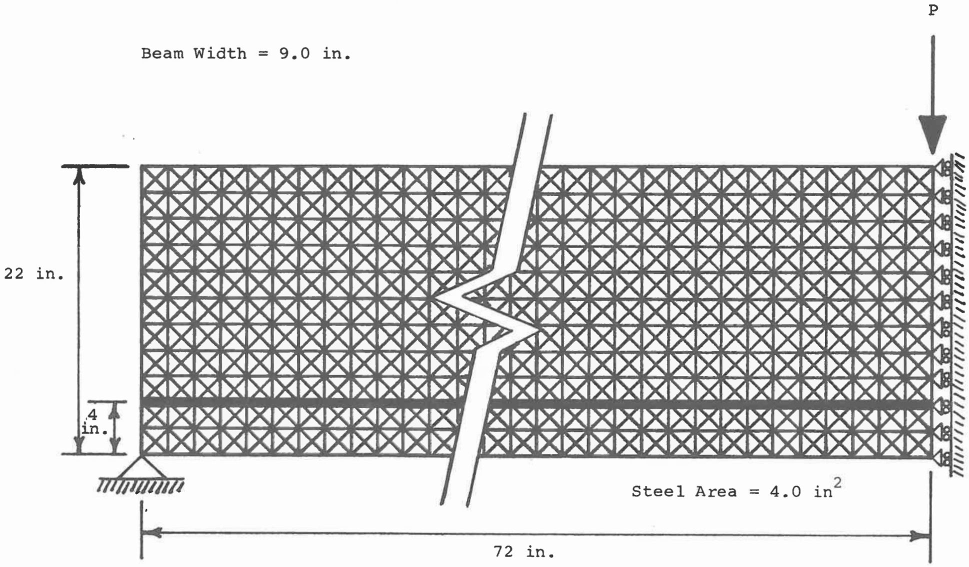


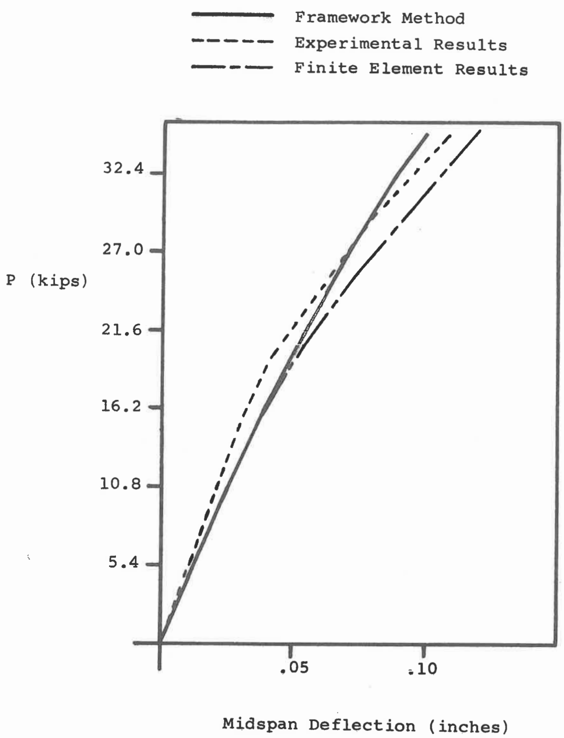
1 Mathematical Model



2 Force-Deformation Relation of Steel



3 Model of Simple Reinforced Concrete Beam



4 Comparison of Results