

THERMORHEOLOGICAL BEHAVIOUR AND COUPLING PROBLEM OF STRUCTURAL MATERIALS

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SUMMARY

The rheological behaviour of structural materials is considerably stimulated in the presence of a temperature field. This influence is manifested by the changes in their thermodynamic characteristics. Our aim is to discover and discuss the problem in connection with thermo-mechanical phenomena in nonlinear bodies of a certain class. These are assumed to be isotropic, homogeneous and incompressible and undergoing small deformations. In particular, we are interested in the analysis of the form of free energy and entropy production for such bodies. Therefore, two alternatives of substantial behaviour are investigated. One is concerned with comparatively small influence of dissipative properties on the amount of internal energy while the other one related to the deformation state characterized by almost total dissipation process.

The above problems mentioned are discussed in connection with the meaning of thermo-mechanical coupling. A double significance may be prescribed to the latter. One follows from the appearance of heat fluxes due to deformation changes and the other is concerned with total or specified responses of the material.

The corresponding constitutive equation for the body considered is derived by using the generalized superposition principle. On the basis of the functional obtained the form of dissipative function is obtained. It follows directly from superposing energetic phenomena of dissipative character. As both the procedures are effected at the differential level, the resulting integral forms are obtained by assuming the integrability conditions to be valid. The results are discussed on the basis of premises which follow from the law of thermodynamics of irreversible processes. It is concluded that dissipative ability of the material may constitute a certain measure of its actual stability. In particular, the amount of dissipated energy may indicate the attainment of certain state of the material in question which should be considered as critical.

The rheological behaviour of structural materials is sensitive to the changes of temperature field and at certain level a small increase of temperature may lead to a significant change of their deformation state. The considerable thermal influence is manifested through the remarkable variation of the thermodynamical characteristics of the materials. It is our aim to discuss this problem in connection with the thermomechanical phenomena being observed in the nonlinear viscoelastic bodies of certain class. In our considerations they are assumed to be isotropic, homogeneous, mechanically incompressible and exhibiting the instantaneous linear elastic response.

In particular, we shall be interested in the analysis of some forms of free energy and entropy production which may be possibly deduced for the bodies in question. The viscoelastic nonlinearity will be assumed as small. It means that the constitutive equation will contain a second order nonlinear term separated from the linear viscoelastic deformation within the additive form. The latter follows from a more general formulation of constitutive thermorheological properties based on the generalized superposition principle. Some particular cases will be specified as regards the amount of free energy and entropy production.

The discussion touches also the problem of thermomechanical viscoelastic coupling in the presence of a variable temperature field. Evidently, it may be understood in two different ways. One follows from the appearance of heat fluxes due to the deformation changes and, therefore, one has to consider the heat conduction in the body. The equation of energy balance then contains the additional term responsible for thermomechanical coupling. It expresses the fact of mutual exchange between the two kinds of energies mechanical and thermal in the rheological process. On the other hand, the second meaning of coupling may be related to the thermomechanical effects as showed by the total response of the material in the presence of mechanical and thermal causes. In general, it indicates that all the physical characteristics should be temperature dependent as the latter appears in the form of constitutive relation as an independent variable. However, it should be mentioned that such coupling occurs between the specified terms of distinctly different physical nature. If these, as being strain contributions, are added, one obtains the total response of the body. As it will be shown later, the particular effects are coupled due to the integrability condition put on the differential form of the constitutive relation.

1. We consider the proposed form of constitutive equation for the nonlinear material with viscoelastic properties. It is obtained by using the generalized superposition principle [1],[2],[3], performed in accordance with the procedure as follows.

Let us denote by $\Delta \epsilon_{ij}^{(n)}$ the increment of small strain tensor related to the observation instant t . It is assumed to be caused by the simultaneous increments of stress deviator s_{ij} and temperature T considered as independent variables. The increments $\Delta s_{ij}^{(n)}$ and $\Delta T^{(n)}$ are taken within the subin-

terval $\omega_n = [\tau_{n-1}, \tau_n]$, $\omega_n \in \omega = [0, t]$. The contribution of the partial state in ω_n to the global strain state at t we assume as follows

$$\varepsilon_{ij}^{(n)}(t, \tau) = K \Delta s_{ij}^{(n)} + k_{ij} \Delta T_n \quad (1)$$

where

$$K = K\{t, \tau_n^*, s_k [t_n^*, T(t_n^*)], T(t_n^*)\} \quad (2)$$

$$k_{ij} = k_{ij}\{t_n^*, s_k [t_n^*, T(t_n^*)], T(t_n^*)\} \quad (3)$$

denote the generalized memory function and the dilatation tensor, respectively. They depend on the specified arguments s_k ($k = 1, 2, 3$), T taken at instant $\tau_n^* \in \omega_n$. Here s_k is the triplet of the stress deviator invariants, s_1 being reserved for the possible pressure which may appear as a result of temperature effects.

By summing up the contributions of eq.(1) and taking the limit for $n \rightarrow \infty$ in ω , we arrive at the constitutive relation

$$\varepsilon_{ij}(t) = (2G)^{-1} s_{ij} - \int_{\omega} s_{ij} \partial_{\tau} K dt + k_{ij} T - \int_{\omega} T \partial_{\tau} k_{ij} dt \quad (4)$$

where

$$K = (2G)^{-1} + H\{t, \tau, s_k [t, T(t)], T(t)\}, \quad \partial_{\tau} = \partial / \partial \tau \quad (5)$$

contains the linear elastic response. The thermal member in eq.(4) is specified as the temperature effects may create the thermal strain and stress independently of mechanical causes. In the case of a simultaneous action of the mechanical and thermal fields the effects are coupled. It means that in such case one cannot separate naturally the particular strain and stress states.

On the other hand, instead of considering increments, as in eq.(1), we can assume the differential constitutive form

$$d\varepsilon_{ij} = K ds_{ij} + k_{ij} dT = \partial_{s_{kl}} \varepsilon_{ij} ds_{kl} + \partial_T \varepsilon_{ij} dT \quad (6)$$

where

$$\partial_{s_{kl}} \varepsilon_{ij} = K \delta_{ik} \delta_{jl}, \quad \partial_T \varepsilon_{ij} = k_{ij} \quad (7)$$

The physical coupling of the qualitatively different functions in eq.(6) occurs if

$$\partial_T K = \partial_{s_{kl}} k_{ij} \delta_{ik} \delta_{jl} \quad (8)$$

and, therefore, one obtains again eq.(4) by integrating of eq.(6). The integrability condition of eq.(8) indicates the additional constitutive relation which exists between the physical characteristics of thermoviscoelastic behaviour. The result obtained points out clearly the significance of the coupling mentioned above.

Alternatively, we rewrite eq.(4) as follows

$$\epsilon_{ij}(t) = (2G)^{-1} s_{ij} + \tilde{K} s_{ij} + \delta_{ij} \tilde{L} \dot{\epsilon} \dot{T} \quad (9)$$

where

$$\tilde{K} s_{ij} = d_t K s_{ij} \quad , \quad \tilde{L} f_{ij} = \int_{\omega} f_{ij}(\tau) d\tau \quad , \quad \dot{T} = dT/dt \quad (10)$$

K and L being the superpositional operator and the integral operator, respectively. Here the dilatation tensor is of the form

$$k_{ij} = \delta_{ij} \dot{\epsilon} \quad (11)$$

where $\dot{\epsilon}$ is the thermal expansion characteristic.

In particular, we assume that the viscoelastic nonlinearity contained in the superpositional operation is of the additive form

$$\tilde{K} s_{ij} = (\tilde{K}_0 + \tilde{K}_*) s_{ij} \quad (12)$$

Here \tilde{K}_0 is a linear operator and then \tilde{K}_* may be considered as a second order nonlinear operation.

If we denote by

$$g_{ij} = \epsilon_{ij} - (2G)^{-1} s_{ij} \quad (13)$$

the viscoelastic strain tensor, eqs.(9) and (12) then give

$$g_{ij}(t) = \tilde{L}[(\tilde{K}_0 + \tilde{K}_*) s_{ij} + \delta_{ij} \dot{\epsilon} \dot{\epsilon} \dot{T}] \quad (14)$$

There are not convincing expressions available in the literature for the free energy and the dissipation in viscoelastic materials. Although, it is possible to derive them rather easily on the basis of rheological models, there are substantial difficulties in their formulation as regards general cases. For example, in references [4] and [5] one finds derived the dissipative functionals in double integral form for linear viscoelastic and nonlinear creeping media undergoing isothermal deformation. In the latter paper the procedure of deriving is founded on the generalized superposition applied to the forms of functionals of the second degree.

It is clear that the central difficulties of the problem lie in the appropriate interpretation of physical properties of the media in question. Moreover, the interpreting depends on the forms of kernels appearing in the operational constitutive relations to describe these properties. Especially, the nonlinearity of the latter introduces some complexity into consideration. Therefore, in most of the papers on dissipative heating problems there are farguing approximations made as regards the distribution of mechanical energy input. For example, it is assumed that this input is totally converted into heat and, hence, the constant level of free energy sustained.

In connection with the above discussion let us consider some admissible forms of the operation \tilde{K} in eq.(9). At first we assume

$$\tilde{K}s_{ij} = s_{ij}f[s(t), T(t)] \partial_t C(t-\tau) = \partial_t Hs_{ij} \quad (15)$$

where we put $s = s_2$ and neglect the influence of the third invariant. Furthermore, we assume

$$f = \exp [s(t) s_0^{-1} a(T)], \quad C = C_0 [1 - \exp -\lambda(t-\tau)] \quad (16)$$

s_0 being certain comparative value of s and C_0, λ physical constants.

As follows from eq.(16) the weak nonlinearity of viscoelastic behavior may be obtained by extending f to obtain

$$f = (1 + \partial_s \bar{f}|_{s=s_0} \cdot s + \dots) \exp a(T_0), \quad \partial_s \bar{f}|_{s=s_0} = a(T_0) s_0^{-1} \quad (17)$$

T_0 representing temperature which corresponds to the level of stress state defined by s_0 . Then by comparing eqs.(15), (17) and (12) we find

$$\tilde{K}_0 s_{ij} = \partial_t \{ \bar{C}_0 [1 - \exp -\lambda(t-\tau)] \} s_{ij}, \quad \bar{C}_0 = C_0 \exp a(T_0) \quad (18)$$

$$\tilde{K}_* s_{ij} = \partial_t \{ \bar{C}_0 [1 - \exp -\lambda(t-\tau)] \} \partial_s \bar{f}|_{s=s_0} \cdot s \cdot s_{ij} \quad (19)$$

the latter operator defining the weak nonlinearity. As it is seen, for the assumed eq.(15) the viscoelastic process with small nonlinearity may be considered as isothermal. In such a case it seems to be reasonable to assume that eq.(19) represents a dissipative term, of course, in approximation.

It should be mentioned that eq.(9) with the substitution of eq.(15) may be easily reduced by differentiation to a nonlinear differential equation with respect to s_{ij} or e_{ij} .

Another interesting case we obtain by putting

$$\tilde{K}s_{ij} = s_{ij}g[s(t), T(t)] \partial_t [B(t) - B(\tau)] = \partial_t Hs_{ij} \quad (20)$$

where we denote

$$f \equiv g, \quad B(t) = B_0 (\exp \xi t - 1). \quad (21)$$

The weak nonlinearity case we obtain by extending the function g , similarly to eq.(17),

$$g = (1 + \partial_s \bar{g}|_{s=s_0} \cdot s + \dots) \exp a(T_0), \quad \partial_s \bar{g}|_{s=s_0} = a(T_0) s_0^{-1}. \quad (22)$$

Thus, by comparing eqs.(20), (22) and (12) we find

$$\tilde{K}_0 s_{ij} = s_{ij} \partial_t \{ \bar{B}_0 [\exp(\xi t) - \exp(\xi \tau)] \}, \quad \bar{B}_0 = B_0 \exp a(T_0) \quad (23)$$

$$\tilde{K}_* s_{ij} = s_{ij} s \cdot \partial_s \bar{g}|_{s=s_0} \cdot \partial_t \{ \bar{B}_0 [\exp(\xi t) - \exp(\xi \tau)] \}. \quad (24)$$

It may be easily proved that eq.(20) is a term of dissipative character. The strain described by the function B is not reversible in time after unloading. As a result of stressing one obtains a permanent deformation of viscoplastic nature.

It is to note that eq.(9) with the substitution of eq.(20) reduces to a nonlinear differential equation for stress by simple differentiation.

We shall use the above results to interpret the dissipative heating phenomenon and thermomechanical coupling.

2. If the considered nonlinear viscoelastic material undergoes a small deformation process, then at $t \in \omega$ stress power defined locally in the small neighbourhood of a particle becomes

$$\dot{W} = s_{kl} \dot{\epsilon}_{kl} = \dot{E} + q_{k,1} \delta_{kl} \quad , \quad q_{k,k} = \partial q_k / \partial x_k \quad (25)$$

Here, \dot{W} is the rate of specific work done by stresses, \dot{E} the rate of specific internal energy and q_i denotes the heat flux vector. For the process in question the internal energy is a function of strain tensor invariants and temperature

$$E = E[\epsilon_k(t), T(t)] \quad (26)$$

and, therefore, its rate may be written as follows

$$\dot{E} = \sum_k \partial_{\epsilon_k} E \cdot \dot{\epsilon}_k + \partial_T E \cdot \dot{T} = \sum_k \partial_{\epsilon_k} E \cdot \dot{\epsilon}_k + C_V \dot{T} \quad , \quad C_V = \partial_T E \quad (27)$$

where ϵ_k are invariants ($k = 1, 2, 3$) and C_V is the specific heat capacity.

The heat flux vector in eq.(25) is expressed through the temperature gradient

$$q_i = -\lambda T_{,i} \quad (28)$$

λ being the coefficient of thermal conductivity of the material.

The equation (25) may be now rewritten by using eqs.(26) and (27)

$$s_{kl} \dot{\epsilon}_{kl} = C_V \dot{T} + \sum_k \partial_{\epsilon_i} E \cdot \partial_{\epsilon_{kl}} \epsilon_i \cdot \dot{\epsilon}_{kl} - \lambda T_{,kl} \delta_{kl} \quad (29)$$

Taking into account the relation between the entropy S and the internal energy at the differential level

$$dE = TdS + s_{kl} d\epsilon_{kl} \quad (30)$$

we find then the specific energy rate as being

$$\dot{E} = T\dot{S} + \dot{W} \quad (31)$$

On the other hand, the rate of entropy is equal to the generation entropy rate less the diffusion flux gradient of entropy denoted by $S_{k,k}^*$

$$\dot{S} = S^* - S_{k,k}^* \quad (32)$$

Substituting eq.(25) into eq.(31) we have

$$\dot{S} = T^{-1}(-q_{k,1}\delta_{kl} + s_{kl}\dot{\epsilon}_{kl}) \quad (33)$$

and then by comparing eqs.(32) and (33) we find

$$S^* = -T^{-1}(q_{k,1}\delta_{kl}T^{-1} - s_{kl}\dot{\epsilon}_{kl}) \quad , \quad S_i^* = T^{-1}q_i \quad (34)$$

representing the generation entropy rate and diffusion flux of entropy, respectively. As we must always have $S^* > 0$, both the terms in the first of eqs. (34) are positive

$$s_{kl}\dot{\epsilon}_{kl} > 0 \quad , \quad q_{k,1}\delta_{kl} < 0 \quad . \quad (35)$$

Thus, S^* gives the sum of entropy production due to heat and that due to deformation.

The free energy being defined by the relation

$$F = E - TS \quad (36)$$

its rate reads

$$\dot{F} = \dot{E} - (T\dot{S} + \dot{TS}) \quad (37)$$

so that by combining eqs.(31) and (37) we obtain

$$\dot{W} = \dot{F} + \dot{TS} + \dot{TS} \quad (38)$$

In particular, if the temperature rate may be disregarded we can write eq.(38) for a constant temperature T_0

$$\dot{W} = \dot{F} + T_0\dot{S} \quad (39)$$

the stress power being defined for an isothermal deformation process. In general, if such is the case the rate of entropy production in eq.(39) cannot be identified with the rate of heat production in the material due to dissipative phenomena. The isothermal process requires, in general, some heat exchange and only when this exchange is considered as small one can interpret the dissipative term in eq.(39) as the rate of heat production.

Returning to our constitutive relations we shall assume that the possible response of the material to the volumetric deformation due to temperature changes in the presence of constraints is of elastic nature. As the symbol s_1 in eq.(5) is reserved for the possible pressure, the first invariant of stress deviator being equal zero, we have $s_1 = \bar{\sigma} = 1/3 \bar{\sigma}_{11}$. Thus, specifying $\bar{\sigma}$ from the function K of eq.(5), by alternating of eq.(9) we find

$$\epsilon = D\bar{\sigma} - \chi(T - T_0) \quad , \quad \epsilon = 1/3 \epsilon_{11} \quad (40)$$

where D denotes the bulk modulus and χ is put a constant. If there are not constraints at all, the temperature change does not introduce any pressure.

Taking into account eq.(40) we rewrite eq.(9) as follows

$$e_{ij}(t) = (2G)^{-1} s_{ij} + \tilde{L}Ks_{ij} \quad (41)$$

where now we neglect the influence of the third invariant s_3 , so that eq.(4) contains

$$H = H\{t, \tau, s[t, T(t)], T(t)\} \quad (42)$$

In the equation (41) e_{ij} is the strain deviator.

On the other hand, the equation of energy rate (29) is modified as follows

$$s_{kl} \dot{e}_{kl} = C_V \dot{T} + 3D^* \alpha (T - T_0) \dot{E} - \lambda T_{,kl} \delta_{kl} + \partial_{\phi} E \cdot \partial_{e_{kl}} e \cdot \dot{e}_{kl} \quad (43)$$

where $D^* = D^{-1}$ and e the second invariant of strain deviator.

The equation (43) expresses the fact of thermomechanical viscoelastic coupling as it describes the reciprocal interchange of mechanical and thermal energy.

From the equation (41) follows that the viscoelastic strain deviator

$$q_{ij}(t) = e_{ij} - (2G)^{-1} s_{ij} = e_{ij} - e_{ij}^0 \quad (44)$$

is expressed as

$$q_{ij}(t) = \tilde{L}Ks_{ij} \quad (45)$$

In particular, if the elastic part of strain deviator is negligible in eq.(44), we have

$$s_{kl} \dot{q}_{kl} = C_V \dot{T} + 3D^* \alpha (T - T_0) \dot{E} - \lambda T_{,kl} \delta_{kl} + \partial_{\phi} E \cdot \partial_{q_{kl}} q \cdot \dot{q}_{kl} \quad (46)$$

where now q denotes the second invariant of viscoelastic strain deviator.

In some specific cases the terms of eq.(46) may be considered as purely dissipative. It occurs, for example, when the material exhibits a pure viscous or viscoplastic behaviour as that expressed by eq.(20).

The equation of energy derived above constitutes the condition of equilibrium defined locally in time for the body in question deforming in a variable temperature field. On the other hand, this condition takes into account the fact of generation of the heat fluxes due to strain state and viscous resistance. In such a way the deformation process is coupled with the heat conduction as the former leads to the heat flow phenomenon inside the body.

3. The problem of main importance is that of specifying the free energy as well the dissipation on the basis of constitutive relations which we have at our disposal. These are used to form the stress power as defined by eq.(25). On the other hand, they give some indications as regards the possible

forms of free energy and entropy production.

Let us form the stress power on the basis of eq.(41). Thus, differentiating with respect to time, we find

$$\dot{s}_{ij}(t) = \dot{s}_{ij}^0(t) + \overset{\circ}{\mathbb{L}}\overset{\circ}{\mathbb{K}}s_{ij} \quad (47)$$

where the dot over the operation is symbolic. Multiplying eq.(47) by the deviator s_{ij} , we have

$$\dot{W} = s_{ij}\dot{s}_{ij}^0 + s_{ij}\overset{\circ}{\mathbb{L}}\overset{\circ}{\mathbb{K}}s_{ij} = (2G)^{-1}\dot{s}_{ij}s_{ij} + s_{ij}\overset{\circ}{\mathbb{L}}\overset{\circ}{\mathbb{K}}s_{ij} \quad (48)$$

If we consider eqs.(2e) and (39) then by the direct comparing the corresponding terms of the equality obtained, we find

$$\dot{F} = (2G)^{-1}\dot{s}_{ij}s_{ij} \quad (49)$$

$$\dot{S} = T_0^{-1}s_{ij}\overset{\circ}{\mathbb{L}}\overset{\circ}{\mathbb{K}}s_{ij} = T_0^{-1}s_{ij}s_{ij}g(s). B_0 \xi \exp \xi t \quad (50)$$

for an isothermal process.

Returning again to eq.(41) we can assume that the expected form of the solution with respect to the deviatoric stress is

$$s_{ij}(t) = 2G e_{ij} + \overset{\circ}{\mathbb{L}}\overset{\circ}{\mathbb{R}}e_{ij} = s_{ij}^0 + \overset{\circ}{\mathbb{L}}\overset{\circ}{\mathbb{R}}e_{ij} \quad (51)$$

where $\overset{\circ}{\mathbb{R}}$ is the resolving operator of the form analogous to eq.(15)

$$\overset{\circ}{\mathbb{R}}e_{ij} = e_{ij} \overset{\circ}{\mathbb{F}}[\overset{\circ}{e}(t), T(t)] \partial_t \overset{\circ}{C}(t-\tau) \quad (52)$$

By using the concept of small nonlinearity, it is possible in analogy to eqs. (18) and (19) write

$$\overset{\circ}{\mathbb{R}}_0 e_{ij} = \partial_t \overset{\circ}{C}(t-\tau) e_{ij} \quad (53)$$

$$\overset{\circ}{\mathbb{R}}_* e_{ij} = \partial_t \overset{\circ}{C}(t-\tau) \partial_e \overset{\circ}{\mathbb{F}}^* \Big|_{e=e_0} \cdot \cdot \cdot e_{ij} \quad (54)$$

e_0 being a comparative value of e .

On the basis of eqs.(47) and (51) the stress power reads

$$\dot{W} = s_{ij}\dot{s}_{ij}^0 + s_{ij}\overset{\circ}{\mathbb{L}}\overset{\circ}{\mathbb{K}}s_{ij} + \dot{s}_{ij}^0 \cdot \overset{\circ}{\mathbb{L}}\overset{\circ}{\mathbb{R}}e_{ij} + \overset{\circ}{\mathbb{L}}\overset{\circ}{\mathbb{K}}s_{ij} \cdot \overset{\circ}{\mathbb{L}}\overset{\circ}{\mathbb{R}}e_{ij} \quad (55)$$

As seen from the form of eq.(55), it is rather difficult to judge and classify the terms appearing there as regards their energetic significance, except the term first. Therefore, it often happens that one has to make some approximative choice in an artificial manner rather in order to simplify energetic expression. Such an approach is often used in the available literature concerned with the problem, even in the linear case.

Alternatively, we can represent eq.(55) in terms of the deviator e_{ij} .

Correspondingly, we can introduce also the deviator of viscoelastic stress defined as follows

$$P_{ij} = s_{ij} - s_{ij}^0 \quad (56)$$

It is seen from the form of eq.(55) that if the elastic components are neglected, for example, in the case when material is subjected to cyclic loadings, we simply obtain

$$\dot{W} = \dot{\tilde{L}}\tilde{K}s_{ij} \cdot \tilde{L}\tilde{R}s_{ij} = \dot{\tilde{Q}}_{ij}P_{ij} \quad (57)$$

From there by integration we find

$$W = \tilde{L}\tilde{L}\tilde{K}s_{ij} \cdot \tilde{L}\tilde{R}s_{ij} = \tilde{L}\tilde{Q}_{ij}P_{ij} \quad (58)$$

the result which indicates the form of stress work to be a functional of second degree. Usually, such forms are assumed in the literature as, for example, in [4] and [6].

The additional term which appears in the equation of energy balance (43) and takes into account the rate of internal energy results from the nonlinearity of strain state. It is clear that it disappears if the material behaves linearly. Thus, the equation (43) simplifies considerably and may be interpreted as the first approximation of a small nonlinear deformation process. The similar equation of energy to the simplified form of eq.(43)

$$s_{kl} \dot{\epsilon}_{kl} = C_v \dot{T} + 3D^* \alpha (T - T_0) - \lambda T,_{kl} \delta_{kl} \quad (59)$$

we find in [5]. However, the right hand side of eq.(59) is considered as purely dissipative.

4. Founding our considerations on the thermodynamical concepts we derived the equation of energy for a nonlinear viscoelastic material. The constitutive relation for such a material was obtained on the basis of the generalized superposition procedure which led to the functional form of nonlinear operator containing stress and temperature as independent variables.

In both the cases the physical meaning of thermorheological coupling of effects was emphasized.

It is, in general, very difficult to make some fargoing conclusions as regards the quantitative judgement of the coupling influence on viscoelastic deformation. It seems, however, that the procedure applied and results obtained may find use in the problems of rheological stability of structural systems as well may be adopted to the investigation of deformation processes of constructions working in the temperature fields. Especially, it is concerned with such problems in which the inertia effects are negligible as assumed here.

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