FINITE ELEMENT STRESS ANALYSIS OF INTERACTING FUEL PELLET AND CANNING UNDER FAST REACTOR CONDITION

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SUMMARY

The design of fuel elements for sodium cooled fast reactors implies the analysis of stresses, strains, displacements and interacting forces in both UO_2 pellet and metal canning. Some authors (see M. Guyette, 1st SMiRT Conf., Paper C2/1*) use a method which can be described as a one-dimensional procedure, where the variables are functions of radius only. The plane strain and axi-symmetric condition are assumed, yielding a close approximation to the stresses in the fuel elements, away from regions of flux perturbation and away from the ends of the pin.

The authors propose the use of a two-dimensional finite element method. A programme has been developed which assumes axial symmetry of geometry and loading (i.e. axi-symmetric temperature and flux distribution in a body of revolution), but discards the plane strain restriction. A standard finite element programme (using triangular elements) has been modified to allow for time dependent phenomena, including complete power cycles from start-up to full operation and shut-down. The interaction effects between fuel pellets and canning are studied by an unorthodox finite element technique, where the nodes of the structures on the interface between the two parts do not match. This approach thus allows for the possibility of frictional relative movement of the two structures.

An important part of the computation described above is the analysis of the temperature distribution in a hollow compounded cylindrical body of the fuel element. Since no equivalent two-dimensional finite element or finite difference code was available, the temperature distribution was considered to be as in the middle of the reactor core, without axial and circumferential variations. Some special effects have been analysed, including the dependance of the thermal conductivity on temperature and dose, and the radial variation of the internal heat generation in a hollow pellet [$\nabla \lambda \nabla T + H = 0$ where $\lambda(T) = 1/(A+BT)$, and $H(r) = C.I_0(\kappa r) + D.K_0(\kappa r)$].

The resulting stress distributions in the canning are included in the form of contours of isobars for various stages of interaction with particular reference to different axial friction coefficients.

1. INTRODUCTION

In the Liquid Metal Fast Breeder Reactor (LMFBR) mechanical interactions may occur between the pellets and the cladding, thereby modifying the stresses in the cladding and pellets. This phenonenon is not unique to the LMFBR, but is known also to occur in other types of reactor. For example, in the Pressurised Light-Water Reactors, creep of the can is known to cause interaction between the can and the fuel pellets. A similar effect occurs in the High Temperature Gas-Cooled Reactor (HTR), in which the fuel compacts exhibit a higher shrinkage rate than do the containing graphite sleeves. Thus in some forms of HTR fuel element, interaction occurs between the fuel compacts and the graphite sleeves.

The HTR problem is one which the authors have studied in detail using 1-dimensional methods (see for example Alujevic and Head [1]). Also using 1-dimensional methods, Guyette [2] has made a detailed study of the problem in the context of the LMFBR fuel pin. The problem should however be analysed by 2-dimensional methods, on account of the short axial length of the fuel pellets and the non-uniform expansion of the pellets due to thermal and irradiation effects.

The authors have attempted to extend their studies of interaction phenomena to include the LMFBR. This paper describes a 2-dimensional analysis of the pellet/cladding interaction using a finite element programme which the authors have previously described (Alujevic and Head [3]). The programme has required modification to permit allowance to be made for relative axial movements between the contacting surfaces of pellet and cladding and for frictional forces associated with such relative movements.

2. COMPUTER CODE

The computer code used for the analysis (see ref. [3]) utilises axi-symmetric triangular linear displacement elements. The theory of these elements is well known (see for example Zienkiewicz [4]) and needs no repetition. The point matching technique commonly used at the boundary between two different materials cannot be used in this case as it would not permit allowance to be made for relative axial movement between pellet and cladding. The authors consider that axial movement may have a significant effect on the structural performance of the fuel pins. A technique has been established therefore to determine nodal forces, applied to pellet and cladding, which are statically equivalent to the interaction pressure (allowing for the axial variation of pressure) and which ensure compatible radial displacements between pellets and cladding over the area of contact. The method permits relative axial displacements to take place.

Fig. 1 shows the mesh used in the development of the code. Only a part of the fuel pin was considered. Appropriate "bending" boundary conditions were assumed, to represent the rotational restraint provided by the adjoining regions of the structure. The code is of course not limited in this respect. It is possible to consider a greater length of fuel pin if it is required to include the effects of axial variations of temperature and neutron flux.

Fig. 2 illustrates the relative axial movements of the pellet and cladding, which occur initially due to relative thermal expansion and which may further develop due to irradiation effects. In using the code to follow the stress history, time steps must be chosen carefully to establish the moment of first and subsequent contacts and/or losses of contact.

Fig. 3 shows a hypothetical power cycle which was assumed for the purpose of programme development. Fig. 4 shows a simplified flow diagram of the code.

3. HEAT TRANSFER CALCULATIONS

The temperature distribution in the fuel pin varies with time, depending on changes of fuel pin power, coolant temperature, material thermal conductivity, pellet/cladding clearance or contact pressure, etc. The temperature distribution must be recalculated at intervals, taking account of these changes, and the modified temperature distribution input to the structural code. Output from the structural code must likewise be input to the heat transfer code. Thus the two analyses must proceed in parallel as a feed-back loop.

The situation requires the use of a 2-dimensional heat transfer code. Such codes exist, for example TESS [5] and HEATRAN [6], but were not available to the authors. A 1-dimensional heat transfer routine was therefore used to provide temperature data for the purpose of testing the structural code. The temperature data is therefore not strictly representative of conditions in the LMFBR fuel pin. The correlations and equations used in the 1-dimensional code are as follows:

Convective heat transfer from cladding to coolant

$$Nu = (7 + 0.025\, Pe^{0.8}) \cdot 0.7 \cdot (d_2 / d_1)^{0.53} \qquad (1)$$

Heat conduction in cladding

$$r^{-1} \cdot d/dr\, (\lambda\, (T) \cdot r \cdot dT/dr) = 0 \qquad (2)$$

where $\lambda\, (T) = 1/\, (a + b \cdot T)$ (3)

Interface heat transfer coefficient

In the case where there is radial clearance, heat transfer is assumed to be by conduction and radiation across the (helium filled) gap. Where the gap is closed, and an interface pressure has developed, the expression of Jankus and Weeks [7] is used

$$h_g = h_o \cdot (1 + p/p_o) \qquad (4)$$

Heat generation and conduction in hollow fuel pellet

$$r^{-1} \cdot d/dr\,(\lambda\,(T) \cdot r \cdot dT/dr) + H\,(r) \;=\; 0 \qquad (5)$$

where
$$\lambda\,(T) \;=\; 1 / (A + B \cdot T) \qquad (6)$$

and
$$H\,(r) \;=\; C \cdot I_o(\varkappa . r) + D \cdot K_o(\varkappa . r) \qquad (7)$$

Integrating equation (5) yields

$$\int \lambda\,(T)\,dT \;=\; -\int \frac{1}{r}\left(\int H(r)\,r\,dr\right) dr \;+\; M \log_e(r) + N \qquad (8)$$

which is evaluated by the method recommended by Johannsen [8].

The calculated temperature distributions for various linear heat ratings in the range 100 to 580 W/cm are shown in Fig. 5. It should be emphasised that these results neglect axial heat transfer and have been used only for the purpose of testing the structural code.

4. STRESS CALCULATIONS

Using the temperature data provided by the routine described in section 3, Keegstra [9] has completed the development of the structural code and made some preliminary stress calculations on a representative LMFBR fuel pin. Fig. 6 gives results in the form of stress contours for several interaction states:

(i) no interaction between fuel pellet and cladding,
(ii) interaction at all nodes.

The second calculation, with full interaction, has been repeated assuming different coefficients of friction between the contacting surfaces:

(a) low friction ($\mu = 0.1$),
(b) intermediate friction ($\mu = 0.5$),
(c) high friction (no relative movement).

Values of initial clearance used in the analysis were zero, 10 μ and 30 μ.

The results given are only the initial (thermoelastic) stresses. Other effects which must be taken into account in any realistic analysis of the stress history include creep, plasticity and swelling. The programme has the capability of including these effects, although programme running times are long. An effect which the (2-dimensional) code cannot include is the radial cracking which is known to occur in the pellet (see Gittus [10]).

5. PLASTIC BEHAVIOUR

The method used to include the effect of plastic strains is that of Yamada and Yoshimura [11], which is an improvement and simplification of the method of Marcal and King [12]. For elements where the yield criterion is reached, an incremental stress/strain relationship is used of the form

$$\{\Delta\,\sigma\} \;=\; 2 \cdot (1+\nu) \cdot G \cdot [D^{p}]\,\{\Delta\,\epsilon\} \qquad (9)$$

where the plastic stress/strain constitutive matrix $[D^p]$ (see ref. [11]) replaces the elastic stress/strain matrix $[D]$. The method permits the propagation of the elastic/plastic interface to be followed, especially in the can when interaction occurs.

6. CONCLUSIONS

The results given indicate the dependence of the thermoelastic stresses, in a representative LMFBR fuel pin, on linear heat rating, on initial pellet/cladding clearance and friction factors. The assumed temperature distributions however are in error, as axial heat transfer has been neglected. When a suitable 2-dimensional heat transfer code is available, the authors will extend the study to cover the full life stress/deformation history of the fuel pin.

7. NOTATION

a, b	...	thermal conductivity coefficients (cladding)
A, B	...	thermal conductivity coefficients (fuel pellets)
C, D	...	heat generation coefficients (fuel pellet)
$[D]$	...	elasticity constitutive matrix
$[D^p]$	...	plasticity constitutive matrix
d_1, d_2	...	diameters of the cooling channel
G	...	modulus of rigidity
H	...	heat generation rate per unit volume
h_g, h_o	...	gap conduction
I_o, K_o	...	modified Bessel functions
M, N	...	integration constants
p, p_o	...	interaction pressure
T	...	temperature
Nu	...	Nusselt number
Pe	...	Peclet number
Δ	...	increment
ϵ	...	strain
$\varkappa$	...	argument
λ	...	thermal conductivity
μ	...	friction factor
ν	...	Poisson's ratio
σ	...	stress
r	...	radius

Note: This paper is based on the M.Sc. thesis by one of the authors (P.N.R.K.), which was awarded the Hinton Prize for Nuclear Engineering in 1972.

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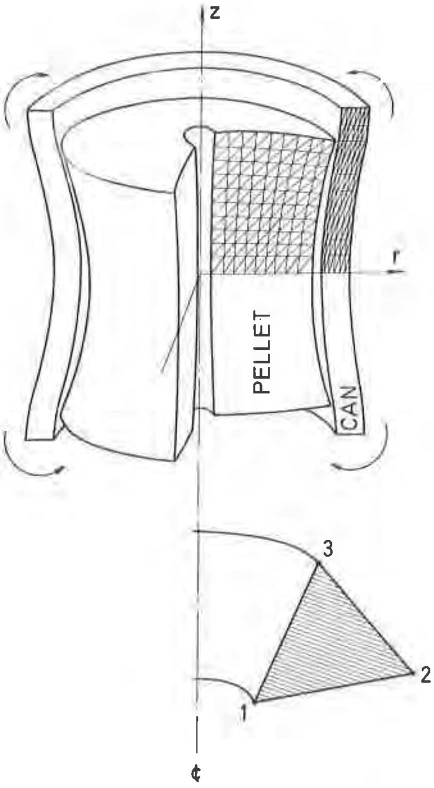
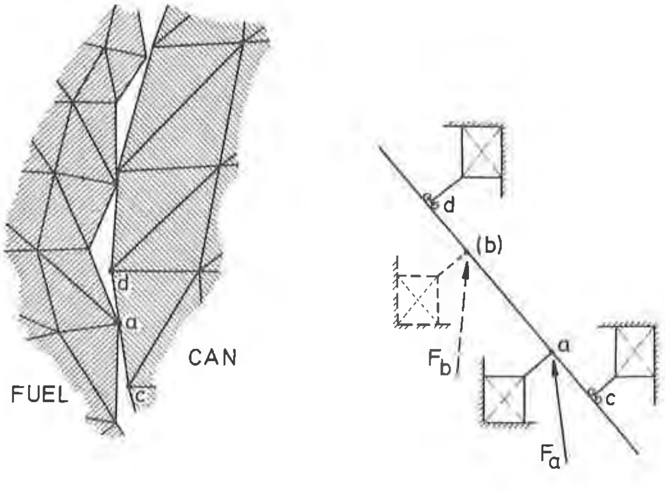


Fig. 1 FINITE ELEMENTS AND MESH



INTERACTION SITUATION INTERACTION MODEL

Fig. 2

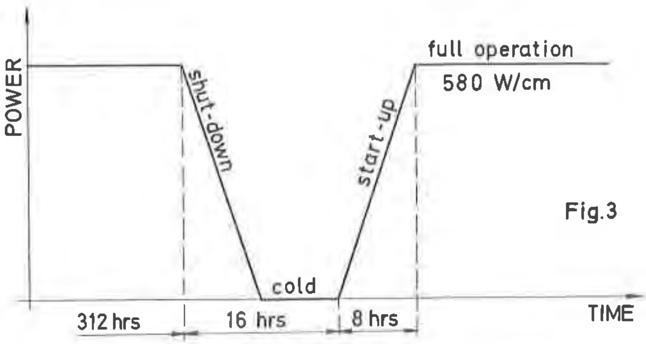
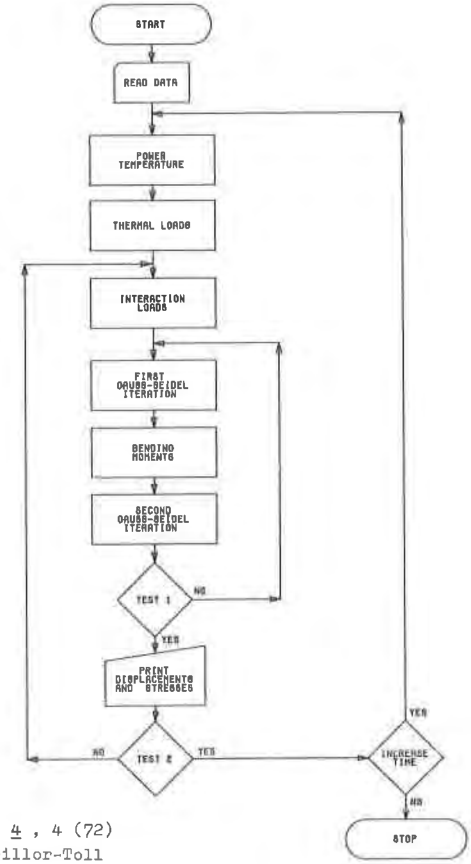


Fig.3 POWER HISTORY



Ref. C.A.D. 4 , 4 (72)
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FIG.4 FUEL PELLET - CANNING FINITE ELEMENT CODE

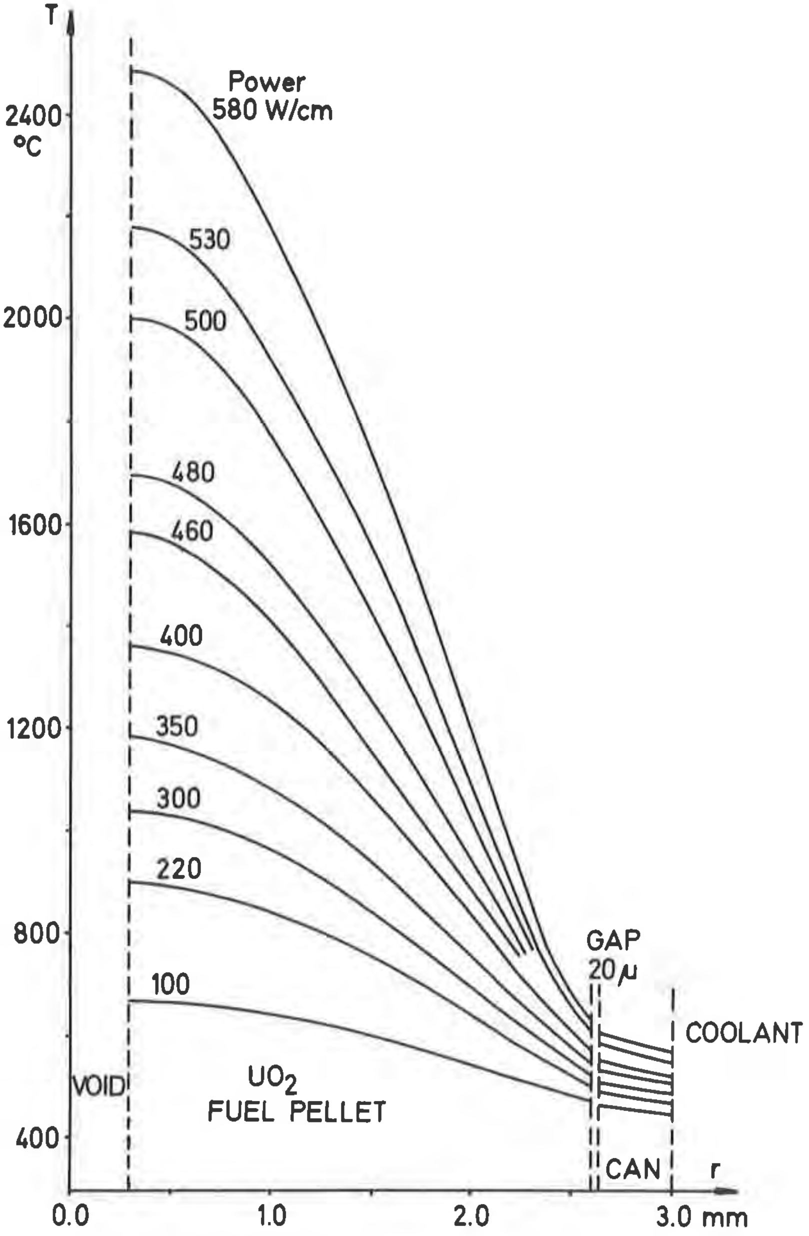
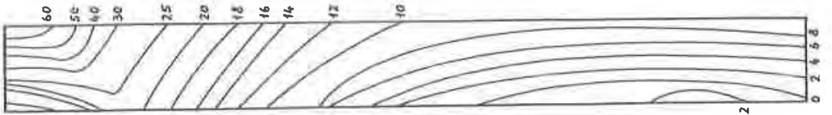
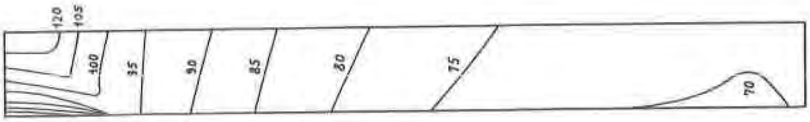


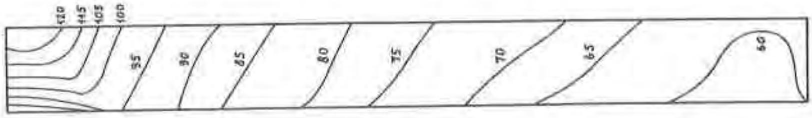
Fig.5 TEMPERATURE DISTRIBUTION



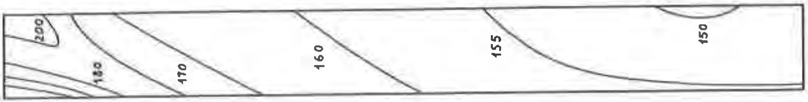
Initial gap 30μ Friction factor 1.0



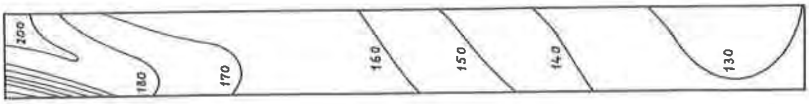
10μ 1.0



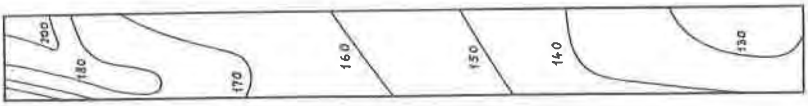
10μ 0.5



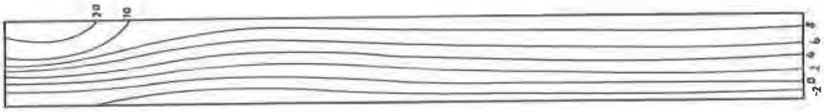
nil 1.0



nil 0.5



nil 0.1



(Non-interacting condition)

Units { 1 kp/mm^2 = 102 bar

Fig. 6 HOOP STRESS AT FULL POWER 580W/cm[GAP CLOSED] IN FUEL ELEMENT CANNING