Order Statistics for a Special Class of Unequally Correlated Multinormal Variates[1]

by

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The moments of the order statistics for independent normal variables are well known and used for many purposes including the determination of the moments of censored normal populations. Selection programs in plant and animal breeding routinely use the average of the expectations of the s top-ranking order statistics as the standardized selection differential in the prediction of genetic gain. In many cases, the assumption of independence may not be satisfied as, for example, when the population under selection is composed of individuals with varying degrees of genetic relationship.

The present study was prompted by the need for some understanding of the effect of unequal correlations on the order statistics, ordering without regard to the genetic relationships. One abstraction of this problem is to assume that the sample of n individuals consists of m full-sib families with k

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individuals per family and that the correlation between two full sibs is the same for all full-sib pairs. This can be represented by the simple between-within classes linear model

$$X_{ij} = \mu + \alpha_i + \epsilon_{ij} \quad , \quad i = 1,2,\ldots,m; \quad j = 1,2,\ldots,k \quad ,$$

where $\alpha_i \sim$ NID $(0,\sigma_\alpha^2)$ and $\epsilon_{ij} \sim$ NID $(0,\sigma^2)$. Then the Y_{ij} are unequally correlated, i.e.,

$$\text{Cov}\ (X_{ij}\ X_{i'j'}) = \begin{cases} \sigma_\alpha^2 & \text{if } i = i',\ j \neq j' \\ 0 & \text{if } i \neq i',\ j \neq j' \end{cases} .$$

Owen and Steck (1962) considered the order statistics for the equi-correlated multivariate normal distribution and showed that all moments and product moments for any correlation ρ $(-\frac{1}{n-1} \leq \rho \leq 1)$ could be obtained from the corresponding moments and product moments for the independence case, $\rho = 0$. Letting $X_{[i;n,\rho]}$ be the ith order statistic in a sample of n equi-correlated multinormal variates with mean 0 and variance-covariance matrix $\Sigma_\rho = \{\sigma_{ij}\}$, where

$$\sigma_{ij} = \begin{cases} 1 \text{ if } i = j \\ \rho \text{ if } i \neq j,\ -\frac{1}{n-1} \leq \rho \leq 1 \end{cases} ,$$

the first moment was shown to be

$$\mathcal{E}(X_{[i;n,\rho]}) = (1-\rho)^{\frac{1}{2}}\ \mathcal{E}(Z_{[i;n]})$$

where $Z_{[i;n]}$ is the ith order statistic in a sample of n under independence. The variance was shown to be

$$\mathcal{V}\{X_{[i;n,\rho]}\} = \rho + (1-\rho)\ \mathcal{V}\{Z_{[i;n]}\} \quad .$$

P. M. Burrows[3], personal communication, provided a proof of results similar to those of Owen and Steck but which dropped the requirement of normality and used a transformation which eliminated the need for considering separately $\rho < 0$ and $\rho > 0$ as is necessary in Owen and Steck's proof. Let $Y_1, Y_2, \ldots, Y_n$ be n independent and identically distributed random variables and let U be stochastically independent of Y_i, $i = 1,2,\ldots,n$, with all variables having zero means and unit variances. Burrows' proof used the transformation

$$X_i = (1-\rho)^{\frac{1}{2}}(Y_i - \bar{Y}) + \lambda^{\frac{1}{2}} U \quad , \quad i = 1,2,\ldots,n \quad ,$$

where

$$n\lambda = \{1+(n-1)\rho\} \quad , \quad -\frac{1}{n-1} \leq \rho \leq 1 \quad , \quad \text{and } \bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \quad .$$

Then $\mathcal{E}(X_i) = 0$ and the variance-covariance matrix is Σ_ρ. As in Owen and Steck's proof, $X_{[i;n,\rho]}$ is identified as

$$X_{[i;n,\rho]} = (1-\rho)^{\frac{1}{2}}[Y_{[i;n]} - \bar{Y}] + \lambda^{\frac{1}{2}} U \quad ,$$

and recognizing that $\bar{Y}$ can be written as $\frac{1}{n}\sum_{i=1}^{n} Y_{[i;n]}$, the first two moments about the origin were shown to be

$$\mathcal{E}\{X_{[i;n,\rho]}\} = (1-\rho)^{\frac{1}{2}}\mathcal{E}\{Y_{[i;n]}\}$$

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and

$$\mathcal{E}\{X^2_{[i;n,\rho]}\} = \rho + (1-\rho)\mathcal{E}\{Y^2_{[i;n]}\} + \frac{2(1-\rho)}{n}\Big[1 - \sum_{\ell=1}^{n} \mathcal{E}\{Y_{[i;n]}, Y_{[\ell;n]}\}\Big]$$

with the variance being

$$\mathcal{V}\{X_{[i;n,\rho]}\} = \rho + (1-\rho)\mathcal{V}\{Y_{[i;n]}\} + \frac{2(1-\rho)}{n}\Big[1 - \sum_{\ell=1}^{n} \mathcal{C}\{Y_{[i;n]}, Y_{[\ell;n]}\}\Big] \quad .$$

These results reduce to those of Owen and Steck with the assumption of normality in which case

$$\sum_{\ell=1}^{n} \mathcal{C}\{Y_{[i;n]}, Y_{[\ell;n]}\} = 1 \quad \text{for all } i.$$

The generalization to be considered in this paper is a relaxation of the equicorrelation to the extent that the set of n variates derive from m independent k-dimensional multinormal distributions with common correlation ρ, $n = mk$. While normality is assumed throughout, the assumption is necessary only at the final stage of performing the numerical integration.

Let X_{ij} designate the j<u>th</u> variate from the i<u>th</u> k-dimensional multinormal distribution ($i = 1,\ldots,m$; $j = 1,\ldots,k$). Arbitrarily taking the mean to be zero and the variance of each to be one gives the variance-covariance matrix

$$\Sigma = \begin{bmatrix} \Sigma_\rho & 0 & \cdots & 0 \\ 0 & \Sigma_\rho & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \Sigma_\rho \end{bmatrix}$$

where each Σ_ρ is a k x k variance-covariance matrix for the equicorrelated case as previously defined. It is easy to show that $-1/(k-1) \le \rho \le 1$ using the same argument as in the equicorrelated case (David, 1970). Note that the X_{ij} for given i are exchangeable variates and, in a like manner, the sets of k-dimensional multinormal variates are exchangeable. The ordering to be considered is of all $n = mk$ variates without regard to group classifications. The ordered variates will be designated $X_{[1;n,m,\rho]}, X_{[2;n,m,\rho]}, \ldots, X_{[n;n,m,\rho]}$. The second and third subscripts, n and m in this case, are sufficient to describe the structure of the sample since $k = n/m$.

Limiting Results

There are several obvious limiting results which provide reference points. (1) When $\rho = 0$, $\mathcal{E}(X^r_{[i;n,m,0]}) = \mathcal{E}(Z^r_{[i;n]})$. (2) When $\rho = 1.0$ all members within each k-variate normal are identical so that the moments for the first k ordered statistics are all equal to those of the largest order statistic in a sample of size m with independence, the next k ordered statistics are identical to those of the second largest order statistic, etc. This can be expressed as $\mathcal{E}(X^r_{[i;n,m,1.0]}) = \mathcal{E}(Z^r_{[i*;m]})$ where i* is the integer equal to or immediately greater than i/k. That is,

$$\mathcal{E}(X^r_{[i;n,m,\rho]}) = \begin{cases} \mathcal{E}(Z^r_{[1;m]}) & \text{for } 1 \le i \le k \\ \mathcal{E}(Z^r_{[2;m]}) & \text{for } k+1 \le i \le 2k \\ \vdots & \\ \mathcal{E}(Z^r_{[m;m]}) & \text{for } (m-1)k+1 \le i \le mk. \end{cases}$$

From (1) and (2) it is clear that, when viewed as a continuous function of ρ, the moments of the order statistics in the structured case as defined must converge in sets of size k from the moments of the order statistics in samples of $n = mk$ independent variates when $\rho = 0.0$ to those in samples of m independent variates when $\rho = 1.0$. (3) When $m = 1$, all variates are equally correlated and the exact results of Owen and Steck (1962) hold. (4) When $m = n$, all variables are independent so that $\mathcal{E}(X^r_{[i;n,n,\rho]}) = \mathcal{E}(Z^r_{[i;n]})$.

Probability Integrals for Equicorrelated Multinormal Variates

Two multinormal probability integrals involving equicorrelated variates will be used repeatedly. In all cases the multinormal variates within a correlated set are assumed to have zero means, unit variances and equal correlations so that the abbreviated notation $\phi_k(\underline{x};\rho) = \phi_k(x_1,x_2,\ldots,x_k;\rho)$ will be used to designate the k-variate multinormal p.d.f. where ρ is the common correlation. ($\phi(x)$ will be used when $k = 1$.) One integral of interest is the probability that precisely s of k variates in a k-variate multinormal will fall below the point h and the remaining k-s variates fall above h. The probability that X_1 to X_s fall below h and X_{s+1} to X_k fall above h is given by

$$L_k(h;s,k-s,\rho) = \int_h^\infty \cdots \int_h^\infty \left[\int_{-\infty}^h \cdots \int_{-\infty}^h \phi_k(\underline{x};\rho)dx_1,\ldots,dx_s\right]dx_{s+1},\ldots,dx_k. \tag{1}$$

Due to exchangeability of the variates within a correlated set, the probability that precisely s of the k variates fall below h (and the remaining k-s above h) is given by

$$\binom{k}{s} L_k(h;s,k-s,\rho) \quad . \tag{2}$$

Using a slight variation of a form given by Gupta (1963) the multiple integral

(1) can be evaluated more simply as

$$L_k(h;s,k-s,\rho) = \int_{-\infty}^{\infty} \phi(y)[\Phi(w)]^s[1 - \Phi(w)]^{k-s}\, dy \tag{3}$$

where

$$\Phi(w) = \int_{-\infty}^{w} \phi(x)\, dx \text{ and } w = \frac{h+\rho^{\frac{1}{2}}y}{(1-\rho)^{\frac{1}{2}}} \quad .$$

The second probability required is the probability that precisely s of the k variates within a correlated subset fall below h, k-s-1 fall above h + δx and the remaining variate falls in $(h,h+\delta x)$, with δx taken to be arbitrarily small. This corresponds to the probability density function of $X_{[k-s;k,\rho]}$, the k-s order statistic in a sample of k equicorrelated multinormal variates, evaluated at h. This can be derived using an argument similar to that of David (1970) for independent variates.

The event $x < X_{[r;k,\rho]} < x + \delta x$ may be realized by $X_i \leq x$ for r-1 variates, $x < X_i \leq x + \delta x$ for one X_i, and $X_i > x + \delta x$ for the remaining k-r variates. The number of ways this can happen is

$$\frac{k!}{(r-1)!1!(k-r)!} = \frac{1}{B(r,k-r+1)}$$

and, because of exchangeability, each has equal probability. Taking X_k to be the variate falling in $(x,x+\delta x)$, the conditional probability that $X_1,\ldots,X_{r-1}$ take values less than x and $X_r,\ldots,X_{k-1}$ take values greater than $x + \delta x$ can be written as

$$\Pr[X_1 \leq x,\ldots,X_{r-1} \leq x, X_r > x+\delta x,\ldots,X_{k-1} > x+\delta x \mid x < X_k \leq x+\delta x] \quad .$$

Regarding δx as small, we have

$$\Pr(x<X_{[r;k,\rho]}\le x+\delta x) =$$

$$\frac{1}{B(r,k-r+1)} \Pr[X_1\le x,\ldots,X_{r-1}\le x, X_r>x+\delta x,\ldots,X_{k-1}>x+\delta x \mid x<X_k<x+\delta x]\phi(x)\delta x + \mathcal{O}(\delta x^2) \quad (4)$$

since the marginal distribution of X_k will be $\phi(x)$ and where $\mathcal{O}(\delta x^2)$ means terms of order $(\delta x)^2$ in which more than one variate falls in $(x,x+\delta x)$. Dividing by δx and letting $\delta x \to 0$ gives the p.d.f. for $X_{[r;k,\rho]}$ as

$$f(X_{(r)};k,\rho) = \frac{1}{B(r,k-r+1)} \Pr[X_1<x,\ldots,X_{r-1}<x,X_r>x,\ldots,X_{k-1}>x \mid X_k=x]\phi(x) \quad (5)$$

It is well known that the k-1 variate conditional distribution, conditional on $X_k = x$, is a multivariate normal with mean ρx and variance-covariance matrix

$$W = (1-\rho^2)\begin{bmatrix} 1 & \frac{\rho}{1+\rho} & \cdots & \frac{\rho}{1+\rho} \\ \frac{\rho}{1+\rho} & 1 & \cdots & \frac{\rho}{1+\rho} \\ \vdots & \vdots & & \vdots \\ \frac{\rho}{1+\rho} & \frac{\rho}{1+\rho} & \cdots & 1 \end{bmatrix}, \text{ (e.g., see Searle, 1971).}$$

Therefore, $\Pr[X_1<x,\ldots,X_{r-1}<x,X_r>x,\ldots,X_{k-1}>x \mid X_k=x]$ is equivalent to

$$\Pr[X_1^*<w,\ldots,X_{r-1}^*<w,X_r^*>w,\ldots,X_{k-1}^*>x \mid X_k=x].$$

where $X_i^* = (X_i-\rho x)/(1-\rho^2)^{\frac{1}{2}}$ and $w = \left(\frac{1-\rho}{1+\rho}\right)^{\frac{1}{2}}$. The joint p.d.f. of the X_i^* is $\phi_{k-1}(\underline{x}^*;\rho^*)$ where $\rho^* = \frac{\rho}{1+\rho}$ so that

$$\Pr[X_1^*<w,\ldots,X_{r-1}^*<w,X_r^*>w,\ldots,X_{k-1}>w \mid X_k=x] = L_{k-1}(w;r-1,k-r;\rho^*)$$

and the p.d.f. of $X_{(r;k,\rho)}$ can be written as

$$f_{(X_{(r)};k,\rho)} = \frac{1}{B(r,k-r+1)} L_{k-1}(w;r-1,k-r;\rho^*)\phi(x) \quad . \tag{6}$$

Probability Density Functions of Order Statistics in a Special Unequally Correlated Case.

The p.d.f. for the $(n-i)^{\underline{th}}$ order statistic, $X_{[n-i;n,m,\rho]}$, in a sample of n variables consisting of m independent sets of k variables with each set having a k-variate multinormal distribution with common correlation ρ can be constructed utilizing products of the probabilities determined in the previous section. In order for $X_{[n-i;n,m,\rho]}$ to take the value x, the $(k-u_m)^{th}$ order statistic, $u_m \le \min(i,k-1)$ of one of the correlated subsets, arbitrarily taken to be set m, must take the value x and the variates in the remaining (m-1) subsets must be distributed about x such that a total of exactly $i-u_m$ variates are greater than x. Let u_j be the number of variates in subset j which are greater than x. Clearly, $\sum_{j=1}^{m-1} u_j = i-u_m$ and $u_j \le \min(i-u_m,k)$ for $j \ne m$. For convenience, the vector $\{u_j\}$ is defined as the p nonzero u_j's arbitrarily ordered so that $j = 1,\ldots,p$; $p \le \min(i-u_m,m-1)$. That is, p is the number of subsets having at least one variable greater than x, m-p-1 subsets have all variables less than x and the remaining set has the $(k-u_m)^{\underline{th}}$ ordered variate taking the value x. The number of ways the m subsets can be divided into the three parcels is

$$\frac{m!}{p!1!(m-p-1)!} = \frac{1}{B(p+1,m-p)} \quad , \tag{7}$$

all allocations having equal probability since the subsets are exchangeable and independent.

Given i and p, there are $\binom{i}{p} = \frac{i!}{p!(i-p)!}$ combinations of values of u_m and $\{u_j\}$ meeting the condition that

$$\sum_{j=1}^{p} u_j + u_m = i \quad .$$

This corresponds, in the language of Whitworth (1942), to the number of ways in which i indifferent things can be distributed into p+1 parcels where, with the exception of one particular parcel, u_m, blank lots are inadmissible. Since $p = 0,1,\ldots,i$, the maximum number of combinations of p, u_m, and $\{u_j\}$ is $\sum_{p=0}^{i}\binom{i}{p} = 2^i$. This is an upper limit since the restrictions $u_m < k$ and $u_j \le k$ have been ignored. These restrictions are taken into account in the final form by applying the usual restriction on combinatorial formulas that $\binom{r}{s} = 0$ if $s > r$.

For each configuration of p, u_m, and $\{u_j\}$, the probability that $X_{[n-i;n,m,\rho]}$ takes the value x is given by the product

$$[\Pr(\text{all elements} < x)]^{m-p-1}\left[\prod_{j=1}^{p} \Pr(\text{exactly } u_j \text{ elements} > x)\right][\Pr(k\text{-}u_m^{\underline{th}} \text{ order statistic in set m equals x})] \qquad (8)$$

Substituting (2) for the quantities in the first and second brackets with $s = k$ and $s = k-u_j$, respectively, and (6) for the quantity in the last bracket with $r = k-u_m$, taking into account the number of ways the m subsets can be divided into the three parcels, and summing over the 2^i possible configurations of p, u_m, $\{u_j\}$ gives the p.d.f. of the $(n-i)^{\underline{th}}$ order statistic, $X_{[n-i;n,m,\rho]}$, as

$$f(X_{[n-i;n,m,\rho]}) = \sum^{2^i} N_{(p,u_m,\{u_j\})} \cdot P_{(x;p,u_m,\{u_j\})} \qquad (9)$$

where

$$N_{(p,u_m,\{u_j\})} = \frac{\prod_{j=1}^{p} \binom{k}{u_j}}{B(p+1,m-p)\ B(u_m+1,k-u_m)}$$

and

$$P_{(x;p,u_m,\{u_j\})} = [L_k(x;k,0,\rho)]^{m-p-1}\left[\prod_{i=1}^{p} L_k(x;k-u_j,u_j,\rho)\right][L_{k-1}(w;k-u_m-1,u_m,\rho^*)]$$

where

$$w = x\left(\frac{1-\rho}{1+\rho}\right)^{\frac{1}{2}} \quad \text{and} \quad \rho^* \text{ and } = \rho/1+\rho \quad .$$

The combinations of p, u_m and $\{u_j\}$ and their associated numbers of combinations and probabilities for the largest five order statistics are shown in Table 1. It is clear that any basic combinations differing only by rearrangements of the integers in $\{u_j\}$ will have the same $N_{(p,u_m,\{u_j\})}$ and $P_{(x;p,u_m,\{u_j\})}$ and can be pooled for simplification of $f(X_{[n-i;n,m,\rho]})$. Note also that the p.d.f.'s for the corresponding smallest order statistics are obtained by interchanging s and r in the $L(x;s,r,\rho)$ functions.

Evaluation of First Two Moments

Numerical integration was used to evaluate

$$\mathcal{E}(X^r_{[n-i;n,m,\rho]}) = \int_{-\infty}^{\infty} X^r_{[n-i;n,m,\rho]}\ f(X_{[n-i;n,m,\rho]})\ dx \qquad (10)$$

for r = 0,1,2 and $i = 0,1,\ldots,\min\left(\frac{n}{2},12\right)$. Two stages of integration are involved; the primary integration shown in (10) and the secondary integration to evaluate the functions $L_{(X;s,r,\rho)}$. The numerical integration in both stages covered the region $-6 < X < 8$ in two equal intervals using 32-point

Gaussian integration for each interval. The approximate accuracy of the integration was measured by the absolute deviations from unity of the integral of the p.d.f., i.e., the integral for $r = 0$.

Appendix Tables 1 and 2 give the means and variances, respectively, of the $(n-i)^{\underline{th}}$ order statistics for $i < \min\left(\frac{n}{2}, 12\right)$ obtained by numerical integration for all possible integer combinations of m and k for $n = (4,6,8,12,16,24,48)$ and for $\rho = (0,0.05,0.25,0.50,0.75,0.95,1.0)$. As specified earlier, all results for $m = 1$ or $m = n$ and for $\rho = 0.0$ or $\rho = 1.00$ are exact results and are included for completeness. (All expectations for normal order statistics under independence were taken from Harter, 1960. Variances of order statistics under independence for $n \leq 20$ were taken from Sarhan and Greenberg, 1956. For $n = 24$ and $n = 48$, $i = 0$, variances were obtained from Ruben (1954); for $i > 0$, variances were computed.) The approximate error of the numerical integration as measured by the absolute deviation from unity of the integral for $r = 0$ is of the order 10^{-5} or less unless otherwise noted by footnotes in Appendix Tables 1 and 2. It will be noted that with only three exceptions all deviations larger than 10^{-5} occurred for $\rho = 0.95$. Whether the absolute errors for $r = 1,2$ are larger or smaller than for $r = 0$ depends on whether the major errors of integration come from the tails of the distribution or from the intermediate values. At the worst, if all the error of integration occurred at $|X| \approx 10$, the absolute error would be of the order 10^{-4} for the mean and 10^{-3} for the variance.

The effect of the intraset correlation on the first moment is strongly nonlinear. The nonlinearity and the convergence by sets to the expectations under independence of the order statistics in samples of size m are illustrated in Figure 1 for $n = 24$ and $m = 6$. (The curves are freehand drawings through the points, shown by dots, at which the expectations were evaluated.) It is clear that the major effect of the correlation occurs when $\rho > 0.8$ and that the extreme order statistics are the most seriously affected by the correlation. Also, as is apparent from Figure 1, the affect is greatest when m is small relative to n, since the order statistics are converging, as $\rho \to 1.0$, to fewer points. Thus, taking the most extreme case of those studied (short of the equicorrelated case, k = 1), the first order statistic for (n,m) = (48,2), the expectations of the normal order statistic under independence is 1.3%, 8.3%, 22.4%, 50.5%, and 125.8% larger than the expectations of the first order statistics for ρ = 0.05,0.25,0.50,0.75 and 0.95, respectively. For purposes of reference, the combination showing the least affect of ρ on the first order statistic was (n,m) = (48,24) in which case the expectation of the first order statistic under independence was only 0.1%, 0.4%, 1.4%, 3.6% and 8.5% larger than the expectations under the corresponding nonzero intraset correlations.

The Owen and Steck result for the equicorrelated case suggests the approximation

$$\mathcal{E}(X_{[i;n,m,\rho]}) \doteq (1-\rho_a)^{\frac{1}{2}}\mathcal{E}(Z_{[i;n]})$$

where $\rho_a = \frac{k-1}{n-1}\rho$ is the average pairwise correlation. This obviously is not a good approximation for large ρ, since it does not provide for convergence to the required points as $\rho \to 1.0$, but it does seem to be a reasonable approximation for small values of ρ. This is illustrated in Figure 2 where the approximations for each order statistic are superimposed on the Figure 1 results. Clearly, the approximation is not good for the extreme order statistic, unless ρ is quite small, but it is a reasonable approximation for the intermediate order statistics if ρ is, say, less than 0.5.

Consistent with the original motivation for this problem, i.e., the prediction of genetic change from truncation selection, the primary interest is in obtaining a good approximation of the mean of the s largest order statistics in a family structured sample of size n. From the genetic point of view, ρ generally will be quite small; for example, the maximum ρ can be in a full sib family structured population, if all variability is genetic in origin, is $\frac{1}{2}$. In such cases, i.e., $\rho \le 0.5$, the approximation $\hat{i}(s,n,m,\rho) = (1-\rho_a)^{\frac{1}{2}}\bar{i}(s,n)$, where $\bar{i}(s,n)$ is the mean of s largest order statistics under independence, becomes an excellent approximation, as s approaches $\frac{n}{2}$, of the mean of the corresponding s largest order statistics under the correlation structure studied, $\tilde{i}(s,n,m,\rho)$. The percentage error from using $\hat{i}(s,n,m,\rho)$ is less than 1% when $s = \frac{n}{2}$ even when $\rho = 0.5$. When s = 1, the percentage error is considerably greater. For the worst case studied, (n,m) = (48,2), the percentage error, when s = 1, is .06% when $\rho = 0.05$, 1.4% when $\rho = 0.25$ and 6.3% when $\rho = 0.5$.

The usefulness of this approximation for small ρ has other desirable implications. First, since the average pairwise correlation is defineable for any sample structure, involving either equal or unequal set sizes and/or equal or unequal intraset correlations, the results of this study suggest that the average pairwise correlation used in the Owen and Steck formulation for expectations may provide reasonable approximations, as long as ρ is not too large, of expectations of order statistics in much more general cases of unequal correlations than the one studied herein. Secondly, the usual formulation of the selection differential in predicting genetic gain is the product of mean of the expectations of the s largest order statistics and the standard deviation among random individuals. Under independence this is $\bar{i}(s,n)\sigma$, where σ is the standard deviation among random individuals. The above approximation suggests that in a correlated system $\bar{i}(s,n)$ be replaced with $(1-\rho_a)^{\frac{1}{2}}\bar{i}(s,n)$ so that the selection differential becomes $\bar{i}(s,n)(1-\rho_a)^{\frac{1}{2}}\sigma$ (approximately). But, $(1-\rho_a)\,\sigma^2$ is the expectation of the total mean square among individuals in the same correlated system. Thus, insofar as the above approximation is satisfactory, the mean of the s largest order statistics under independence, $\bar{i}(s,n)$, can be used to estimate the selection differential as long as the standard deviation is adjusted to reflect the correlation structure. The latter is accomplished by using the estimated total mean square for the sample. No explicit information on the family structure or the magnitudes of the intraset correlations is required.

The effect of the intraset correlations on the variances of the order statistics (Appendix Table 2) is more nearly linear than is the effect on the expectations. All the variances are monotonically increasing functions of ρ with positive second derivatives and converging, again, to those of the order statistics in samples of size m under independence. The average rate of change in the variances with changing ρ is much greater, however, than that predicted by substituting the average intraset correlation in Owen and Steck's result for the variances under equicorrelation. No particular effort was devoted to finding an approximation for the variances.

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Table 1. Terms of the p.d.f.'s for the Five Largest Order Statistics in a Combined Sample of m Independent Samples of k Equicorrelated Variates Each.

Order Statistic	Configuration	No. of sets having all X's < x (m-1-p)	No. of X's > x U_0	$\{U_\ell\}$	$N_{(p,U_0,\{U_\ell\})}$	$P_{(x;p,U_0,\{U_\ell\})}$
n	1	m-1	0	ϕ	$\binom{m}{1}\binom{k}{1} = n$	$[L_k(x;k,0,\rho)]^{m-1}L_{k-1}(x^*;k-1,0,\rho^*)\ \phi(x)$
n-1	1	m-1	1	ϕ	$\binom{m}{1}\binom{k}{1,1} = n(k-1)$	$[L_k(x;k,0,\rho)]^{m-1}L_{k-1}(x^*;k-2,1,\rho^*)\ \phi(x)$
	2	m-2	0	{1}	$\binom{m}{1,1}\binom{k}{1}\binom{k}{1} = nk(m-1)$	$[L_k(x;k,0,\rho)]^{m-2}L_{k-1}(x^*;k-1,0,\rho^*)\ L_k(x;k-1,1,\rho)\ \phi(x)$
n-2	1	m-1	2	ϕ	$\binom{m}{1}\binom{k}{1,2} = \frac{n(k-1)(k-2)}{2}$	$[L_k(x;k,0,\rho)]^{m-1}L_{k-1}(x^*;k-3,2,\rho^*)\ \phi(x)$
	2	m-2	1	{1}	$\binom{m}{1,1}\binom{k}{1,1}\binom{k}{1} = n(m-1)k(k-1)$	$[L_k(x;k,0,\rho)]^{m-2}L_{k-1}(x^*;k-2,1,\rho^*)L_k(x;k-1,1,\rho)\ \phi(x)$
	3	m-2	0	{2}	$\binom{m}{1,1}\binom{k}{1}\binom{k}{2} = \frac{n(m-1)k(k-1)}{2}$	$[L_k(x;k,0,\rho)]^{m-2}L_{k-1}(x^*;k-1,0,\rho^*)L_k(x;k-2,2,\rho)\ \phi(x)$
	4	m-3	0	{1,1}	$\binom{m}{1,2}\binom{k}{1}^3 = \frac{n(m-1)(m-2)k^2}{2}$	$[L_k(x;k,0,\rho)]^{m-3}L_{k-1}(x^*;k-1,0,\rho^*)[L_k(x;k-1,1,\rho]^2\ \phi(x)$
n-3	1	m-1	3	ϕ	$\binom{m}{1}\binom{k}{1,3} = \frac{n(k-1)(k-2)(k-3)}{6}$	$[L_k(x;k,0,\rho)]^{m-1}L_{k-1}(x^*;k-4,3,\rho^*)\ \phi(x)$
	2	m-2	2	{1}	$\binom{m}{1,1}\binom{k}{1,2}\binom{k}{1} = \frac{n(m-1)k(k-1)(k-2)}{2}$	$[L_k(x;k,0,\rho)]^{m-2}L_{k-1}(x^*;k-3,2,\rho^*)L_k(x;k-1,1,\rho)\ \phi(x)$
	3	m-2	1	{2}	$\binom{m}{1,1}\binom{k}{1,1}\binom{k}{2} = \frac{n(m-1)k(k-1)^2}{2}$	$[L_k(x;k,0,\rho)]^{m-2}L_{k-1}(x^*;k-2,1,\rho^*)L_k(x;k-2,2,\rho)\ \phi(x)$
	4	m-2	0	{3}	$\binom{m}{1,1}\binom{k}{1}\binom{k}{3} = \frac{n(m-1)k(k-1)(k-2)}{6}$	$[L_k(x;k,0,\rho)]^{m-2}L_{k-1}(x^*;k-1,0,\rho^*)L_k(x;k-3,3,\rho)\ \phi(x)$
	5	m-3	1	{1,1}	$\binom{m}{1,2}\binom{k}{1,1}\binom{k}{1}^2 = \frac{n(m-1)(m-2)k^2(k-1)}{2}$	$[L_k(x;k,0,\rho)]^{m-3}L_{k-1}(x^*;k-2,1,\rho^*)[L_k(x;k-1,1,\rho)]^2\ \phi(x)$

Table 1. Terms of the p.d.f.'s for the Five Largest Order Statistics in a Combined Sample of m Independent Samples of k Equicorrelated Variates Each. (cont.)

Order Statistic	Configuration	No. of sets having all X's < x (m-1-p)	No. of X's > x U_0	No. of X's > x $\{U_\ell\}$	$N_{(p,U_0,\{U_\ell\})}$	$P_{(x;p,U_0,\{U_\ell\}}$
n-3	6	m-3	0	{2,1}	$\binom{m}{1,2}\binom{k}{1}^2\binom{k}{2} = \frac{n(m-1)(m-2)k^2(k-1)}{4}$	$[L_k(x;k,0,\rho)]^{m-3}L_{k-1}(x^*;k-1,0,\rho^*)L_k(x;k-2,2,\rho)L_k(x;k-1,1,\rho)\ \phi(x)$
	7	m-3	0	{1,2}	$\binom{m}{1,2}\binom{k}{1}^2\binom{k}{2} = \frac{n(m-1)(m-2)k^2(k-1)}{4}$	$[L_k(x;k,0,\rho)]^{m-3}L_{k-1}(x^*;k-1,0,\rho^*)L_k(x;k-2,2,\rho)L_k(x;k-1,1,\rho)\ \phi(x)$
	8	m-4	0	{1,1,1}	$\binom{m}{1,3}\binom{k}{1}^4 = \frac{n(m-1)(m-2)(m-3)k^3}{6}$	$[L_k(x;k,0,\rho)]^{m-4}L(x^*;k-1,0,\rho)[L_k(x;k-1,1,\rho)]^3\ \phi(x)$
n-4	1	m-1	4	ϕ	$\binom{m}{1}\binom{k}{1,4}$	$[L_k(x;k,0,\rho)]^{m-1}L_{k-1}(x^*;k-5,4,\rho^*)\ \phi(x)$
	2	m-2	3	{1}	$\binom{m}{1,1}\binom{k}{1,3}\binom{k}{1}$	$[L_k(x;k,0,\rho)]^{m-2}L_{k-1}(x^*;k-4,3,\rho^*)L_k(x;k-1,1,\rho)\ \phi(x)$
	3	m-2	2	{2}	$\binom{m}{1,1}\binom{k}{1,2}\binom{k}{2}$	$[L_k(x;k,0,\rho)]^{m-2}L_{k-1}(x^*;k-3,2,\rho^*)L_k(x;k-2,2,\rho)\ \phi(x)$
	4	m-2	1	{3}	$\binom{m}{1,1}\binom{k}{1,1}\binom{k}{3}$	$[L_k(x;k,0,\rho)]^{m-2}L_{k-1}(x^*;k-2,1,\rho^*)L_k(x;k-3,3,\rho)\ \phi(x)$
	5	m-2	0	{4}	$\binom{m}{1,1}\binom{k}{1}\binom{k}{4}$	$[L_k(x;k,0,\rho)]^{m-2}L_{k-1}(x^*;k-1,0,\rho^*)L_k(x;k-4,4,\rho)\ \phi(x)$
	6	m-3	2	{1,1}	$\binom{m}{1,2}\binom{k}{1,2}\binom{k}{1}^2$	$[L_k(x;k,0,\rho)]^{m-3}L_{k-1}(x^*;k-3,2,\rho^*)[L_k(x;k-1,1,\rho)]^2\ \phi(x)$
	7	m-3	1	{2,1}	$\binom{m}{1,2}\binom{k}{1,1}\binom{k}{2}\binom{k}{1}$	$[L_k(x;k,0,\rho)]^{m-3}L_{k-1}(x^*;k-2,1,\rho^*)L_k(x;k-1,1,\rho)L_k(x;k-2,2,\rho)\ \phi(x)$
	8	m-3	1	{1,2}	$\binom{m}{1,2}\binom{k}{1,1}\binom{k}{2}\binom{k}{1}$	$[L_k(x;k,0,\rho)]^{m-3}L_{k-1}(x^*;k-2,1,\rho^*)L_k(x;k-1,1,\rho)L_k(x;k-2,2,\rho)\ \phi(x)$
	9	m-3	0	{2,2}	$\binom{m}{1,2}\binom{k}{1}\binom{k}{2}^2$	$[L_k(x;k,0,\rho)]^{m-3}L_{k-1}(x^*;k-1,0,\rho^*)[L_k(x;k-2,2,\rho)]^2\ \phi(x)$

Table 1. Terms of the p.d.f.'s for the Five Largest Order Statistics in a Combined Sample of m Independent Samples of k Equicorrelated Variates Each. (cont.)

Order Statistic	Configuration	No. of sets having all X's < x (m-1-p)	No. of X's > x U_0	No. of X's > x $\{U_\ell\}$	$N_{(p,U_0,\{U_\ell\})}$	$P_{(x;p,U_0,\{U_\ell\})}$
n-4	10	m-3	0	{3,1}	$\binom{m}{1,2}\binom{k}{1}^2\binom{k}{3}$	$[L_k(x;k,0,\rho)]^{m-3}L_{k-1}(x^*;k-1,0,\rho^*)L_k(x;k-1,1,\rho)L_k(x;k-3,3,\rho)\ \phi(x)$
	11	m-3	0	{1,3}	$\binom{m}{1,2}\binom{k}{1}^2\binom{k}{3}$	$[L_k(x;k,0,\rho)]^{m-3}L_{k-1}(x^*;k-1,0,\rho^*)L_k(x;k-1,1,\rho)L_k(x;k-3,3,\rho)\ \phi(x)$
	12	m-4	1	{1,1,1}	$\binom{m}{1,3}\binom{k}{1,1}\binom{k}{1}^3$	$[L_k(x;k,0,\rho)]^{m-4}L_{k-1}(x^*;k-2,1,\rho^*)[L_k(x;k-1,1,\rho)]^3\ \phi(x)$
	13	m-4	0	{2,1,1}	$\binom{m}{1,3}\binom{k}{1}^3\binom{k}{2}$	$[L_k(x;k,0,\rho)]^{m-4}L_{k-1}(x^*;k-1,0,\rho^*)[L_k(x;k-1,1,\rho)]^2L_k(x;k-2,2,\rho)\ \phi(x)$
	14	m-4	0	{1,2,1}	$\binom{m}{1,3}\binom{k}{1}^3\binom{k}{2}$	$[L_k(x;k,0,\rho)]^{m-4}L_{k-1}(x^*;k-1,0,\rho^*)[L_k(x;k-1,1,\rho)]^2L_k(x;k-2,2,\rho)\ \phi(x)$
	15	m-4	0	{1,1,2}	$\binom{m}{1,3}\binom{k}{1}^3\binom{k}{2}$	$[L_k(x;k,0,\rho)]^{m-4}L_{k-1}(x^*;k-1,0,\rho^*)[L_k(x;k-1,1,\rho)]^2L_k(x;k-2,2,\rho)\ \phi(x)$
	16	m-5	0	{1,1,1,1}	$\binom{m}{1,4}\binom{k}{1}^5$	$[L_k(x;k,0,\rho)]^{m-5}L_{k-1}(x^*;k-1,0,\rho^*)[L_k(x;k-1,1,\rho)]^4\ \phi(x)$

Figure 1: Expectations of 12 largest order statistics for (n,m) = (24,6) as functions of the intraset correlations.

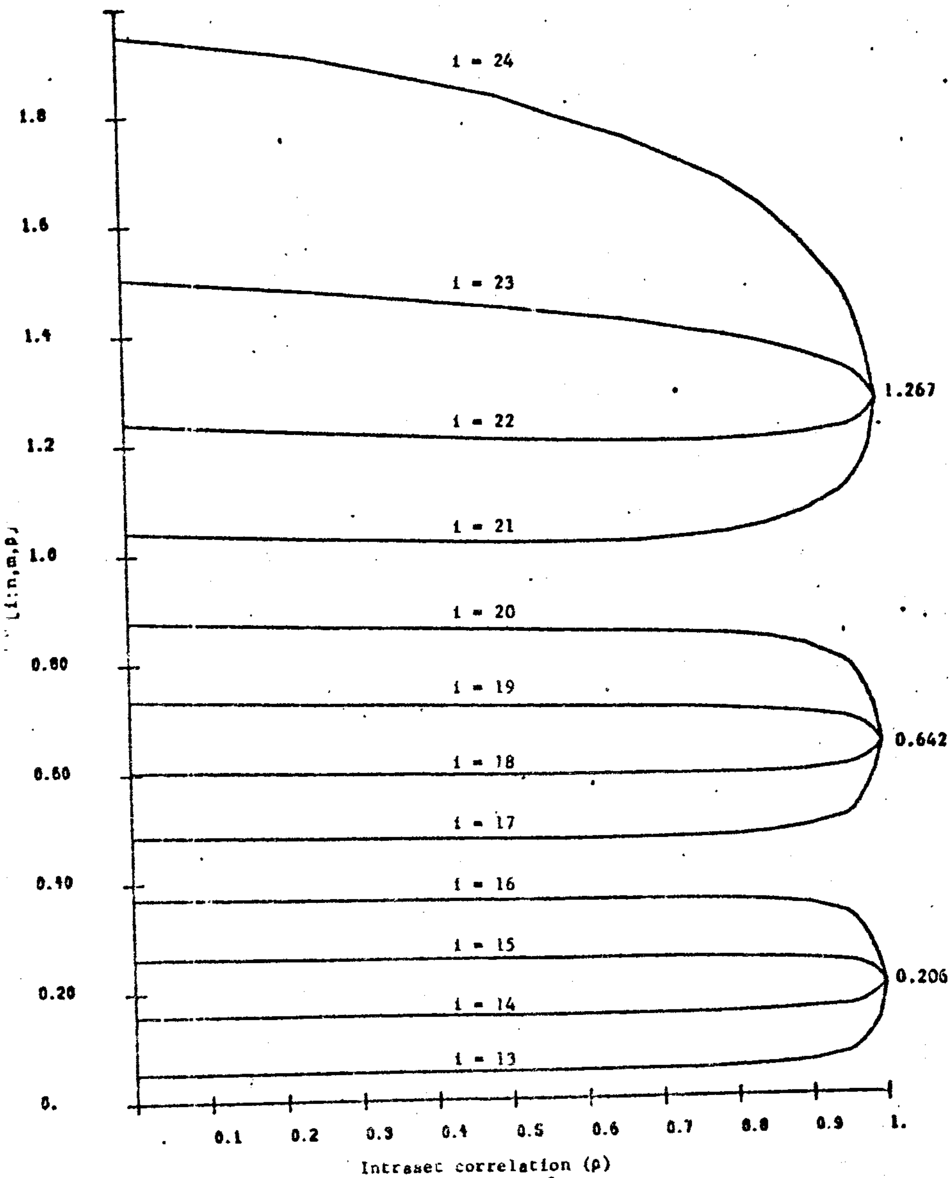
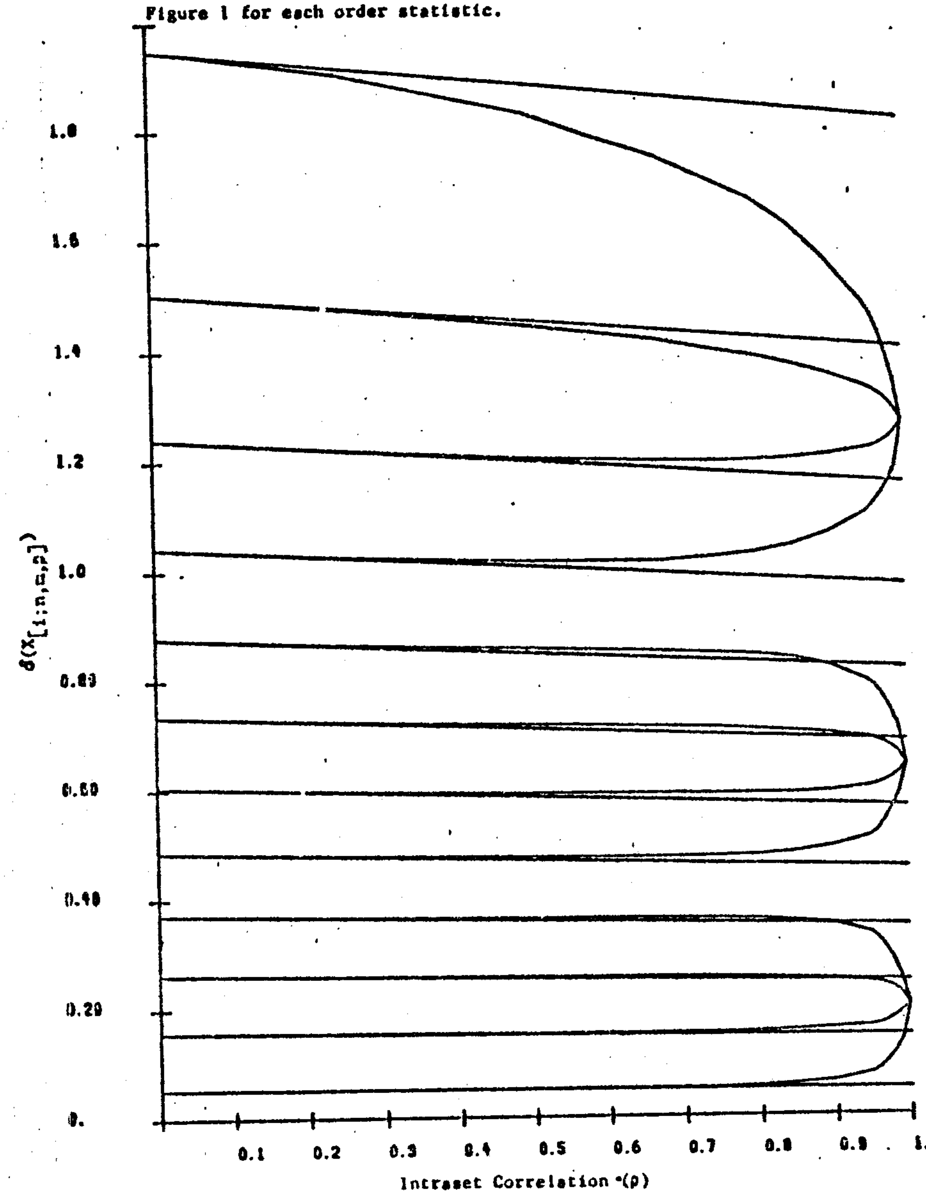


Figure 2: Values of the function $(1-\rho_a)^{\frac{1}{2}}\,\delta(Z_{[i;n]})$ superimposed on Figure 1 for each order statistic.



Appendix Table 1. Expectations of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k^c	0.0^a	0.05	0.25	0.50	0.75	0.95	1.00^b
1	4	4	1	1.02938	1.02938	1.02938	1.02938	1.02938	1.02938	1.02938
		2	2	1.02938	1.02058	0.98057	0.91618[1]	0.82335	0.68584	0.56419
		1	4	1.02938	1.00332	0.89147	0.72788	0.51469	0.23018	0.00000
2	4	4	1	0.29701	0.29701	0.29701	0.29701	0.29701	0.29701	0.29701
		2	2	0.29701	0.29483	0.29227	0.30611	0.35091	0.45156	0.56419
		1	4	0.29701	0.28949	0.25722	0.21002	0.14851	0.06641	0.00000
1	6	6	1	1.26721	1.26721	1.26721	1.26721	1.26721	1.26721	1.26721
		3	2	1.26721	1.26065	1.22958	1.17629	1.09445	0.96572	0.84628
		2	3	1.26721	1.25417	1.19412	1.09645	0.95533	0.74717	0.56419
		1	6	1.26721	1.23512	1.09744	0.89605	0.63361	0.28336	0.00000
2	6	6	1	0.64176	0.64176	0.64176	0.64176	0.64176	0.64176	0.64176
		3	2	0.64176	0.63871	0.63024	0.63159	0.65831	0.73963	0.84628
		2	3	0.64176	0.63554	0.61419	0.59339	0.57715	0.56657	0.56419
		1	6	0.64176	0.62551	0.55578	0.45379	0.32088	0.14350	0.00000

[a] Exact result = $z_{[n-i;n]}$. [b] Exact result = $z_{[n-i^*;m]}$ where i^* = integer immediately greater than i/k.

[c] Exact result = $(1-\rho)^{\frac{1}{2}} z_{[n-i;n]}$ when $k = n$.

[1,2,3] Absolute deviation of integral of p.d.f. from unity is of the order 10^{-5}, 10^{-4}, or 10^{-3}, respectively.

Appendix Table 1. Expectations of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k^c	0.0^a	0.05	0.25	0.50	0.75	0.95	1.00^b
3	6	6	1	0.20155	0.20155	0.20155	0.20155	0.20155	0.20155	0.02155
		3	2	0.20155	0.20061	0.19745	0.19088	0.16935	0.10126	0.00000
		2	3	0.20155	0.19965	0.19614	0.20894	0.26219	0.39908	0.56419
		1	6	0.20155	0.19645	0.17455	0.14252	0.10078	0.04507	0.00000
1	8	8	1	1.42360	1.42360	1.42360	1.42360	1.42360	1.42360	1.42360
		4	2	1.42360	1.41830	1.39237	1.34568	1.27059	1.14739	1.02938
		2	4	1.42360	1.40787	1.33473	1.21495	1.04178	0.78715	0.56419
		1	8	1.42360	1.38755	1.23287	1.00664	0.71180	0.31833	0.00000
2	8	8	1	0.85222	0.85222	0.85222	0.85222	0.85222	0.85222	0.85222
		4	2	0.85222	0.84927	0.83985	0.83691	0.85546	0.92636	1.02938
		2	4	0.85222	0.84322	0.80837	0.76283	0.70608	0.62917[1]	0.56419
		1	8	0.85222	0.83064	0.73804	0.60261	0.42611	0.19056	0.00000
3	8	8	1	0.47282	0.47282	0.47282	0.47282	0.47282	0.47282	0.47282
		4	2	0.47282	0.47122	0.46619	0.46047	0.44608	0.39211	0.29701
		2	4	0.47282	0.46793	0.45262	0.44580	0.45903	0.50609[1]	0.56419
		1	8	0.47282	0.46085	0.40947	0.33433	0.23641	0.10573	0.00000
4	8	8	1	0.15251	0.15251	0.15251	0.15251	0.15251	0.15251	0.15251
		4	2	0.15251	0.15200	0.15052	0.15058	0.15921	0.20533	0.29701
		2	4	0.15251	0.15095	0.14744	0.15816	0.21223	0.36675[1]	0.56419
		1	8	0.15251	0.14865	0.13208	0.10784	0.07626	0.03410	0.00000

Appendix Table 1. Expectations of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
1	12	12	1	1.62923	1.62923	1.62923	1.62923	1.62923	1.62923	1.62923
		6	2	1.62923	1.62533	1.60526	1.56640	1.49961	1.38339	1.26721
		4	3	1.62923	1.62146	1.58249	1.51086	1.39528	1.20730	1.02938
		3	4	1.62923	1.61763	1.56074	1.46046	1.30592	1.06568	0.84628
		2	6	1.62923	1.61005	1.51972	1.37056	1.15494	0.83924	0.56419
		1	12	1.62923	1.58798	1.41095	1.15204	0.81462	0.36431	0.00000
2	12	12	1	1.11573	1.11573	1.11573	1.11573	1.11573	1.11573	1.11573
		6	2	1.11573	1.11322	1.10422	1.09839	1.10928	1.16865	1.26721
		4	3	1.11573	1.11070	1.09138	1.06920	1.04889	1.03337	1.02938
		3	4	1.11573	1.10816	1.07785	1.03683	0.98422	0.91050[1]	0.84628
		2	6	1.11573	1.10303	1.04990	0.97094	0.86164	0.70302[1]	0.56419
		1	12	1.11573	1.08748	0.96625	0.78894	0.55787	0.24948	0.00000
3	12	12	1	0.79284	0.79284	0.79284	0.79284	0.79284	0.79284	0.79284
		6	2	0.79284	0.79110	0.78530	0.77979	0.77045	0.72985	0.64176
		4	3	0.79284	0.78934	0.77770	0.77165	0.79008	0.88074	1.02938
		3	4	0.79284	0.78757	0.76935	0.75506	0.75686	0.79185[1]	0.84628
		2	6	0.79284	0.78396	0.75064	0.71061	0.66543	0.60918[3]	0.56419
		1	12	0.79284	0.77276	0.68662	0.56062	0.39642	0.17728	0.00000

Appendix Table 1. Expectations of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
$i+1$	n	m	k^c	0.0^a	0.05	0.25	0.50	0.75	0.95	1.00^b
4	12	12	1	0.53684	0.53684	0.53684	0.53684	0.53684	0.53684	0.53684
		6	2	0.53684	0.53568	0.53199	0.52924	0.53108	0.56008	0.64176
		4	3	0.53684	0.53450	0.52699	0.51992	0.50452	0.43499	0.29701
		3	4	0.53684	0.53331	0.52213	0.51861	0.54568	0.66250[1]	0.84628
		2	6	0.53684	0.53089	0.51088	0.49656	0.49824	0.52573[2]	0.56419
		1	12	0.53684	0.52325	0.46492	0.37960	0.26842	0.12004	0.00000
5	12	12	1	0.31225	0.31225	0.31225	0.31225	0.31225	0.31225	0.31225
		6	2	0.31225	0.31158	0.30949	0.30773	0.30441	0.27951	0.20155
		4	3	0.31225	0.31090	0.30667	0.30322	0.30054	0.29793[1]	0.29701
		3	4	0.31225	0.31021	0.30377	0.29670	0.27400	0.17548[1]	0.00000
		2	6	0.31225	0.30881	0.29849	0.30041	0.33617	0.43871[2]	0.56419
		1	12	0.31225	0.30434	0.27042	0.22079	0.15613	0.06982	0.00000
6	12	12	1	0.10259	0.10259	0.10259	0.10259	0.10259	0.10259	0.10259
		6	2	0.10259	0.10237	0.10170	0.10127	0.10296	0.12533	0.20155
		4	3	0.10259	0.10215	0.10079	0.10046	0.10845	0.16544[1]	0.29701
		3	4	0.10259	0.10192	0.09980	0.09689	0.08721	0.05303[1]	0.00000
		2	6	0.10259	0.10146	0.09849	0.10607	0.15679	0.32703[1]	0.56419
		1	12	0.10259	0.09999	0.08885	0.07254	0.05130	0.02294	0.00000
1	16	16	1	1.76599	1.76599	1.76599	1.76599	1.76599	1.76599	1.76599
		8	2	1.76599	1.76287	1.74615	1.71198	1.65034	1.53857	1.42360
		4	4	1.76599	1.75668	1.70913	1.62057	1.47759	1.24648	1.02938
		2	8	1.76599	1.74458	1.64279	1.47387	1.22984	0.87360	0.56419
		1	16	1.76599	1.72127	1.52939	1.24874	0.88300	0.39489	0.00000

Appendix Table 1. Expectations of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
2	16	16	1	1.28474	1.28474	1.28474	1.28474	1.28474	1.28474	1.28474
		8	2	1.28474	1.28260	1.27439	1.26774	1.27497	1.32783	1.42360
		4	4	1.28474	1.27829	1.25163	1.21414	1.16457	1.09308[1]	1.02938
		2	8	1.28474	1.26962	1.20413	1.10268	0.95897	0.74859[2]	0.56419
		1	16	1.28474	1.25221	1.11262	0.90845	0.64237	0.28728	0.00000
3	16	16	1	0.99027	0.99027	0.99027	0.99027	0.99027	0.99027	0.99027
		8	2	0.99027	0.98866	0.98301	0.97759	0.97017	0.93620	0.85222
		4	4	0.99027	0.98540	0.96774	0.95160	0.94859	0.97716[1]	1.02938
		2	8	0.99027	0.97878	0.93282	0.86918	0.78532	0.66676[2]	0.56419
		1	16	0.99027	0.96520	0.85760	0.70023	0.49514	0.22143	0.00000
4	16	16	1	0.76317	0.76317	0.76317	0.76317	0.76317	0.76317	0.76317
		8	2	0.76317	0.76194	0.75787	0.75446	0.76432	0.77605	0.85222
		4	4	0.76317	0.75946	0.74697	0.73927	0.75567	0.85366[1]	1.02938
		2	8	0.76317	0.75440	0.72183	0.68438	0.64484	0.59399[2]	0.56419
		1	16	0.76317	0.74385	0.66092	0.53964	0.38159	0.17065	0.00000
5	16	16	1	0.57001	0.57001	0.57001	0.57001	0.57001	0.57001	0.57001
		8	2	0.57001	0.56910	0.56618	0.56374	0.56127	0.54384	0.47282
		4	4	0.57001	0.56726	0.55831	0.55049	0.53592	0.46067[1]	0.29701
		2	8	0.57001	0.56350	0.54103	0.52284	0.51852	0.53658[2]	0.56419
		1	16	0.57001	0.55558	0.49364	0.40306	0.28501	0.12746	0.00000

Appendix Table 1. Expectations of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
$i+1$	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
6	16	16	1	0.39622	0.39622	0.39622	0.39622	0.39622	0.39622	0.39622
		8	2	0.39622	0.39559	0.39362	0.39204	0.39181	0.40529	0.47282
		4	4	0.39622	0.39432	0.38826	0.38296	0.37500	0.34708[1]	0.29701
		2	8	0.39622	0.39172	0.37724	0.37288	0.39648	0.47160[2]	0.56419
		1	16	0.39622	0.38619	0.34314	0.28017	0.19811	0.08860	0.00000
7	16	16	1	0.23375	0.23375	0.23375	0.23375	0.23375	0.23375	0.23375
		8	2	0.23375	0.23338	0.23223	0.23131	0.23027	0.21796	0.15251
		4	4	0.23375	0.23263	0.22912	0.22661	0.22836	0.24955[2]	0.29701
		2	8	0.23375	0.23110	0.22315	0.22724	0.27033	0.39947[2]	0.56419
		1	16	0.23375	0.22783	0.20243	0.16529	0.11688	0.05227	0.00000
8	16	16	1	0.07729	0.07729	0.07729	0.07729	0.07729	0.07729	0.07729
		8	2	0.07729	0.07717	0.07679	0.07650	0.07678	0.08809	0.15251
		4	4	0.07729	0.07692	0.07577	0.07532	0.08214	0.14217[1]	0.29701
		2	8	0.07729	0.07641	0.07395	0.07964	0.12616	0.30257[1]	0.56419
		1	16	0.07729	0.07533	0.06694	0.05465	0.03865	0.01728	0.00000
1	24	24	1	1.94767	1.94767	1.94767	1.94767	1.94767	1.94767	1.94767
		12	2	1.94767	1.94540	1.93249	1.90392	1.84869	1.74267	1.62923
		8	3	1.94767	1.94313	1.91796	1.86497	1.76901	1.59730	1.42360
		6	4	1.94767	1.94089	1.90400	1.82948	1.70085	1.48135	1.26721
		4	6	1.94767	1.93643	1.87748	1.76594	1.58600	1.29768	1.02938
		3	8	1.94767	1.93202	1.85252	1.70950	1.48945	1.15128	0.84628
		2	12	1.94767	1.92335	1.80625	1.61081	1.32886	0.91887	0.56419
		1	24	1.94767	1.89835	1.68673	1.37721	0.97384	0.43551	0.00000

Appendix Table 1. Expectations of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
$i+1$	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
2	24	24	1	1.50338	1.50338	1.50338	1.50338	1.50338	1.50338	1.50338
		12	2	1.50338	1.50172	1.49482	1.48792	1.49158	1.53695	1.62923
		8	3	1.50338	1.50005	1.48585	1.46679	1.44658	1.42876[1]	1.42360
		6	4	1.50338	1.49839	1.47665	1.44406	1.39870	1.33023	1.26721
		4	6	1.50338	1.49506	1.45803	1.39822	1.30788	1.16443	1.02938
		3	8	1.50338	1.49173	1.43943	1.35393	1.22579	1.02765	0.84628
		2	12	1.50338	1.48508	1.40302	1.27151	1.08273	0.80602	0.56419
		1	24	1.50338	1.46531	1.30197	1.06305	0.75169	0.33617	0.00000
3	24	24	1	1.23924	1.23924	1.23924	1.23924	1.23924	1.23924	1.23924
		12	2	1.23924	1.23790	1.23287	1.22772	1.22188	1.19477	1.11573
		8	3	1.23924	1.23656	1.22638	1.21709	1.22236	1.28705[1]	1.42360
		6	4	1.23924	1.23521	1.21966	1.20319	1.19610	1.21774[1]	1.26721
		4	6	1.23924	1.23251	1.20561	1.17048	1.12854	1.07406[1]	1.02938
		3	8	1.23924	1.22980	1.19103	1.13546	1.05951	0.94734[2]	0.84628
		2	12	1.23924	1.22435	1.16122	1.06558	0.93144	0.73585[2]	0.56419
		1	24	1.23924	1.20786	1.07321	0.87628	0.61962	0.27710	0.00000
4	24	24	1	1.04091	1.04091	1.04091	1.04091	1.04091	1.04091	1.04091
		12	2	1.04091	1.03980	1.03587	1.03221	1.03077	1.04582	1.11573
		8	3	1.04091	1.03869	1.03075	1.02323	1.01507	0.97307	0.85222
		6	4	1.04091	1.03757	1.02554	1.01546	1.02277	1.10118	1.26721
		4	6	1.04091	1.03533	1.01462	0.99365	0.98265	0.99560[1]	1.02938
		3	8	1.04091	1.03307	1.00305	0.96683	0.92747	0.88136[1]	0.84628
		2	12	1.04091	1.02851	0.97849	0.90820	0.81434	0.68104[3]	0.56419
		1	24	1.04091	1.01455	0.90145	0.73603	0.52046	0.23275	0.00000

Appendix Table 1. Expectations of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order $i+1$	Sample Size n	Number of Groups m	Number of Individuals Per Group k[c]	Intra-Class Correlation (ρ) 0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
5	24	24	1	0.87682	0.87682	0.87682	0.87682	0.87682	0.87682	0.87682
		12	2	0.87682	0.87589	0.87274	0.86991	0.86778	0.85637	0.79284
		8	3	0.87682	0.87497	0.86860	0.86288	0.85875	0.85437[1]	0.85222
		6	4	0.87682	0.87403	0.86440	0.85579	0.84615	0.79204[1]	0.64176
		4	6	0.87682	0.87216	0.85576	0.84280	0.85036	0.91622[2]	1.02938
		3	8	0.87682	0.87027	0.84656	0.82360	0.81142	0.82225[3]	0.84628
		2	12	0.87682	0.86644	0.82648	0.77576	0.71439	0.63187[3]	0.56419
		1	24	0.87682	0.85462	0.75935	0.62000	0.43841	0.19606	0.00000
6	24	24	1	0.73354	0.73354	0.73354	0.73354	0.73354	0.73354	0.73354
		12	2	0.73354	0.73277	0.73022	0.73802	0.72686	0.73409[1]	0.79284
		8	3	0.73354	0.73200	0.72686	0.72247	0.72127	0.74546[1]	0.85222
		6	4	0.73354	0.73122	0.72345	0.71665	0.71004	0.68842[1]	0.64176
		4	6	0.73354	0.72966	0.71650	0.70724	0.71995	0.82144[2]	1.02938
		3	8	0.73354	0.72809	0.70923	0.69520	0.70271	0.75976[1]	0.84628
		2	12	0.73354	0.72490	0.69308	0.65809	0.62409	0.58969[2]	0.56419
		1	24	0.73354	0.71496	0.63526	0.51869	0.36677	0.16402	0.00000
7	24	24	1	0.60399	0.60399	0.60399	0.60399	0.60399	0.60399	0.60399
		12	2	0.60399	0.60336	0.60131	0.59959	0.59845	0.59211	0.53684
		8	3	0.60399	0.60273	0.59860	0.59514	0.59241	0.57286	0.47282
		6	4	0.60399	0.60209	0.59586	0.59067	0.58852	0.59999[3]	0.64176
		4	6	0.60399	0.60081	0.59027	0.58156	0.56906	0.49114[1]	0.29701
		3	8	0.60399	0.59952	0.58456	0.57593	0.59528	0.69204[3]	0.84628
		2	12	0.60399	0.59690	0.57190	0.54973	0.53925	0.54803[3]	0.56419
		1	24	0.60399	0.58869	0.52307	0.42708	0.30199	0.13506	0.00000

Appendix Table 1. Expectations of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, n = mk.

Order i+1	Sample Size n	Number of Groups m	Number of Individuals Per Group k[c]	Intra-Class Correlation (ρ) 0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
8	24	24	1	0.48391	0.48391	0.48391	0.48391	0.48391	0.48391	0.48391
		12	2	0.48391	0.48340	0.48180	0.48047	0.47974	0.48424	0.53684
		8	3	0.48391	0.48290	0.47966	0.47699	0.47534	0.47380[1]	0.47282
		6	4	0.48391	0.48239	0.47750	0.47356	0.47353	0.50737[1]	0.64176
		4	6	0.48391	0.48137	0.47310	0.46593	0.45420	0.40202[2]	0.29701
		3	8	0.48391	0.48034	0.46865	0.46222	0.48189	0.60693[2]	0.84628
		2	12	0.48391	0.47824	0.45908	0.44726	0.45697	0.50434[3]	0.56419
		1	24	0.48391	0.47165	0.41907	0.34217	0.24195	0.10820	0.00000
9	24	24	1	0.37047	0.37047	0.37047	0.37047	0.37047	0.37047	0.37047
		12	2	0.37047	0.37008	0.36887	0.36788	0.36725	0.36296	0.31225
		8	3	0.37047	0.36970	0.36725	0.36527	0.36457	0.37889	0.47282
		6	4	0.37047	0.36931	0.36562	0.36263	0.35964	0.33054[1]	0.20155
		4	6	0.37047	0.36853	0.36229	0.35703	0.35074	0.33126[1]	0.29701
		3	8	0.37047	0.36774	0.35891	0.35163	0.33459	0.22957[2]	0.00000
		2	12	0.37047	0.36614	0.35207	0.34826	0.37482	0.45849[2]	0.56419
		1	24	0.37047	0.36108	0.32083	0.26196	0.18523	0.08284	0.00000
10	24	24	1	0.26163	0.26163	0.26163	0.26163	0.26163	0.26163	0.26163
		12	2	0.26163	0.26136	0.26051	0.25983	0.25946	0.26291	0.31225
		8	3	0.26163	0.26109	0.25938	0.25801	0.25691	0.24376	0.15251
		6	4	0.26163	0.26082	0.25824	0.25616	0.25419	0.24212	0.20155
		4	6	0.26163	0.26027	0.25591	0.25255	0.25235	0.26500[2]	0.29701
		3	8	0.26163	0.25971	0.25352	0.24732	0.22872	0.14585[2]	0.00000
		2	12	0.26163	0.25858	0.24902	0.25081	0.29013	0.41002[3]	0.56419
		1	24	0.26163	0.25501	0.22658	0.18500	0.13082	0.05850	0.00000

Appendix Table 1. Expectations of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, n = mk.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
11	24	24	1	0.15583	0.15583	0.15583	0.15583	0.15583	0.15583	0.15583
		12	2	0.15583	0.15567	0.15517	0.15476	0.15450	0.15108	0.10259
		8	3	0.15583	0.15551	0.15450	0.15369	0.15321	0.15281	0.15251
		6	4	0.15583	0.15534	0.15382	0.15262	0.15251	0.16232[1]	0.20155
		4	6	0.15583	0.15502	0.15244	0.15073	0.15511	0.19720[2]	0.29701
		3	8	0.15583	0.15469	0.15102	0.14694	0.13401	0.08351[2]	0.00000
		2	12	0.15583	0.15401	0.14850	0.15308	0.19900	0.35374[1]	0.56419
		1	24	0.15583	0.15188	0.13495	0.11019	0.07791	0.03484	0.00000
12	24	24	1	0.05176	0.05176	0.05176	0.05176	0.05176	0.05176	0.05176
		12	2	0.05176	0.05170	0.05154	0.05140	0.05135	0.05452	0.10259
		8	3	0.05176	0.05165	0.05131	0.05105	0.05117	0.06303	0.15251
		6	4	0.05176	0.05160	0.05109	0.05071	0.05156	0.07673[1]	0.20155
		4	6	0.05176	0.05149	0.05063	0.05019	0.05524	0.15519[1]	0.29701
		3	8	0.05176	0.05138	0.05016	0.04874	0.04419	0.02755[2]	0.00000
		2	12	0.05176	0.05115	0.04937	0.05304	0.09264	0.27234[2]	0.56419
		1	24	0.05176	0.05045	0.04482	0.03660	0.02588	0.01157	0.00000
1	48	48	1	2.23312	2.23312	2.23312	2.23312	2.23312	2.23312	2.23312
		24	2	2.23312	2.23181	2.22354	2.20239	2.15624	2.05872	1.94767
		16	3	2.23312	2.23051	2.21431	2.17483	2.09427	1.93607	1.76599
		12	4	2.23312	2.22921	2.20539	2.14962	2.04130	1.83893	1.62923
		8	6	2.23312	2.22664	2.18833	2.10428	1.95223	1.68625	1.42360
		6	8	2.23312	2.22408	2.17216	2.06387	1.87758	1.56567	1.26721
		4	12	2.23312	2.21903	2.14193	1.99304	1.75398	1.37621	1.02938

Appendix Table 1. Expectations of Order Statistics (i < 12) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, n = mk.

Order i+1	Sample Size n	Number of Groups m	Number of Individuals Per Group k[c]	Intra-Class Correlation (ρ) 0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
1	48	3	16	2.23312	2.21406	2.11392	1.93130	1.65156	1.22616	0.84628
		2	24	2.23312	2.20432	2.06282	1.82509	1.48326	0.98918	0.56419
		1	48	2.23312	2.17658	1.93394	1.57905	1.11656	0.49934	0.00000
2	48	48	1	1.83655	1.83655	1.83655	1.83655	1.83655	1.83655	1.83655
		24	2	1.83655	1.83553	1.83073	1.82450	1.82465	1.86041[1]	1.94767
		16	3	1.83655	1.83451	1.82477	1.80948	1.79081	1.77196[1]	1.76599
		12	4	1.83655	1.83350	1.81873	1.79359	1.75484	1.69120[1]	1.62923
		8	6	1.83655	1.83146	1.80661	1.76164	1.68643	1.55559[1]	1.42360
		6	8	1.83655	1.82943	1.79456	1.73072	1.62450	1.44420[1]	1.26721
		4	12	1.83655	1.82539	1.77093	1.67297	1.51656	1.26508[2]	1.02938
		3	16	1.83655	1.82136	1.74815	1.62017	1.42389	1.12099[3]	0.84628
		2	24	1.83655	1.81339	1.70495	1.52598	1.26773	0.89099[2]	0.56419
		1	48	1.83655	1.79005	1.59050	1.29864	0.91828	0.41067	0.00000
3	48	48	1	1.60860	1.60860	1.60860	1.60860	1.60860	1.60860	1.60860
		24	2	1.60860	1.60773	1.60403	1.59959	1.59514	1.57567	1.50338
		16	3	1.60860	1.60685	1.59940	1.59063	1.59031	1.63875	1.76599
		12	4	1.60860	1.60598	1.59469	1.58019	1.57010	1.58344[1]	1.62923
		8	6	1.60860	1.60423	1.58508	1.55688	1.51981	1.46787[1]	1.42360
		6	8	1.60860	1.60249	1.57531	1.53241	1.46853	1.36590[2]	1.26721
		4	12	1.60860	1.59899	1.55561	1.48379	1.37322	1.19632[2]	1.02938
		3	16	1.60860	1.59550	1.53604	1.43732	1.28817	1.05749	0.84628
		2	24	1.60860	1.58854	1.49792	1.35164	1.14120	0.83447[2]	0.56419
		1	48	1.60860	1.56787	1.39309	1.13745	0.80430	0.35969	0.00000

Appendix Table 1. Expectations of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
$i+1$	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
4	48	48	1	1.44224	1.44224	1.44224	1.44224	1.44224	1.44224	1.44224
		24	2	1.44224	1.44147	1.43841	1.43503	1.43286	1.44150	1.50338
		16	3	1.44224	1.44070	1.43453	1.42766	1.42156	1.39345[1]	1.28474
		12	4	1.44224	1.43993	1.43060	1.42042	1.42016	1.47644[1]	1.62923
		8	6	1.44224	1.43838	1.42258	1.40332	1.38858	1.39310[1]	1.42360
		6	8	1.44224	1.43683	1.41435	1.38393	1.34768	1.30221[2]	1.26721
		4	12	1.44224	1.43373	1.39747	1.34279	1.26424	1.14248[3]	1.02938
		3	16	1.44224	1.43062	1.38033	1.30159	1.18608	1.00865[3]	0.84628
		2	24	1.44224	1.42439	1.34616	1.22304	1.04727	0.78992[3]	0.56419
		1	48	1.44224	1.40572	1.24902	1.01982	0.72112	0.32249	0.00000
5	48	48	1	1.30827	1.30827	1.30827	1.30827	1.30827	1.30827	1.30827
		24	2	1.30827	1.30758	1.30495	1.30220	1.30019	1.29376	1.23924
		16	3	1.30827	1.30688	1.30159	1.29603	1.29186	1.28753[1]	1.28474
		12	4	1.30827	1.30619	1.29820	1.28980	1.28289	1.24891[1]	1.11573
		8	6	1.30827	1.30480	1.29129	1.27703	1.27445	1.31938[3]	1.42360
		6	8	1.30827	1.30341	1.28420	1.26180	1.24451	1.25046[3]	1.26721
		4	12	1.30827	1.30061	1.26951	1.22715	1.17299	1.09194[3]	1.20938
		3	16	1.30827	1.29781	1.25434	1.19065	1.10154	0.96346[3]	0.84682
		2	24	1.30827	1.29217	1.22348	1.11849	0.97043[1]	0.75520[3]	0.56419
		1	48	1.30827	1.27514	1.13300	0.92509	0.65414	0.29254	0.00000
6	48	48	1	1.19439	1.19439	1.19439	1.19439	1.19439	1.19439	1.19439
		24	2	1.19439	1.19376	1.19145	1.18915	1.18760	1.19040[1]	1.23924
		16	3	1.19439	1.19313	1.18849	1.18384	1.18107	1.19321[1]	1.28474
		12	4	1.19439	1.19250	1.18550	1.17846	1.17284	1.15793[2]	1.11573

Appendix Table 1. Expectations of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
6	48	8	6	1.19439	1.19124	1.17944	1.16791	1.16869	1.23557[2]	1.42360
		6	8	1.19439	1.18998	1.17323	1.15605	1.15130	1.18942[3]	1.26721
		4	12	1.19439	1.18744	1.16032	1.12720	1.09232	1.05644[3]	1.02938
		3	16	1.19439	1.18489	1.14683	1.09502	1.02768[1]	0.92579[3]	0.84682
		2	24	1.19439	1.17976	1.11887	1.02883	0.90409	0.72193[3]	0.56419
		1	48	1.19439	1.16414	1.03436	0.84455	0.59719	0.26707	0.00000
7	48	48	1	1.09420	1.09420	1.09420	1.09420	1.09420	1.09420	1.09420
		24	2	1.09420	1.09363	1.09159	1.08961	1.08828	1.08546	1.04091
		16	3	1.09420	1.09306	1.08895	1.08497	1.08226	1.07291	0.99027
		12	4	1.09420	1.09249	1.08629	1.08028	1.07644	1.08062[a]	1.11573
		8	6	1.09420	1.09134	1.08090	1.07085	1.06323	1.01965[2]	0.85222
		6	8	1.09420	1.09019	1.07540	1.06155	1.06357	1.12532[3]	1.26721
		4	12	1.09420	1.08787	1.06396	1.03789	1.01844	1.02146[3]	1.02938
		3	16	1.09420	1.08554	1.05193	1.00979	0.96082	0.89540[3]	0.84682
		2	24	1.09420	1.08085	1.02657	0.94928	0.84485[1]	0.69179[2]	0.56419
		1	48	1.09420	1.06650	0.94761	0.77372	0.54710	0.24467	0.00000
8	48	48	1	1.00396	1.00396	1.00396	1.00396	1.00396	1.00396	1.00396
		24	2	1.00396	1.00344	1.00161	0.99990	0.99878	0.99984[1]	1.04091
		16	3	1.00396	1.00292	0.99925	0.99579	0.99353	0.99179[1]	0.99027
		12	4	1.00396	1.00240	0.99686	0.99166	0.98878	1.00400[1]	1.11573
		8	6	1.00396	1.00135	0.99203	0.98326	0.97567	0.94438[2]	0.85222
		6	8	1.00396	1.00030	0.98712	0.97530	0.97778	1.05561[2]	1.26721

Appendix Table 1. Expectations of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
8	48	4	12	1.00396	0.99818	0.97694	0.95624	0.94894	0.98004[3]	1.02938
		3	16	1.00396	0.99605	0.96618	0.93202	0.89880[1]	0.86895[3]	0.84682
		2	24	1.00396	0.99175	0.94320	0.87702	0.79070[1]	0.66495[3]	0.56419
		1	48	1.00396	0.97854	0.86945	0.70991	0.50198	0.22449	0.00000
9	48	48	1	0.92125	0.92125	0.92125	0.92125	0.92125	0.92125	0.92125
		24	2	0.92125	0.92077	0.91913	0.91763	0.91666	0.91517	0.87682
		16	3	0.92125	0.92029	0.91700	0.91398	0.91209	0.91654[1]	0.99027
		12	4	0.92125	0.91982	0.91485	0.91030	0.90733	0.89551	0.79284
		8	6	0.92125	0.91886	0.91051	0.90283	0.89692	0.88326[2]	0.85222
		6	8	0.92125	0.91790	0.90609	0.89548	0.88755	0.83484[2]	0.64176
		4	12	0.92125	0.91596	0.89699	0.88033	0.88210	0.93354[3]	1.02938
		3	16	0.92125	0.91401	0.88738	0.85983	0.84020[1]	0.84173[3]	0.84682
		2	24	0.92125	0.91007	0.86660	0.81022	0.74030[1]	0.64138[3]	0.56419
		1	48	0.92125	0.89792	0.79782	0.65142	0.46062	0.20600	0.00000
10	48	48	1	0.84442	0.84442	0.84442	0.84442	0.84442	0.84442	0.84442
		24	2	0.84442	0.84398	0.84251	0.84118	0.84034	0.84078	0.87682
		16	3	0.84442	0.84355	0.84058	0.83792	0.83623	0.83193	0.76317
		12	4	0.84442	0.84311	0.83864	0.83464	0.83203	0.82608[1]	0.79284
		8	6	0.84442	0.84223	0.83473	0.82799	0.82388	0.82660[2]	0.85222
		6	8	0.84442	0.84135	0.83075	0.82127	0.81224	0.76695[1]	0.64176
		4	12	0.84442	0.83959	0.82259	0.80884	0.81655	0.88819[3]	1.02938
		3	16	0.84442	0.83780	0.81400	0.79191	0.78393[1]	0.81055[3]	0.84682
		2	24	0.84442	0.83420	0.79526	0.74765	0.69266[1]	0.62054[3]	0.56419
		1	48	0.84442	0.82304	0.73129	0.59709	0.42221	0.18882	0.00000

Appendix Table 1. Expectations of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order i+1	Sample Size n	Number of Groups m	Number of Individuals Per Group k[c]	Intra-Class Correlation (ρ) 0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
11	48	48	1	0.77228	0.77228	0.77228	0.77228	0.77228	0.77228	0.77228
		24	2	0.77228	0.77189	0.77056	0.76939	0.76865	0.76775	0.73354
		16	3	0.77228	0.77149	0.76882	0.76647	0.76500	0.76410[1]	0.76317
		12	4	0.77228	0.77109	0.76707	0.76353	0.76137	0.76330[2]	0.79284
		8	6	0.77228	0.77029	0.76354	0.75761	0.75477	0.77130[2]	0.85222
		6	8	0.77228	0.76949	0.75996	0.75154	0.74373	0.71458[2]	0.64176
		4	12	0.77228	0.76788	0.75263	0.74086	0.75080	0.84327[2]	1.02938
		3	16	0.77228	0.76625	0.74496	0.72733	0.72908	0.77549[3]	0.84682
		2	24	0.77228	0.76296	0.72814	0.68839	0.64708[1]	0.60128[3]	0.56419
		1	48	0.77228	0.75273	0.66882	0.54609	0.38614	0.17269	0.00000
12	48	48	1	0.70397	0.70397	0.70397	0.70397	0.70397	0.70397	0.70397
		24	2	0.70397	0.70361	0.70242	0.70138	0.70073	0.70092	0.73354
		16	3	0.70397	0.70325	0.70085	0.69877	0.69749	0.69958	0.76317
		12	4	0.70397	0.70289	0.69928	0.69614	0.69429	0.70068[1]	0.79284
		8	6	0.70397	0.70216	0.69610	0.69085	0.68846	0.71028[2]	0.85222
		6	8	0.70397	0.70143	0.69287	0.68544	0.67960	0.66659[2]	0.64176
		4	12	0.70397	0.69997	0.68629	0.67573	0.68269	0.78126[3]	1.02938
		3	16	0.70397	0.69848	0.67945	0.66537	0.67495[1]	0.73915[3]	0.84682
		2	24	0.70397	0.69549	0.66442	0.63177	0.60311[1]	0.58212[2]	0.56419
		1	48	0.70397	0.68615	0.60966	0.49778	0.35199	0.15741	0.00000

Appendix Table 2. Variances of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
$i+1$	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
1	4	4	1	0.49172	0.49172	0.49172	0.49172	0.49172	0.49172	0.49172
		2	2	0.49172	0.50024	0.53545	0.58170	0.63030	0.67106	0.68169
		1	4	0.49172	0.51713	0.61879	0.74586	0.87293	0.97459	1.00000
2	4	4	1	0.36046	0.36046	0.36046	0.36046	0.36046	0.36046	0.36046
		2	2	0.36046	0.37125	0.41762	0.48521	0.56866	0.65463	0.68169
		1	4	0.36046	0.39243	0.52034	0.68023	0.84011	0.96802	1.00000
1	6	6	1	0.41593	0.41593	0.41593	0.41593	0.41593	0.41593	0.41593
		3	2	0.41593	0.42184	0.44692	0.48118	0.51853	0.55087	0.55947
		2	3	0.41593	0.42772	0.47682	0.54176	0.60998	0.66696	0.68169
		1	6	0.41593	0.44513	0.56195	0.70796	0.85398	0.97080	1.00000
2	6	6	1	0.27958	0.27958	0.27958	0.27958	0.27958	0.27958	0.27958
		3	2	0.27958	0.28694	0.31998	0.37249	0.44481	0.52954	0.55947
		2	3	0.27958	0.29421	0.35759	0.44873	0.55484	0.65366	0.68169
		1	6	0.27958	0.31560	0.45968	0.63979	0.81989	0.96398	1.00000

[a] Exact result = $\sigma^2_{Z[n-i;n]}$ for $n \le 20$, taken from Sarhan and Greenberg (1956) for all i; for $n > 20$, $i = 0$, from Rubin (1954); for $n > 20$ and $i > 0$ variances were computed.

[b] Exact result = $\sigma^2_{Z[n-i^*;m]}$ where i^* = integer immediately greater than i/k.

[c] Exact result = $\rho + (1-\rho)\,\sigma^2_{Z[n-i;n]}$ when $k = n$.

[1, 2, 3] Absolute deviation of integral of p.d.f. from unity is of the order 10^{-5}, 10^{-4}, or 10^{-3}, respectively.

Appendix Table 2. Variances of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
$i+1$	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
3	6	6	1	0.24621	0.24621	0.24621	0.24621	0.24621	0.24621	0.24621
		3	2	0.24621	0.25379	0.28504	0.32733	0.37677	0.42967	0.44867
		2	3	0.24621	0.26135	0.32396	0.41154	0.52068	0.64085	0.68169
		1	6	0.24621	0.28390	0.43466	0.62311	0.81155	0.96231	1.00000
1	8	8	1	0.37290	0.37290	0.37290	0.37290	0.37290	0.37290	0.37290
		4	2	0.37290	0.37746	0.39723	0.42514	0.45646	0.48422	0.49172
		2	4	0.37290	0.38649	0.44347	0.51913	0.59854	0.66465	0.68169
		1	8	0.37290	0.40425	0.52967	0.68645	0.84322	0.96864	1.00000
2	8	8	1	0.23940	0.23940	0.23940	0.23940	0.23940	0.23940	0.23940
		4	2	0.23940	0.24498	0.27085	0.31439	0.37877	0.46062	0.49172
		2	4	0.23940	0.25599	0.32817	0.43112	0.54798	0.65249[1]	0.68169
		1	8	0.23940	0.27743	0.42955	0.61970	0.80985	0.96197	1.00000
3	8	8	1	0.20077	0.20077	0.20077	0.20077	0.20077	0.20077	0.20077
		4	2	0.20077	0.20654	0.23093	0.26472	0.30390	0.34507	0.36046
		2	4	0.20077	0.21803	0.29092	0.39516	0.52073	0.64460[1]	0.68169
		1	8	0.20077	0.24073	0.40058	0.60038	0.80019	0.96004	1.00000
4	8	8	1	0.18719	0.18719	0.18719	0.18719	0.18719	0.18719	0.18719
		4	2	0.18719	0.19301	0.21695	0.24977	0.29031	0.33954	0.36046
		2	4	0.18719	0.20464	0.27587	0.37283	0.49315	0.63172[1]	0.68169
		1	8	0.18719	0.22783	0.39039	0.59359	0.79680	0.95936	1.00000

Appendix Table 2. Variances of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
1	12	12	1	0.32364	0.32364	0.32364	0.32364	0.32364	0.32364	0.32364
		6	2	0.32364	0.32679	0.34100	0.36213	0.38691	0.40965	0.41593
		4	3	0.32364	0.32993	0.35774	0.39755	0.44214	0.48127	0.49172
		3	4	0.32364	0.33305	0.37398	0.43088	0.49275	0.54560	0.55947
		2	6	0.32364	0.33924	0.40527	0.49333	0.58559	0.66207	0.68169
		1	12	0.32364	0.35745	0.49273	0.66182	0.83091	0.96618	1.00000
2	12	12	1	0.19726	0.19726	0.19726	0.19726	0.19726	0.19726	0.19726
		6	2	0.19726	0.20105	0.21940	0.25287	0.30723	0.38396	0.41593
		4	3	0.19726	0.20480	0.24026	0.29896	0.37840	0.46425	0.49172
		3	4	0.19726	0.20853	0.26011	0.33989	0.43780	0.53216[1]	0.55947
		2	6	0.19726	0.21539	0.29749	0.41288	0.54070	0.65204[1]	0.68169
		1	12	0.19726	0.23740	0.39795	0.59863	0.79932	0.95986	1.00000
3	12	12	1	0.15798	0.15798	0.15798	0.15798	0.15798	0.15798	0.15798
		6	2	0.15798	0.16189	0.17923	0.20498	0.23594	0.26768	0.27958
		4	3	0.15798	0.16578	0.20044	0.25515	0.33499	0.44487	0.49172
		3	4	0.15798	0.16966	0.22121	0.30071	0.40588	0.52143[1]	0.55947
		2	6	0.15798	0.17737	0.26099	0.38099	0.51956	0.64638[2]	0.68169
		1	12	0.15798	0.20008	0.36848	0.57899	0.78949	0.95790	1.00000
4	12	12	1	0.13981	0.13981	0.13981	0.13981	0.13981	0.13981	0.13981
		6	2	0.13981	0.14376	0.16053	0.18443	0.21542	0.25787	0.27958
		4	3	0.13981	0.14771	0.18111	0.22770	0.28146	0.33782	0.36046
		3	4	0.13981	0.15165	0.20175	0.27460	0.37325	0.50421[1]	0.55947
		2	6	0.13981	0.15950	0.24241	0.36069	0.50252	0.64082[2]	0.68169
		1	12	0.13981	0.18282	0.35486	0.56991	0.78495	0.95699	1.00000

Appendix Table 2. Variances of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
5	12	12	1	0.13061	0.13061	0.13061	0.13061	0.13061	0.13061	0.13061
		6	2	0.13061	0.13459	0.15100	0.17352	0.20061	0.23194	0.24621
		4	3	0.13061	0.13855	0.17130	0.21620	0.27137	0.33573[1]	0.36046
		3	4	0.13061	0.14252	0.19146	0.25708	0.33191	0.41451[1]	0.44867
		2	6	0.13061	0.15044	0.23192	0.34504	0.48458	0.63346[2]	0.68169
		1	12	0.13061	0.17408	0.34796	0.56531	0.78265	0.95653	1.00000
6	12	12	1	0.12664	0.12664	0.12664	0.12664	0.12664	0.12664	0.12664
		6	2	0.12664	0.13062	0.14687	0.16889	0.19565	0.22919	0.24621
		4	3	0.12664	0.13459	0.16702	0.21076	0.26379	0.32953[1]	0.36046
		3	4	0.12664	0.13857	0.18708	0.25237	0.33099	0.41783[1]	0.44867
		2	6	0.12664	0.14651	0.22681	0.33286	0.46209	0.61958[1]	0.68169
		1	12	0.12664	0.17031	0.34498	0.56332	0.78166	0.95633	1.00000
1	16	16	1	0.29501	0.29501	0.29501	0.29501	0.29501	0.29501	0.29501
		8	2	0.29501	0.29744	0.30870	0.32618	0.34737	0.36726	0.37290
		4	4	0.29501	0.30226	0.33473	0.38160	0.43397	0.47961	0.49172
		2	8	0.29501	0.31175	0.38309	0.47842	0.57815	0.66059	0.68169
		1	16	0.29501	0.33026	0.47126	0.64750	0.82375	0.96475	1.00000
2	16	16	1	0.17439	0.17439	0.17439	0.17439	0.17439	0.17439	0.17439
		8	2	0.17439	0.17727	0.19170	0.21957	0.26779	0.34067	0.37290
		4	4	0.17439	0.18296	0.22370	0.29056	0.37737	0.46531	0.49172
		2	8	0.17439	0.19412	0.28092	0.40307	0.53672	0.65150[2]	0.68169
		1	16	0.17439	0.21567	0.38080	0.58720	0.79360	0.95872	1.00000

Appendix Table 2. Variances of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
3	16	16	1	0.13634	0.13634	0.13634	0.13634	0.13634	0.13634	0.13634
		8	2	0.13634	0.13930	0.15295	0.17448	0.20157	0.22923	0.23940
		4	4	0.13634	0.14520	0.18564	0.25151	0.34423	0.45346[1]	0.49172
		2	8	0.13634	0.15684	0.24617	0.37400	0.51848	0.64694[2]	0.68169
		1	16	0.13634	0.17952	0.35225	0.56817	0.78408	0.95682	1.00000
4	16	16	1	0.11787	0.11787	0.11787	0.11787	0.11787	0.11787	0.11787
		8	2	0.11787	0.12086	0.13399	0.15354	0.17971	0.21776	0.23940
		4	4	0.11787	0.12683	0.16603	0.22593	0.31148	0.43465[1]	0.49172
		2	8	0.11787	0.13868	0.22789	0.35599	0.50514	0.64297[2]	0.68169
		1	16	0.11787	0.16197	0.33840	0.55893	0.77947	0.95589	1.00000
5	16	16	1	0.10735	0.10735	0.10735	0.10735	0.10735	0.10735	0.10735
		8	2	0.10735	0.11036	0.12312	0.14126	0.16364	0.18925	0.20077
		4	4	0.10735	0.11637	0.15448	0.20787	0.26938	0.33339[1]	0.36046
		2	8	0.10735	0.12833	0.21654	0.34231	0.49284	0.63902[2]	0.68169
		1	16	0.10735	0.15198	0.33051	0.55368	0.77684	0.95537	1.00000
6	16	16	1	0.10105	0.10105	0.10105	0.10105	0.10105	0.10105	0.10105
		8	2	0.10105	0.10406	0.11659	0.13391	0.15537	0.18392	0.20077
		4	4	0.10105	0.11008	0.14751	0.19889	0.26123	0.33291[1]	0.36046
		2	8	0.10105	0.12210	0.20906	0.33075	0.47979	0.63410[2]	0.68169
		1	16	0.10105	0.14600	0.32579	0.55053	0.77526	0.95505	1.00000

Appendix Table 2. Variances of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, n = mk.

Order i+1	Sample Size n	Number of Groups m	Number of Individuals Per Group k[c]	Intra-Class Correlation (ρ) 0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
7	16	16	1	0.09740	0.09740	0.09740	0.09740	0.09740	0.09740	0.09740
		8	2	0.09740	0.10042	0.11280	0.12963	0.14998	0.17453	0.18719
		4	4	0.09740	0.10645	0.14345	0.19362	0.25539	0.32998[2]	0.36046
		2	8	0.09740	0.11850	0.20429	0.32060	0.46442	0.62566[2]	0.68169
		1	16	0.09740	0.14253	0.32305	0.54870	0.77435	0.95487	1.00000
8	16	16	1	0.09572	0.09572	0.09572	0.09572	0.09572	0.09572	0.09572
		8	2	0.09572	0.09874	0.11105	0.12765	0.14772	0.17294	0.18719
		4	4	0.09572	0.10478	0.14156	0.19085	0.24986	0.32337[1]	0.36046
		2	8	0.09572	0.11684	0.20136	0.31244	0.44473	0.61162[1]	0.68169
		1	16	0.09572	0.14093	0.32179	0.54786	0.77393	0.95479	1.00000
1	24	24	1	0.26151	0.26151	0.26151	0.26151	0.26151	0.26151	0.26151
		12	2	0.26151	0.26319	0.27135	0.28483	0.30204	0.31883	0.32364
		8	3	0.26151	0.26486	0.28084	0.30615	0.33676	0.36509	0.37290
		6	4	0.26151	0.26652	0.29006	0.32609	0.36810	0.40576	0.41593
		4	6	0.26151	0.26982	0.30780	0.36309	0.42462	0.47774	0.49172
		3	8	0.26151	0.27309	0.32480	0.39726	0.47596	0.54226	0.55947
		2	12	0.26151	0.27956	0.35717	0.46111	0.56995	0.65889	0.68169
		1	24	0.26151	0.29843	0.44613	0.63076	0.81538	0.96308	1.00000
2	24	24	1	0.14899	0.14899	0.14899	0.14899	0.14899	0.14899	0.14899
		12	2	0.14899	0.15094	0.16127	0.18295	0.22381	0.29138	0.32364
		8	3	0.14899	0.15288	0.17296	0.21141	0.27164	0.34633	0.37290
		6	4	0.14899	0.15481	0.18416	0.23665	0.31047	0.39060[1]	0.41593

Appendix Table 2. Variances of Order Statistics (i < 12) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, n = mk.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
2	24	4	6	0.14899	0.15864	0.20537	0.28128	0.37574	0.46600	0.49172
		3	8	0.14899	0.16243	0.22534	0.32107	0.43232	0.53230	0.55947
		2	12	0.14899	0.16990	0.26261	0.39228	0.53231	0.65084	0.68169
		1	24	0.14899	0.19154	0.36174	0.57449	0.78725	0.95745	1.00000
3	24	24	1	0.11337	0.11337	0.11337	0.11337	0.11337	0.11337	0.11337
		12	2	0.11337	0.11538	0.12517	0.14203	0.16500	0.18886	0.19726
		8	3	0.11337	0.11738	0.13684	0.17181	0.23007	0.32486	0.37290
		6	4	0.11337	0.11937	0.14827	0.19919	0.27716	0.37778[1]	0.41593
		4	6	0.11337	0.12334	0.17032	0.24782	0.35089	0.45854[1]	0.49172
		3	8	0.11337	0.12727	0.19131	0.29080	0.41201	0.52665[2]	0.55947
		2	12	0.11337	0.13505	0.23067	0.36665	0.51702	0.64727[2]	0.68169
		1	24	0.11337	0.15770	0.33503	0.55669	0.77834	0.95567	1.00000
4	24	24	1	0.09568	0.09568	0.09568	0.09568	0.09568	0.09568	0.09568
		12	2	0.09568	0.09771	0.10707	0.12210	0.14328	0.17613	0.19726
		8	3	0.09568	0.09973	0.11839	0.14823	0.18650	0.22488	0.23940
		6	4	0.09568	0.10174	0.12965	0.17561	0.24610	0.35790	0.41593
		4	6	0.09568	0.10576	0.15176	0.22643	0.33052	0.45064[1]	0.49172
		3	8	0.09568	0.10975	0.17315	0.27176	0.39669	0.52189[1]	0.55947
		2	12	0.09568	0.11768	0.21370	0.35137	0.50681	0.64316[3]	0.68169
		1	24	0.09568	0.14090	0.32176	0.54784	0.77392	0.95478	1.00000

Appendix Table 2. Variances of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
5	24	24	1	0.08111	0.08111	0.08111	0.08111	0.08111	0.08111	0.08111
		12	2	0.08111	0.08714	0.09620	0.10996	0.12793	0.14893	0.15798
		8	3	0.08111	0.08917	0.10724	0.13469	0.17088	0.21772[1]	0.23940
		6	4	0.08111	0.09120	0.11824	0.15920	0.20942	0.25931[1]	0.27958
		4	6	0.08111	0.09524	0.14010	0.21021	0.31004	0.44030[2]	0.49172
		3	8	0.08111	0.09927	0.16157	0.25729	0.38251	0.51631[3]	0.55947
		2	12	0.08111	0.10727	0.20282	0.34019	0.49839	0.64120[3]	0.68169
		1	24	0.08111	0.12706	0.31083	0.54056	0.77028	0.95406	1.00000
6	24	24	1	0.07817	0.07817	0.07817	0.07817	0.07817	0.07817	0.07817
		12	2	0.07817	0.08020	0.08902	0.10194	0.11860	0.14189[1]	0.15798
		8	3	0.07817	0.08224	0.09985	0.12564	0.15932	0.20815[1]	0.23940
		6	4	0.07817	0.08427	0.11064	0.14913	0.19759	0.25541[1]	0.27958
		4	6	0.07817	0.08832	0.13215	0.19715	0.28666	0.42143[2]	0.49172
		3	8	0.07817	0.09237	0.15351	0.24520	0.36780	0.51008[1]	0.55947
		2	12	0.07817	0.10042	0.19514	0.33105	0.49056	0.63824[2]	0.68169
		1	24	0.07817	0.12426	0.30862	0.53908	0.76954	0.95391	1.00000
7	24	24	1	0.07336	0.07336	0.07336	0.07336	0.07336	0.07336	0.07336
		12	2	0.07336	0.07540	0.08404	0.09635	0.11179	0.13051	0.13981
		8	3	0.07336	0.07743	0.09470	0.11926	0.14972	0.18447	0.20077
		6	4	0.07336	0.07946	0.10533	0.14211	0.18895	0.25038[2]	0.27958
		4	6	0.07336	0.08353	0.12652	0.18700	0.25672	0.32800[1]	0.36046
		3	8	0.07336	0.08758	0.14766	0.23468	0.35122	0.50004[3]	0.55947
		2	12	0.07336	0.09567	0.18941	0.32304	0.48272	0.63668[3]	0.68169
		1	24	0.07336	0.11969	0.30502	0.53668	0.76834	0.95367	1.00000

Appendix Table 2. Variances of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
8	24	24	1	0.06995	0.06995	0.06995	0.06995	0.06995	0.06995	0.06995
		12	2	0.06995	0.07199	0.08050	0.09238	0.10709	0.12649	0.13981
		8	3	0.06995	0.07402	0.09103	0.11472	0.14404	0.18152[1]	0.20077
		6	4	0.06995	0.07606	0.10154	0.13699	0.18125	0.24201[1]	0.27958
		4	6	0.06995	0.08012	0.12248	0.18089	0.25042	0.32909[2]	0.36046
		3	8	0.06995	0.08418	0.14336	0.22559	0.33104	0.48255[2]	0.55947
		2	12	0.06995	0.09229	0.18502	0.31570	0.47440	0.63471[3]	0.68169
		1	24	0.06995	0.11645	0.30246	0.53497	0.76749	0.95350	1.00000
9	24	24	1	0.06753	0.06753	0.06753	0.06753	0.06753	0.06753	0.06753
		12	2	0.06753	0.06957	0.07799	0.08955	0.10368	0.12105	0.13061
		8	3	0.06753	0.07161	0.08842	0.11148	0.13973	0.17682	0.20077
		6	4	0.06753	0.07364	0.09884	0.13331	0.17476	0.22210[1]	0.24621
		4	6	0.06753	0.07771	0.11960	0.17673	0.24645	0.32839[1]	0.36046
		3	8	0.06753	0.08178	0.14027	0.21864	0.30618	0.40229[2]	0.44867
		2	12	0.06753	0.08990	0.18166	0.30882	0.46518	0.63021[3]	0.68169
		1	24	0.06753	0.14416	0.30065	0.53377	0.76688	0.95338	1.00000
10	24	24	1	0.06588	0.06588	0.06588	0.06588	0.06588	0.06588	0.06588
		12	2	0.06588	0.06791	0.07626	0.08761	0.10137	0.11895	0.13061
		8	3	0.06588	0.06995	0.08663	0.10925	0.13657	0.16933	0.18719
		6	4	0.06588	0.07199	0.09697	0.13080	0.17177	0.22231	0.24621
		4	6	0.06588	0.07606	0.11761	0.17377	0.24283	0.32619[2]	0.36046
		3	8	0.06588	0.08012	0.13815	0.21539	0.30535	0.40811[2]	0.44867
		2	12	0.06588	0.08825	0.17916	0.30233	0.45453	0.62339[3]	0.68169
		1	24	0.06588	0.11258	0.29941	0.53294	0.76647	0.95329	1.00000

Appendix Table 2. Variances of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, n = mk.

Order i+1	Sample Size n	Number of Groups m	Number of Individuals Per Group k[c]	Intra-Class Correlation (ρ) 0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
11	24	24	1	0.06484	0.06484	0.06484	0.06484	0.06484	0.06484	0.06484
		12	2	0.06484	0.06687	0.07518	0.08638	0.09990	0.11667	0.12664
		8	3	0.06484	0.06891	0.08550	0.10784	0.13482	0.16896	0.18719
		6	4	0.06484	0.07094	0.09580	0.12922	0.16969	0.22090[1]	0.24621
		4	6	0.06484	0.07502	0.11635	0.17175	0.23909	0.32300[2]	0.36046
		3	8	0.06484	0.07908	0.13682	0.21375	0.30588	0.41060[2]	0.44867
		2	12	0.06484	0.08722	0.17748	0.29641	0.44169	0.61534[1]	0.68169
		1	24	0.06484	0.11159	0.29863	0.53242	0.76621	0.95324	1.00000
12	24	24	1	0.06433	0.06433	0.06433	0.06433	0.06433	0.06433	0.06433
		12	2	0.06433	0.06637	0.07465	0.08579	0.09920	0.11601	0.12664
		8	3	0.06433	0.06841	0.08495	0.10716	0.13386	0.16708	0.18719
		6	4	0.06433	0.07044	0.09524	0.12844	0.16820	0.21736[1]	0.24621
		4	6	0.06433	0.07451	0.11574	0.17065	0.23547	0.32582[1]	0.36046
		3	8	0.06433	0.07858	0.13618	0.21305	0.30629	0.41041[2]	0.44867
		2	12	0.06433	0.08672	0.17661	0.29176	0.42563	0.60134[2]	0.68169
		1	24	0.06433	0.11112	0.29825	0.53217	0.76608	0.95322	1.00000
1	48	48	1	0.21787	0.21787	0.21787	0.21787	0.21787	0.21787	0.21787
		24	2	0.21787	0.21875	0.22350	0.23236	0.24477	0.25768	0.26151
		16	3	0.21787	0.21964	0.22895	0.24555	0.26745	0.28893	0.29501
		12	4	0.21787	0.22052	0.23425	0.25784	0.28766	0.31577[1]	0.32364
		8	6	0.21787	0.22228	0.24445	0.28047	0.32354	0.36242	0.37290
		6	8	0.21787	0.22402	0.25423	0.30131	0.35557	0.40328	0.41593
		4	12	0.21787	0.22747	0.27282	0.33945	0.41287	0.47544	0.49172

Appendix Table 2. Variances of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order i+1	Sample Size n	Number of Groups m	Number of Individuals Per Group k[c]	Intra-Class Correlation (ρ) 0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
1	48	3	16	0.21787	0.23088	0.29042	0.37449	0.46462	0.54004	0.55947
		2	24	0.21787	0.23759	0.32357	0.43885	0.55859	0.65675	0.68169
		1	48	0.21787	0.25698	0.41340	0.60894	0.80447	0.96089	1.00000
2	48	48	1	0.11795	0.11795	0.11795	0.11795	0.11795	0.11795	0.11795
		24	2	0.11795	0.11896	0.12487	0.13923	0.17043	0.22979[1]	0.26151
		16	3	0.11795	0.11997	0.13151	0.15727	0.20382	0.26951[1]	0.29501
		12	4	0.11795	0.12098	0.13791	0.17327	0.23036	0.29994[1]	0.32364
		8	6	0.11795	0.12297	0.15010	0.20144	0.27379	0.34978[1]	0.37290
		6	8	0.11795	0.12495	0.16163	0.22631	0.31041	0.39231[1]	0.41593
		4	12	0.11795	0.12887	0.18321	0.27021	0.37333	0.46616[2]	0.49172
		3	16	0.11795	0.13273	0.20331	0.30932	0.42859	0.53185[3]	0.55947
		2	24	0.11795	0.14031	0.24048	0.37936	0.52698	0.65003[2]	0.68169
		1	48	0.11795	0.16205	0.33846	0.55898	0.77949	0.95590	1.00000
3	48	48	1	0.08669	0.08669	0.08669	0.08669	0.08669	0.08669	0.08669
		24	2	0.08669	0.08772	0.09332	0.10463	0.12249	0.14245	0.14899
		16	3	0.08669	0.08875	0.09986	0.12286	0.16638	0.24763[1]	0.29501
		12	4	0.08669	0.08978	0.10627	0.13979	0.19872	0.28632[1]	0.32364
		8	6	0.08669	0.09183	0.11870	0.17008	0.24861	0.34164[1]	0.37290
		6	8	0.08669	0.09386	0.13061	0.19684	0.28889	0.38632[2]	0.41593
		4	12	0.08669	0.09790	0.15308	0.24372	0.35599	0.46202[2]	0.49171
		3	16	0.08669	0.10189	0.17411	0.28506	0.41371	0.52820	0.55947
		2	24	0.08669	0.10973	0.21302	0.35830	0.51505	0.64623[2]	0.68169
		1	48	0.08669	0.13235	0.31501	0.54334	0.77167	0.95433	1.00000

Appendix Table 2. Variances of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order $i+1$	Sample Size n	Number of Groups m	Number of Individuals Per Group k[c]	Intra-Class Correlation (ρ) 0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
4	48	48	1	0.07109	0.07109	0.07109	0.07109	0.07109	0.07109	0.07109
		24	2	0.07109	0.07213	0.07746	0.08736	0.10291	0.12914	0.14899
		16	3	0.07109	0.07317	0.08378	0.10353	0.13289	0.16397[1]	0.17439
		12	4	0.07109	0.07420	0.09004	0.12003	0.17192	0.26620[1]	0.32364
		8	6	0.07109	0.07627	0.10234	0.15093	0.22879	0.33271[1]	0.37290
		6	8	0.07109	0.07832	0.11429	0.17877	0.27278	0.38036[2]	0.41593
		4	12	0.07109	0.08241	0.13709	0.22768	0.34392	0.45892[3]	0.49172
		3	16	0.07109	0.08645	0.15860	0.27067	0.40388	0.52614[3]	0.55947
		2	24	0.07109	0.09443	0.19855	0.34633	0.50778	0.64530[3]	0.68169
		1	48	0.07109	0.11753	0.30331	0.53554	0.76777	0.95355	1.00000
5	48	48	1	0.06162	0.06162	0.06162	0.06162	0.06162	0.06162	0.06162
		24	2	0.06162	0.06266	0.06779	0.07674	0.08989	0.10660	0.11337
		16	3	0.06162	0.06371	0.07393	0.09181	0.11812	0.15496[1]	0.17439
		12	4	0.06162	0.06474	0.08003	0.10684	0.14485	0.18365[1]	0.19726
		8	6	0.06162	0.06682	0.09210	0.13722	0.21023	0.32219[3]	0.37290
		6	8	0.06162	0.06888	0.10395	0.16556	0.25839	0.36887[3]	0.41593
		4	12	0.06162	0.07299	0.12683	0.21595	0.33379	0.45866[3]	0.49172
		3	16	0.06162	0.07706	0.14860	0.26029	0.39601	0.52540[3]	0.55947
		2	24	0.06162	0.08512	0.18928	0.33802	0.50236[1]	0.64498[3]	0.68169
		1	48	0.06162	0.10854	0.29622	0.53081	0.76541	0.95308	1.00000
6	48	48	1	0.05522	0.05522	0.05522	0.05522	0.05522	0.05522	0.05522
		24	2	0.05522	0.05627	0.06123	0.06952	0.08133	0.09891[1]	0.11337
		16	3	0.05522	0.05731	0.06722	0.08377	0.10760	0.14519[1]	0.17439

Appendix Table 2. Variances of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, $n = mk$.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
6	48	12	4	0.05522	0.05835	0.07317	0.09797	0.13275	0.17688[2]	0.19726
		8	6	0.05522	0.06042	0.08501	0.12673	0.19117	0.30297[2]	0.37290
		6	8	0.05522	0.06249	0.09671	0.15506	0.24428	0.36554[3]	0.41593
		4	12	0.05522	0.06661	0.11953	0.20650	0.32443	0.45073[3]	0.49172
		3	16	0.05522	0.07070	0.14145	0.25198	0.38902[1]	0.52412[3]	0.55947
		2	24	0.05522	0.07880	0.18266	0.33157	0.49785	0.64511[3]	0.68169
		1	48	0.05522	0.10246	0.29142	0.52761	0.76381	0.95276	1.00000
7	48	48	1	0.05059	0.05059	0.05059	0.05059	0.05059	0.05059	0.05059
		24	2	0.05059	0.05163	0.05647	0.06425	0.07502	0.08906	0.09567
		16	3	0.05059	0.05268	0.06233	0.07788	0.09932	0.12541	0.13634
		12	4	0.05059	0.05372	0.06817	0.09148	0.12394	0.17061[2]	0.19726
		8	6	0.05059	0.05579	0.07980	0.11860	0.16982	0.21988[2]	0.23940
		6	8	0.05059	0.05786	0.09133	0.14637	0.22961	0.35740[3]	0.41593
		4	12	0.05059	0.06199	0.11401	0.19842	0.31525	0.44369[3]	0.49172
		3	16	0.05059	0.06609	0.13598	0.24490	0.38242	0.52042[3]	0.55947
		2	24	0.05059	0.07423	0.17760	0.32620	0.49385[1]	0.64429[2]	0.68169
		1	48	0.05059	0.09806	0.28794	0.52530	0.76265	0.95253	1.00000
8	48	48	1	0.04709	0.04709	0.04709	0.04709	0.04709	0.04709	0.04709
		24	2	0.04709	0.04813	0.05286	0.06025	0.07025	0.08416[1]	0.09567
		16	3	0.04709	0.04917	0.05862	0.07338	0.09334	0.12035[1]	0.13634
		12	4	0.04709	0.05021	0.06436	0.08649	0.11665	0.16200[1]	0.19726
		8	6	0.04709	0.05229	0.07580	0.11257	0.16069	0.21589[2]	0.23940
		6	8	0.04709	0.05436	0.08717	0.13910	0.21359	0.33671[2]	0.41593
		4	12	0.04709	0.05848	0.10967	0.19128	0.30590	0.44080[3]	0.49172

Appendix Table 2. Variances of Order Statistics ($i < 12$) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, n = mk.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
8	48	3	16	0.04709	0.06259	0.13163	0.23860	0.37595[1]	0.51563[3]	0.55947
		2	24	0.04709	0.07075	0.17354	0.32152	0.49014[1]	0.64223[3]	0.68169
		1	48	0.04709	0.09474	0.28532	0.52355	0.76177	0.95235	1.00000
9	48	48	1	0.04435	0.04435	0.04435	0.04435	0.04435	0.04435	0.04435
		24	2	0.04435	0.04539	0.05003	0.05710	0.06649	0.07872	0.08511
		16	3	0.04435	0.04643	0.05570	0.06983	0.08860	0.11494[1]	0.13634
		12	4	0.04435	0.04747	0.06136	0.08254	0.11050	0.14348	0.15798
		8	6	0.04435	0.04954	0.07263	0.10785	0.15411	0.21263[2]	0.23940
		6	8	0.04435	0.05161	0.08386	0.13311	0.19471	0.25392[2]	0.27958
		4	12	0.04435	0.05574	0.10615	0.18484	0.29607	0.43860[3]	0.49172
		3	16	0.04435-	0.05985	0.12806	0.23285	0.36941[1]	0.51177[3]	0.55947
		2	24	0.04435	0.06803	0.17019	0.31729	0.48654[1]	0.63932[3]	0.68169
		1	48	0.04435	0.09214	0.28326	0.52218	0.76109	0.95222	1.00000
10	48	48	1	0.04215	0.04215	0.04215	0.04215	0.04215	0.04215	0.04215
		24	2	0.04215	0.04319	0.04776	0.05457	0.06347	0.07539	0.08511
		16	3	0.04215	0.04423	0.05335	0.06697	0.08472	0.10699	0.11787
		12	4	0.04215	0.04527	0.05893	0.07934	0.10591	0.13996[1]	0.15798
		8	6	0.04215	0.04734	0.07007	0.10402	0.14857	0.20873[3]	0.23940
		6	8	0.04215	0.04941	0.08117	0.12850	0.18742	0.25199[1]	0.27958
		4	12	0.04215	0.05354	0.10325	0.17899	0.28546	0.43166[3]	0.49172
		3	16	0.04215	0.05765	0.12508	0.22750	0.36263[1]	0.50958[3]	0.55947
		2	24	0.04215	0.06585	0.16735	0.31339	0.48298[1]	0.63630[3]	0.68169
		1	48	0.04215	0.09004	0.28161	0.52108	0.76054	0.95211	1.00000

Appendix Table 2. Variances of Order Statistics (i < 12) in Correlated Samples of m Independent Sets of k Equicorrelated (ρ) Multinormal Variates, n = mk.

Order	Sample Size	Number of Groups	Number of Individuals Per Group	Intra-Class Correlation (ρ)						
i+1	n	m	k[c]	0.0[a]	0.05	0.25	0.50	0.75	0.95	1.00[b]
11	48	48	1	0.04036	0.04036	0.04036	0.04036	0.04036	0.04036	0.04036
		24	2	0.04036	0.04140	0.04590	0.05250	0.06100	0.07198	0.07817
		16	3	0.04036	0.04243	0.05143	0.06462	0.08159	0.10389[1]	0.11787
		12	4	0.04036	0.04347	0.05695	0.07671	0.10216	0.13649[2]	0.15798
		8	6	0.04036	0.04554	0.06797	0.10084	0.14356	0.20385[2]	0.23940
		6	8	0.04036	0.04761	0.07895	0.12479	0.18237	0.25072[2]	0.27958
		4	12	0.04036	0.05174	0.10082	0.17369	0.27372	0.41960[3]	0.49172
		3	16	0.04036	0.05586	0.12254	0.22246	0.35548	0.50778[3]	0.55947
		2	24	0.04036	0.06406	0.16490	0.30970	0.47941[1]	0.63396[3]	0.68169
		1	48	0.04036	0.08834	0.28027	0.52018	0.76009	0.95202	1.00000
12	48	48	1	0.03888	0.03888	0.03883	0.03888	0.03888	0.03888	0.03888
		24	2	0.03888	0.03992	0.04436	0.05078	0.05895	0.06964	0.07817
		16	3	0.03888	0.04095	0.04984	0.06266	0.07898	0.10062	0.11787
		12	4	0.03888	0.04199	0.05530	0.07452	0.09897	0.13197[1]	0.15798
		8	6	0.03888	0.04406	0.06621	0.09818	0.13895	0.19611[2]	0.23940
		6	8	0.03888	0.04612	0.07710	0.12172	0.17818	0.24857[2]	0.27958
		4	12	0.03888	0.05025	0.09878	0.16896	0.26023	0.40213[3]	0.49172
		3	16	0.03888	0.05437	0.12036	0.21768	0.34782[1]	0.50431[3]	0.55947
		2	24	0.03888	0.06258	0.16276	0.30618	0.47579[1]	0.63282[2]	0.68169
		1	48	0.03888	0.08694	0.27916	0.51944	0.75972	0.95194	1.00000