MODELLING TECHNIQUES FOR PIPE WHIP ANALYSIS

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SUMMARY

The problem of modelling a pipe and its restraining units during the pipe whip accident is discussed; the following main points (which are available in the FRUSTA program) are considered:

(a) A complete dynamic model of the pipe is considered only for the part between the postulated rupture and the first elbow, the remaining part is considered only to simulate its stiffness; such a simplification is demonstrated to be generally adequate.
(b) Both a circumferential (pipe only on one side of the rupture) and a longitudinal (pipe on both the sides of the rupture) breakage can be considered.
(c) The pipe mass is lumped in each node (max. 20)
(d) The pipe stiffness is simulated by means of a particular element, which can account for the elastic, plastic, ovalization, instability and unloading behaviour of the pipe; the energy stored in each subelement (max. 20 for each element) is the controlling quantity to determine the behaviour of the element: a check is performed at each step in order to control that no excessive distance from theoretical material behaviour results during the transitory.
(e) The restraint is simulated by a proper element with a bilinear characteristic after a "no load" phase.
(f) A damper can be simulated by means of an element, whose reaction force is proportional to the velocity of the application point.
(g) A dummy (fully indefinitely elastic) element can be introduced in order to simulate end conditions, rigid body motions, etc.
(h) The crushing behaviour of the pipe when hitting a restraining surface can be approximately simulated by introducing a proper restraint, which accounts for the pipe elasticity and reduction of moment capability.
(i) A step by step integration procedure is followed: a Gauss-Euler algorithm is used.
(j) Maxima of the kinematics, stress and strain characteristics are given.

1. Introduction

The general aspects of the pipe whip phoenomenon and of its safety implications have been discussed in the companion paper F5/5 (9); here the problem of modelling the pipe and the restraint units will be discussed.

It is assumed that the pipe motion will be restrained by either restraints or dampers placed at given locations along the pipe.

The absorption of energy by the restraints (or the dampers) and the pipe itself is a very complicated phoenomenon, as both of them will generally react in the plastic range. So it was made en effort to write a computer code named FRUSTA (Forza e reazioni nell'urto tra strutture e tubazioni accelerate) in order to be able tu study the whole pheenomenon.

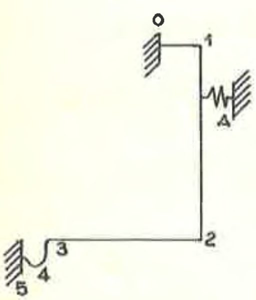


fig. 1 piping sketch

The typical situation of a pipe in a commonly design nuclear power reactor is sketchly illustrated in fig.1.Let us assume that such event takes place at location 1 and that the force will accelerate the pipe towards the restraint located at A. A first problem to be solved was whether it should be necessary to model the whole pipe from 0 to 5 location or the modelling of a reduced part was enough. A compromise solution resulted by assuming a complete dynamic model of the 0-1-2 span, while the remaining part was simulated only statically by an element placed at the end of the span. Consequently the two situations, which can be considered are the ones depicted in figs.2a - 2b.

The validity of this assumption has been widely tested and it has always been found that the displacements are generally large only in the vicinity of the force application point and of the restraint; so neglecting the inertia forces in the 2-5 span does not generally cause a large error.

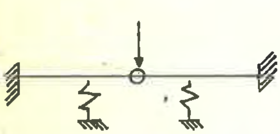
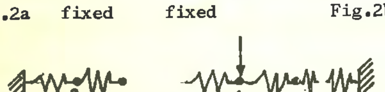


fig.2a fixed fixed Fig.2b cantilever



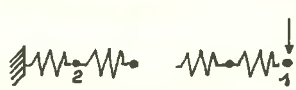


fig.3a fixed fixed fig.3b cantilever

A lumped masses and spring approch was chosen in order to describe the pipe-restraint (damper) complex; the forces (inertia, jet loads, restraint loads, damping loads) can be considered only at the nodes.

Typical situations are dipicted in figs.3a, 3b; a dummy element can be interposed every where, e.g. at the ends of the elements chain so accounting for possible end conditions different from the "built in" case.

2. Solution Procedure

A plastic stepwise elastic analysis method has been chosen; so at each time step the whole system has been considered as fully elastic, while the rigidity may change from time step to time step. Such being tha case at each step the motion equation will be:

(1) $$(K)\cdot|X| + (C)\,|\dot{X}| + (m)\,|\ddot{X}| = F(t)$$

where

(K) stiffness matrix, given by (K) = (Kp) + (Kr) where

(Kp) pipe stiffness matrix

(Kr) restraint stiffness matrix

(C) damping diagonal matrix

(m) lumped mass diagonal matrix

F(t) jet load vector

|X| displacement vector

and where • denotes differentation with respect to time.

Eq. (1) has been solved numerically by means of a step integration procedure; in order to reduce instability problems the "constant average acceleration procedure"(Euler-Gauss) was selected.

The stiffness matrix can be changed at each time step in order to account for the plastic behaviour of the material: the so called "tangent stiffness" method is so used.

3. Models of the various parts

3.1 - Model of the pipe

The pipe is modelled in a serie of elements, which are subsequently assembled in order to obtain (Kp) the pipe stiffness matrix; (see ref.(2)).

Each element has its own 2 x 2 stiffness matrix correlating applied loads to the relative displacements, see fig.4.

element geometry
fig.4

The deformation has been computed assuming that:

a) the deformation is due to moment only

b) the stress strain law is $\sigma = E \cdot \varepsilon$

(σ = stress, ε = strain, E = modulus of elasticity) in the elastic range or during unloading and $\sigma = A \cdot \varepsilon^m$ (m = strain exponent) during loading in the plastic range.

c) plane sections remain plane

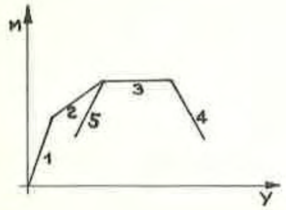
d) ovalization occurs when the total maximum strain in the pipe section is larger than:

(2) $\varepsilon = K \cdot 0.6 \cdot t/r$

where t = pipe thickness, r = pipe radius, 0,6 = gradient factor K = experimental factor. The studies by Gerard, Donnel, Flugge (3, 4, 5,6) can be quoted in order to justify (2) expression and to compute the K factor (which typically ranges in the 1.0 ÷ 2.0 zone).

e) instability occurs when the total accumulated energy is larger than a predetermined amount.

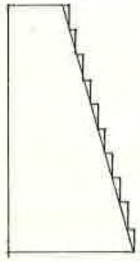
In fig. 5 the moment-rotation relationship is depicted, where the various phases are as follows:



Moment-rotation relationship
fig.5

1) elastic
2) plastic
3) ovalization
4) instability
5) unloading

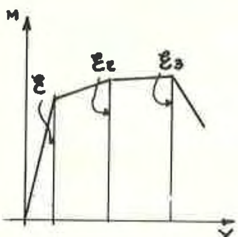
Each element (i.e. pipe span between two consecutive nodes) is devided in a number of subelements (maximum 20), where the moment is mediated (see fig.5); the internal degrees of freedom are condensed. Each of the subelement has an indipendent behaviour so that the control on loading or unloading is relatively rather strict; in each subelement the energy is computed and is assumed as the control parameter for the subelement stiffness computation; the complete procedure ia as follows



moment in the element

fig. 5b

$\mathcal{E}_{se}$ = subelement energy) and it has been depicted in fig.6.



Energy levels in the pipe

fig.6

During first loading:

$\mathcal{E}_{se} < \mathcal{E}_1$	elastic
$\mathcal{E}_{se} > \mathcal{E}_1 \quad \dot{\mathcal{E}}_{se} > 0$	plastic
$\mathcal{E}_{se} > \mathcal{E}_1 \quad \dot{\mathcal{E}}_{se} < 0$	elastic unloading
$\mathcal{E}_{se} > \mathcal{E}_2 \quad \dot{\mathcal{E}}_{se} > 0$	ovalization
$\mathcal{E}_{se} > \mathcal{E}_2 \quad \dot{\mathcal{E}}_{se} < 0$	elastic unloading
$\mathcal{E}_{se} > \mathcal{E}_3$	instability

. denotes derivation with respect to time

When unloading occurred, the behaviour of the subelement will be elastic both if it loads or unloads provided that the total energy of the subelement is less than $\mathcal{E}^*$, where $\mathcal{E}^*$ is the energy when unloading occurred.

This behaviour is comparatively easy to be simulated by means of FORTRAN programming procedure; a $\mathcal{E}^*$ value is defined which is equal to $\mathcal{E}_1$ before initial overcoming of the elastic limit is reached, it is equal to 0 during unloading and it is equal to the stored energy, when unloading occurs, so that:

$$(3)\qquad \begin{array}{lll} \mathcal{E}_{se} < \mathcal{E}_1 & & \text{elastic} \\ \mathcal{E}_{se} > \mathcal{E}_1 & \dot{\mathcal{E}}_{se} > 0 & \text{plastic (or instability etc.)} \\ \mathcal{E}_{se} > \mathcal{E}_1 & \dot{\mathcal{E}}_{se} < 0 & \text{elastic} \end{array}$$

The total energy of each subelement is besides the controlling parameter in order to compute the stiffness in the plastic range; the increment in the displacement and rotation for each subelement is given by:

$$\Delta W = \frac{\partial W}{\partial T}\Delta T + \frac{\partial W}{\partial M}\Delta M \qquad \Delta\theta = \frac{\partial \theta}{\partial T}\Delta T + \frac{\partial \theta}{\partial M}\Delta M$$

The tangent stiffness matrix is

$$(4)\quad (K_p) = \begin{vmatrix} \frac{\partial W}{\partial T} & \frac{\partial W}{M} \\ \frac{\partial \theta}{\partial T} & \frac{\partial \theta}{\partial M} \end{vmatrix}$$

The various terms are given by:

$$
(5)\quad
\begin{cases}
\dfrac{\partial W}{\partial T} = \dfrac{l^2}{4}\left(\dfrac{1}{mEI_m}\right)^{\frac{2m}{1+m}}\left(\dfrac{1}{m}+1\right)^{\frac{1-m}{1+m}}\mathcal{E}_{se}^{\frac{1-m}{1+m}} \\[2ex]
\dfrac{\partial W}{\partial M} = \dfrac{\partial \theta}{\partial T} = \dfrac{1}{2}\left(\dfrac{1}{mEI_m}\right)^{\frac{2m}{1+m}}\left(\dfrac{1}{m}+1\right)^{\frac{1-m}{1+m}}\mathcal{E}_{se}^{\frac{1-m}{1+m}} \\[2ex]
\dfrac{\partial \theta}{\partial M} = \left(\dfrac{1}{mEI_m}\right)^{\frac{2m}{1+m}}\left(\dfrac{1}{m}+1\right)^{\frac{1-m}{1+m}}\mathcal{E}_{se}^{\frac{1-m}{1+m}} \\[2ex]
EI_m = r\left(2Ar^2 t \displaystyle\int_0^{\pi} \sin^{1+m}\theta \, d\theta\right)^{1/m} \qquad l = \text{subelement length}
\end{cases}
$$

In the elastic range the stiffness matrix is simply given by:

$$
(6)\quad
\begin{cases}
\dfrac{\partial W}{\partial T} = \dfrac{l^3}{4EI} \\[2ex]
\dfrac{\partial W}{\partial M} = \dfrac{l^2}{2EI} \\[2ex]
\dfrac{\partial \theta}{\partial M} = \dfrac{1}{EI}
\end{cases}
$$

The espression given by (6) are used in the ovalization range too, simply by a modification of the E value, which is as low as possible in the ovalization range and it is negative in the instability one.

When instability occurs the element is unloaded and hinge is afterwards assumed. Besides it should be mentioned that , during the time integration procedure, some departures from the theoretical moment-rotation relationship could occur still an equilibrium check is performed at each step anyhow while the computation of the stiffness is based on energy only, a control of the moment resulting from external equilibrium and internal stresses at each step is performed and the maximum and minimum value of their ratio is recorded. It is consequently possible to have an idea of the computation accurancy at the end by inspecting this ratio value; its difference from one typically ranges between 1 ÷ 2% or even less if a low time step value is selected.

3.2 Dummy element

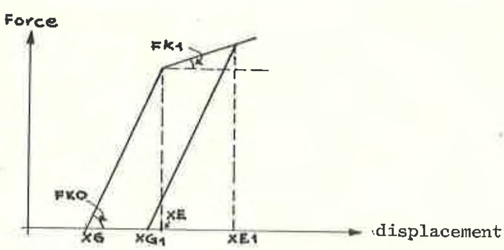
A dummy element is considered whenever it is necessary to account for end condition different from "built in" or for other special features (such as valve etc.); an indefinitely elastic behaviour of the element is considered and the stiffness is computed by the use of (7) (so accounting for moment variation in the element):

(7)

$$\begin{cases} \dfrac{\partial W}{\partial T} = \dfrac{l^3}{3EI} \\[2ex] \dfrac{\partial W}{M} = \dfrac{\partial \theta}{\partial T} = \dfrac{l^2}{2EI} \\[2ex] \dfrac{\partial \theta}{\partial M} = \dfrac{1}{EI} \end{cases}$$

3.3 Restraint element

A large variety of structural restraint elments could be designed in order to prevent excessive pipe movements, consequently a comparatively rough but simple and flexible element has been devised. The force applied by the restraint is assumed to be parallel to the jet force one, so assuming a large lateral flexibility of the restraint (as it generally is). A sketchy behaviour of the element is depicted in fig.7.



restraint behaviour
fig.7

The stiffness is:

$Kr = 0$ if $x < x_G$ (or XG1 after impact and unloading)

(8) $Kr = FKO$ if $x_G < x < x_E$ (or $XG1 < x < x_{E1}$ after unloading)

$Kr = FK1$ if $x > x_E$ (or X_{E1} after reloading)

3.4 Damper element

A damper element can be simulated by adding at each node a viscous force given by (9) plus a standard restraint

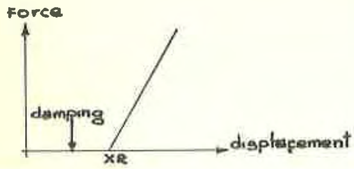
(9) $Fd = C\,\dot{x}$

where C is the damping coefficient of element.

It should be noted that under this assumption the damper will act continuosly and with a costant damping coefficient..

Besides one should consider that, as the force is continuosly directed in one direction, the damper is, by itself, unable to prevent the motion, but only to limit its velocity. The motion will be then countinuos till either the force will drop to a value the pipe can sustain or the damper will finish its free run and then will act as a standard restraint. The way in which the damper works is illustrated by fig.8 and by the following formulas:

$$(10)\quad \begin{array}{ll} X < X_R & F = C_D \dot{X} \\ X > X_R & F = K\,(X - X_R) \end{array}$$



Damper behaviour fig.8

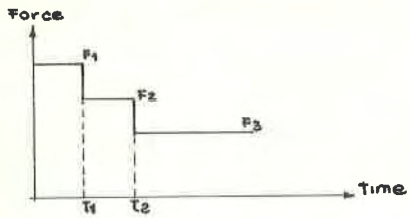
If the drop in the force is high enough it is possible that the pipe will sustain the motion by itself and the restraint action displacement of the damper is unnecessary.

3.5 Break point

The point in which the breakage takes place is simulated in the model by means of an hinge; it is actually quite difficult to correctly simulate the action of the part of the pipe where the jet force is acting (breakage) and the hinge hypothesis can be justified on the basis of its semplicity. So it will be supposed that in the case of fig. 2 the two pipe sections will trasmit only a shear force the one to the other. Obviously the cantilever option for each side of the piping is also possible.

3.6 Forcing function

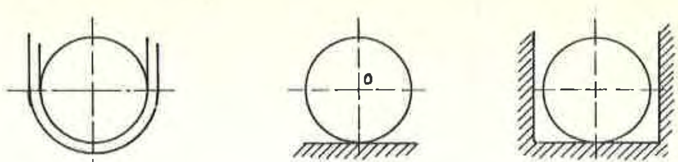
An extensive discussion of the method by which the forces from the breakage are computed is illustrated in ref. 8 (as an example).
The forces shall be given to the program in a stepwise manner as in fig. 10.



forcing function fig. 10

3.7 Pipe crushing

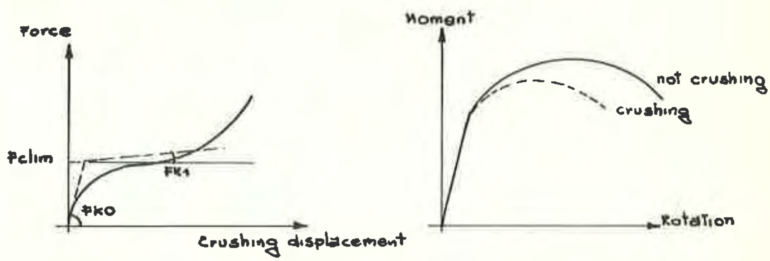
So far no mention has been made of the pipe crushing due to the impact of the restraint. This phoenomenon is actually possible and its extent will be largely dependent from the shape of the restraint and from the forces applied. Whenever the restraint effectively embraces the pipe providing lateral support (see fig. 9a) the tendency to crushing will be quite small.



Not crushing fig.9a crushing fig.9b limited crushing fig.9c

The opposite is obviously true for the case depicted in fig. 9b, where nothing prevents the pipe from lateral displacements; somewhat intermediate is the situation depicted in fig. 9c.

If the pipe crushes there will be some absorption of energy, so that the pipe acts, by itself, as a restraint, besides the pipe bending capacity could be effected, in fig.10a the idealized force-crushing displacement curve is shown, while in fig.10b a possible moment-rotation relationship is depicted.



crushed pipe idealized behaviour fig. 10

Experimental data are obviously necessary in order to simulate the crushing pipe behaviour, anyhow the FRUSTA code assumes a fictitious restraint to represent the cru - shing behaviour of the pipe; besides no reduction of the bending capacity is assumed if the contact force remains lower than the specified Fceim value. A reduction of the maximum moment the pipe can sustain and of the maximum energy which can be absorbed is assumed if the contact force overcomes the Fceim value.

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