Transactions of the 14th International Conference on Structural Mechanics in Reactor Technology (SMiRT 14), Lyon, France, August 17-22, 1997

M05/5

The impact of quality on failure probabilities

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ABSTRACT

Results of the impact of increased quality, as measured in terms of a reduction of the coefficient of variation, on the required design capacities for normal and lognormal load and resistance density functions are obtained. It is observed that dramatic reductions of design capacities could be obtained by the incorporation of quality in the constructed facility. However, under certain conditions continued imposition of quality requirements does not translate into improved reliability. The interaction of quality, reliability, loads and capacities in complex.

INTRODUCTION

The objective of quality assurance requirements is to ensure that risks are minimized and that safety, reliability, and performance are maximized. Most existing critical facilities were designed and constructed without the benefit of quality assurance programs presently accorded for the design and construction of commercial nuclear power facilities; and for the latter, more often than not, quality is invoked as an indirect contributor to reliability. Reliability comparisons are often made between these two classes of facilities. To be useful, these comparisons must explicitly consider the role of quality in enhancing safety. The emphasis herein is on design, procurement, and construction as distinct from risk minimization by management actions. Treatment of design and construction errors is beyond the scope of this paper.

QUALITY AND FAILURE PROBABILITY/RELIABILITY INDEX

The ultimate objective of any quality program is to reduce uncertainties in the expected performance of structures, systems and components. As a direct result of the reduction of uncertainties, failure probabilities are reduced. Alternatively, for a target failure probability (e.g. in design), the reduction of uncertainties leads to a reduction in the required design capacities. The latter case is depicted in Fig. 1 for the normal probability density function (PDF). Herein, the loading (S) is considered the independent variable and a designer has the opportunity to achieve the target reliability by the choice of a mean resistance ($\overline{R}$) with an associated variability. From a large number of possibilities, three candidate resistances are shown in Fig. 1. All three resistance PDFs would produce the same failure probability for the prescribed loading S.

Variability is usually measured in terms of the Coefficient of Variation (COV), which is the standard deviation of a variate normalized by its mean:

$$COV = \frac{\sigma}{\overline{m}} \tag{1}$$

In Eq. 1, σ is the standard deviation and $\overline{m}$ is the mean. A prime objective of a quality assurance program is to reduce COV_R, the coefficient of variation of the resistance PDF. For a rational utilization of a quality program it is necessary to understand the impact that increased quality would have on failure probability. Herein, increased quality will be measured in terms COV_R. As quality is increased, COV_R is reduced. Inversely, reducing quality leads to increased COV_R. The three resistance density functions of Fig. 1, with COV_R of 0.1, 0.15 and 0.2 and their respective means are equally acceptable in that, for the assumed loading, they all lead to the same failure probability, as measured by the reliability index β=2.5. In Fig. 1 it is interesting to note the sensitivity of $\overline{R}$ to changes in COV_R.

Failure probability is computed by the appropriate convolving of the load and resistance density functions of Fig. 1. Alternatively, the reliability index, β, is used to designate the level of reliability. The following are values of β for commonly specified failure probabilities, and failure probabilities for commonly specified β:

P_f	$2.3\text{x}10^{-2}$	10^{-2}	$6.2\text{x}10^{-3}$	$5\text{x}10^{-3}$	$1.3\text{x}10^{-3}$	10^{-3}	10^{-4}	10^{-5}
β	2.0	2.33	2.5	2.57	3.0	3.1	3.72	4.25

β for Normal PDF

β is a function of the means and variances of the load and resistance density functions, and for the normal PDF is given by [e.g. 1 and 2]

$$\beta = \frac{\overline{R} - \overline{S}}{\sqrt{\sigma_R 2 + \sigma_S 2}} \tag{2}$$

Eq. 2 can be manipulated (Appendix A) and a relationship derived for the Central Factor of Safety (CFS), Eq. A3. CFS is defined as the ratio of mean resistance, $\overline{R}$, to mean loading $\overline{S}$:

$$CFS = \frac{\overline{R}}{\overline{S}} \tag{3}$$

CFS is a measure of the relative positions of the load and resistance density functions as, for example, shown in Fig. 1 for the three candidate resistance density functions.

Since the objective of design is to achieve a target reliability, in principle, quality should be optional in that *increased* quality should lead to a *reduced* required CFS to achieve the *prescribed* reliability. For the candidate resistances of Fig. 1 the following parameter values are required to achieve the same prescribed reliability of β = 2.5, when $\overline{S}$ = 2.0 and COV_S = 0.15.

As quality increases, the required CFS is reduced and, hence, the required mean resistances:

COV_R	0.20	0.15	0.10
CFS	2.13	1.76	3.52
$\overline{R}$	4.26	3.52	3.07

Clearly then, quality should reduce the cost of the constructed environment for any specified reliability. However, since achieving quality also has its own intrinsic cost, it should be possible to find the optimal combination of $\overline{R}$ and COV_R for any prescribed reliability. Usually, though, this is not done explicitly and quality is used as a surrogate for achieving an unquantified additional reliability.

β for Lognormal PDF

As the lognormal PDF is a commonly assumed model for variates that are known to be strictly positive, it would be useful to extend the above discussion to the lognormal PDF. For this case the reliability index β* is given by [e.g. 1 and 2]

$$\beta^* = \frac{\ln(R_m / S_m)}{\sqrt{\zeta_R^2 + \zeta_S^2}} \tag{4}$$

where ζ is the logarithmic standard deviation (which approximately equals to COV when ≤ 0.3), and R_m and S_m are the median values of the variate. The ratio R_m / S_m is called the Median Factor of Safety (MFS):

$$MFS = \frac{R_m}{S_m} \tag{5}$$

In Fig. 2, the lognormal PDFs (full lines) are compared with those of the corresponding normal distributions (dashed lines) of Fig. 1. Correspondence is obtained by requiring that $\beta = 2.5$ for both distributions, and the parameter COV_R is set successively to equal 0.1, 0.15 and 0.2. Clearly, the shapes of the PDFs tend to become dissimilar and significant differences in the distribution modes evolve as COV_R increases.

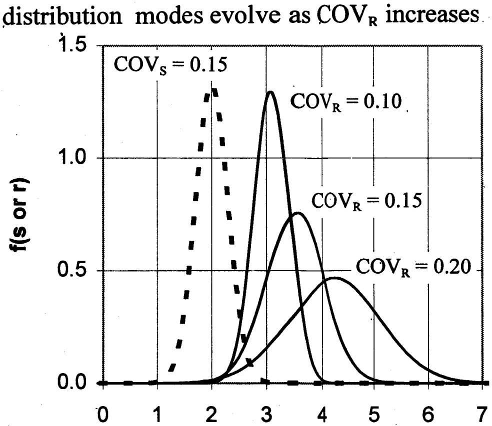

Fig. 1 Normal probability density functions of Load and three Resistances for $\beta = 2.5$

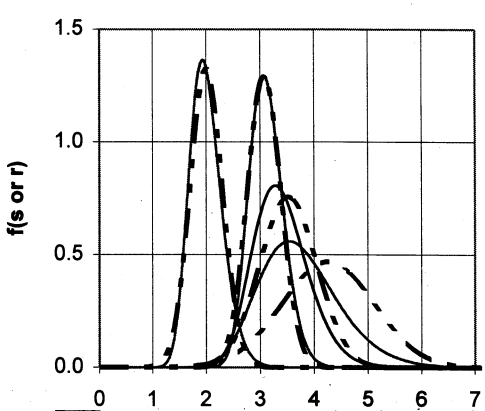

Fig. 2 Comparison of Fig. 1 functions with their lognormal counterparts (full)

Table 1 lists the modal, median and central factors of safety for the examples of Figs. 1 and 2. Also shown are the lognormal to normal ratios to help in the comparison. In general, the differences between the lognormal and the normal increase with increasing COV_R.

Table 1 Comparison of normal and lognormal factors of safety

	Modal FS			MFS			CFS		
COV_R	Norm.	LogN	Ratio*	Norm.	LogN	Ratio*	Norm.	LogN	Ratio*
0.10	1.54	1.59	1.03	1.54	1.57	1.02	1.54	1.56	1.01
0.15	1.76	1.69	0.96	1.76	1.68	0.95	1.76	1.69	0.96
0.20	2.13	1.83	0.86	2.13	1.86	0.87	2.13	1.87	0.88
0.25	2.77	1.98	0.71	2.77	2.05	0.74	2.77	2.09	0.75

* Ratio of Lognormal to Normal

The required safety factors for the lognormal PDF in comparison with those of the normal PDF could become significantly smaller; therefore, the ad hoc assumption of lognormal distributions for structural reliability calculations should be justified more than is the practice today.

RESULTS - *Normal Distribution*

Figure 3a is a graphical representation of the relation between COV_R and β, for two selected CFS (2 and 4) and a set of COV_S (0.1-0.5). It is to be noted that for the normal PDF the impact of quality is relatively small when COV_R is large (≥ 0.25). And in this range the variability in the load, COV_S, has a minimal influence on reliability: all five selected COV_S tend to the same β as COV_R increases. Similar observations were made in an earlier paper [3]. The impact of quality (reduced COV_R) on reliability becomes significant when $COV_R \leq 0.25$ and the influence of the variability of the load becomes more pronounced. This is an important observation in that when COV_R and COV_S are large, the better solution is to increase the CFS rather than to increase quality.

For structural reliability purposes the useful range of β is from 2 to 5. This range translates into a failure probability from about 2×10^{-2} to 3×10^{-6}. Therefore it would be more useful to evaluate the relationships of Fig. 3a within this range. Fig. 3b is simply an enlarged cutout from Fig. 3a. Clearly, for large load variability ($COV_S \geq 0.5$), the CFS should be greater than 2. As the CFS increases, the impact of the load variability diminishes. The usefulness of increased quality is dependent both on CFS and COV_S. For example, for CFS = 4, reliability tends to increase as COV_R is reduced for all COV_S. For CFS of 4.0 and $COV_S = 0.2$, reducing the COV_R by a quality program from 0.370 to 0.295 to 0.245 would increase β from 2.0 to 2.5 to 3.0, an overall gain of more than an order of magnitude in reliability. On the other hand, for CFS = 2, the same tendency exists for $COV_S \leq 0.3$. For $COV_S \geq 0.3$ the usefulness of increasing quality deteriorates. Fig. 3 strongly suggests the usefulness of a minimum quality program as well as the futility to improve performance reliability with more quality requirements when CFS is small and COV_S is large .

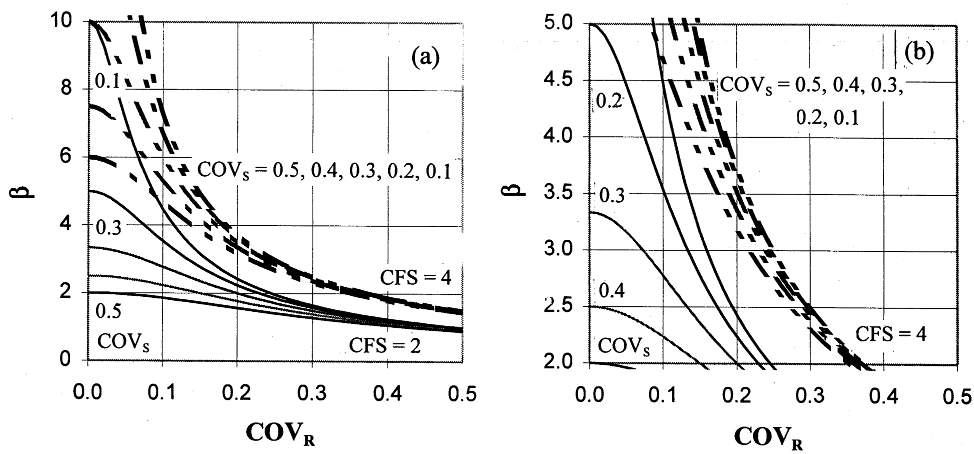

Fig. 3 β vs. COV_R for CFS of 2 & 4 and COV_S (0.1 to 0.5) - Normal PDF

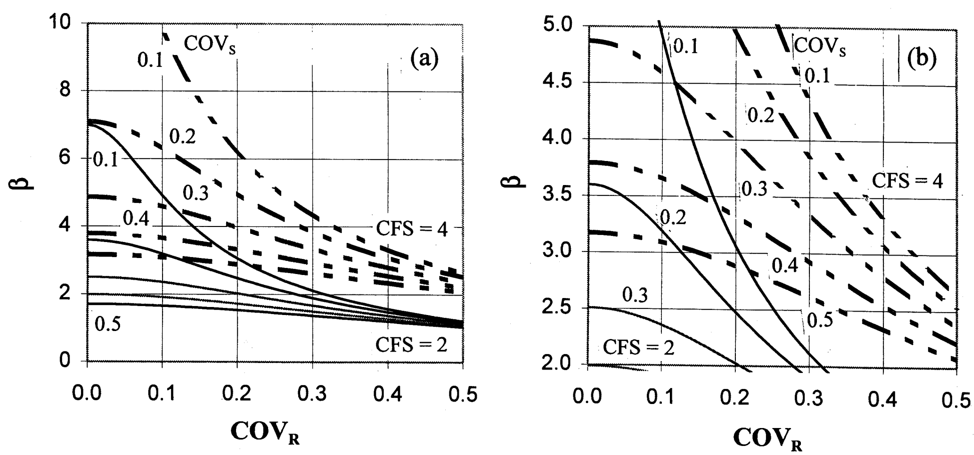

Fig. 4 β vs. COV_R for CFS of 2 & 4 and COV_S (0.1 to 0.5) - Lognormal PDF

RESULTS - *Lognormal Distribution*

Fig. 4a is the counterpart of Fig. 3a for the lognormal PDF. The same trend as for the normal PDF is to be noted as COV_R increases, albeit at a slower rate. However, the impact of the load variability is almost independent of CFS. For the larger COV_S (≥ 0.3) the impact of reducing COV_R on β is small. However, when the COV_S is small, significant increases in reliability can be achieved by increased quality. Again, a more useful evaluation can be made if β is limited to the range of 2 to 5 as shown in Fig. 4b. Clearly the influence of the load variability is more evident and the reliabilities improve faster (by increased quality) when the load variability is small. Unlike the normal PDF, load variability is an important parameter for all CFSs. The demarcation for the efficient use of quality is at about $COV_S = 0.4$ for CFS = 4 and at about 0.2 for CFS = 2. A simple relationship seems to exist between CFS and COV_S for the determination of this demarcation line.

RESULTS - *Comparison of Normal and Lognormal Distributions*

Fig. 5 compares the normal and lognormal PDFs for CFS equal to 2 and 4, and for three selected COV_S (0.1, 0.3 and 0.5). This comparison is necessary from the viewpoint of the significant different reliability results that could be obtained from the normal and lognormal PDFs.

Fig. 5a (COV_S = 0.1, e.g. for dead load only)shows that large reliabilities could be obtained for both the lognormal and normal for small changes in COV_R. And for the same reliability, the lognormal is relatively insensitive to changes in the resistance function, meaning that quality control should be significant to reduce CFS appreciably.

In Fig. 5b (COV_S = 0.3, e.g. for dead plus live loads), the CFS curves are similar in their impact on β. In general though the curves are flatter, meaning that larger reductions in COV_R are required to increase reliability. However, the CFS curves are distinctly different. For a given β, say 3.0, the CFS for the normal changes from 4 to 2 by a change in COV_R from 0.24 to 0.07. For the lognormal a reduction from CFS 4 to 2 is impossible by any reduction of COV_R.

And lastly, Fig. 5c (COV_S = 0.5 for highly uncertain loads) shows, as expected, that larger CFS are required and the trends of the two distributions cross over at about COV_R = 0.24. More importantly, the normal is by far the more sensitive to increasing reliability as a result of small changes in COV_R. And for the lognormal, it really does not justify to have further reductions in COV_R below about 0.15. Under these circumstances, the only useful way to increase reliability would be by increasing the CFS. A comparison of Figs. 5a and 5c is revealing: whereas the normal curves show similar trends, the lognormal curves are drastically affected by changes in COV_S.

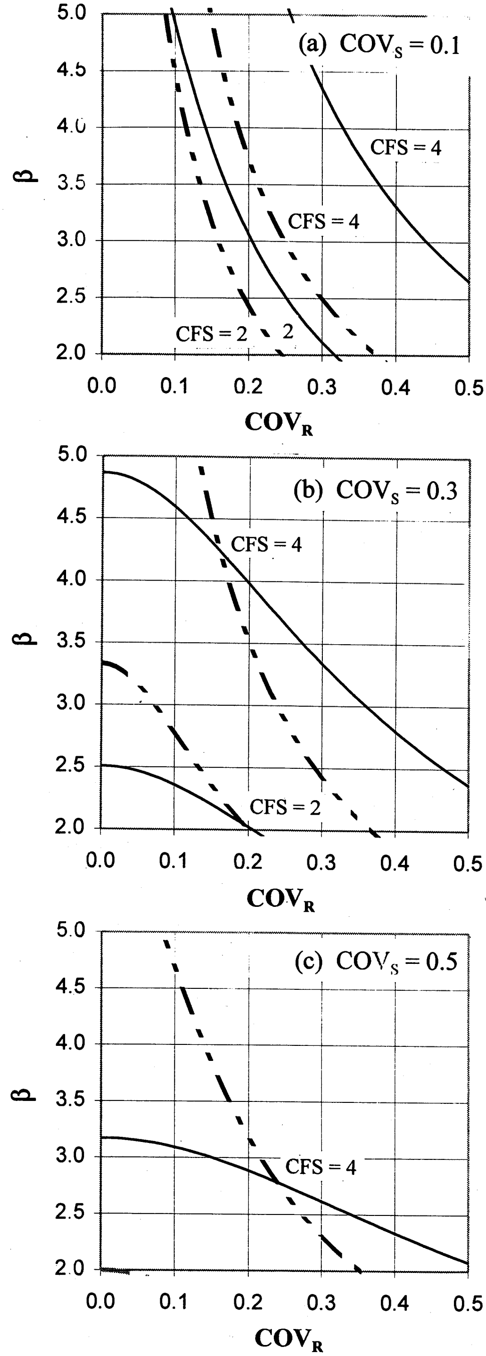

Fig. 5 Comparison of normal (dashed) and lognormal (full) PDF β vs. COV_R results for CFS of 2 & 4

OBSERVATIONS

- The interaction of quality, reliability, loads and capacities is highly complex and is significantly different for the two distributions examined.

- The impact of the variability of the load, COV_S, on reliability could be significant.

- Reductions in variability by a quality program should be explicitly considered, such that some measure of the increased reliability can be calculated and the cost effectiveness of the quality program assessed. Otherwise, a non-quantified improvement in reliability may lead to an unjustified sense of security, as the β vs. COV_R curves tend to flatten out as COV_R is reduced.

- For existing facilities both $\overline{R}$ and COV_R are frozen, and hence β, for any given COV_S. $\overline{R}$ and COV_R can be estimated from the general practice of design and construction that existed during the building of the facility. Figs. 3 and 4, or alternately Appendix A, can then be used to relate reliabilities of these existing facilities *relative* to those of the commercial nuclear facilities using input data deemed appropriate for the latter. Sensitivity studies using different PDFs should be undertaken to establish robust results. Alternatively, robust justification should be presented for the use of the selected distribution.

- Although drastic improvements in reliability could be obtained by the incorporation of quality in the constructed facility, under certain conditions continued imposition of quality requirements does not translate into improved reliability.

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APPENDIX A

Equations for Central and Median Factors of Safety

a) Normal Probability Density Function

The reliability Index, β, is given by

$$\beta = \frac{\bar{R} - \bar{S}}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad \textbf{(A1)}$$

Rearranging Eq. A1 as follows leads to Eq. A3

$$\bar{R} - \bar{S} = \beta\sqrt{\sigma_R^2 + \sigma_S^2}$$

$$\frac{\bar{R}}{\bar{S}} - 1 = \beta\sqrt{\frac{\sigma_R^2}{\bar{S}^2} + \frac{\sigma_S^2}{\bar{S}^2}}$$

$$CFS - 1 = \beta\sqrt{\frac{\sigma_R^2}{\bar{S}^2} + COV_S^2}$$

$$CFS - 1 = \beta\sqrt{COV_R^2 \cdot \frac{\bar{R}^2}{\bar{S}^2} + COV_S^2}$$

$$CFS - 1 = \beta\sqrt{CFS^2 \cdot COV_R^2 + COV_S^2}$$

$$(CFS - 1)^2 = \beta^2 (CFS^2 \cdot COV_R^2 + COV_S^2) \qquad \textbf{(A2)}$$

$$CFS^2 - 2CFS + 1 = \beta^2 CFS^2 COV_R^2 + \beta^2 COV_S^2$$

$$CFS^2(1 - \beta^2 COV_R^2) - 2CFS + (1 - \beta^2 COV_S^2) = 0 \qquad \textbf{(A3)}$$

The above equation (A3) is quadratic in CFS. Thus, CFS can be obtained from

$$CFS = \frac{1 \pm \sqrt{1 - \left(1 - \beta^2 COV_S^2\right)\left(1 - \beta^2 COV_R^2\right)}}{1 - \beta^2 COV_R^2} \qquad \textbf{(A4)}$$

From Eq. (A2) the following steps lead to an expression for COV_R

$$\left(\frac{CFS - 1}{\beta}\right)^2 = CFS^2 \cdot COV_R^2 + COV_S^2$$

$$COV_R^2 \cdot CFS^2 = \left(\frac{CFS - 1}{\beta}\right)^2 - COV_S^2$$

$$COV_R = \frac{1}{CFS}\sqrt{\left(\frac{CFS - 1}{\beta}\right)^2 - COV_S^2} \qquad \textbf{(A5)}$$

Thus, given β and CFS, the required COV_R can be calculated from A5.

b) Lognormal Probability Density Function

The reliability Index, β, is given by

$$\beta^* = \frac{\ln\left(R_m / S_m\right)}{\sqrt{\zeta_R^2 + \zeta_S^2}} \qquad \textbf{(A6)}$$

Thus,

$$\ln MFS = \beta^* \sqrt{\zeta_R^2 + \zeta_2^2}$$

and

$$MFS = \exp\left(\beta^* \sqrt{\zeta_R^2 + \zeta_S^2}\right) \qquad \textbf{(A7)}$$