

## FAILURE ANALYSIS OF PRESTRESSED CONCRETE BEAM UNDER IMPACT LOADING

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### ABSTRACT

This paper presents a failure analysis of prestressed concrete (PC) beam under impact loading. At first, the failure analysis of PC beam section is performed by using the discrete section element method in order to obtain the dynamic bending moment-curvature relation. Secondary, the failure analysis of PC beam is performed by using the rigid panel-spring model. Finally, the numerical calculation is executed and is compared with the experimental results. It is found that this approach can simulate well the experiments at the local and overall failure of the PC beam as well as the impact load and the displacement-time relations.

### 1. INTRODUCTION

On July 16, 1989 the collapse accident of the prestressed concrete (PC) rock-shed structure has occurred at Fukui Prefecture in Japan by falling rocks. Taking this opportunity the safety assessment [1],[2] of the PC rock-shed structure has become more important from the viewpoint of the fundamental study. The PC rock-shed is currently designed by the allowable stress design method by elastic analysis in which the impact (dynamic) load is replaced into static load. However, nobody knows concerning the safety of the PC rock-shed, for instance, until how much can the PC rock-shed withstand for the weight and height of the falling rocks. Little attempt has been made so far on the safety assessment of the PC rock-shed from the viewpoint of the structural failure under impact loading.

This study presents a failure analysis of the PC beams under impact loading as the fundamental step [3],[4]. Firstly, the failure analysis is performed at the section level by using the discrete section element method to find the dynamic bending moment( $M$ )-curvature( $\phi$ ) relation. Secondary, the failure analysis is also performed at the beam level by using the rigid panel-spring model [5] connecting with the discrete section elements. Finally, the numerical calculation results are compared with the experimental results of impact test.

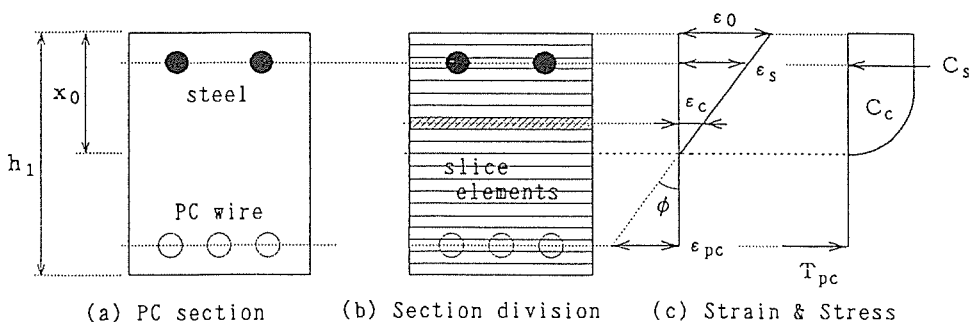


Fig.1 Discrete section model

2. FAILURE ANALYSIS OF CROSS SECTION OF PC BEAM

The failure analysis of the cross section of PC beam is performed by using the discrete section element method to find the bending moment(M)-curvature ( $\phi$ ) relation. The cross section of PC beam is divided into the slice elements as shown in Fig. 1.

2.1 Analysis Procedure

① The initial stress is introduced into the section by the prestressing of PC wire and the initial strain of each element is calculated.

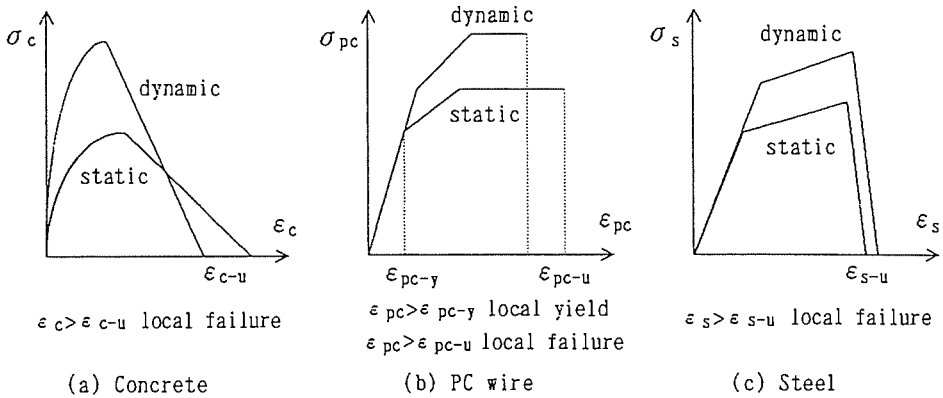


Fig.2 Relationship of dynamic stress-strain of each material and definition of local failure

② The curvature and the strain by the external force are calculated by assuming the position of neutral axis. Therefore, the total strain at each element of section is found by adding the initial strain to the strain due to the external force.

③ The stress at each element of section is obtained by using the stress-strain relation of each material considering the strain-rate effects of concrete, steel and PC wire as shown in Fig. 2.

④ The exact position of neutral axis can be determined by the equilibrium condition that the compressive force is equal to the tensile force.

⑤ The exact stress and strain at each element of section are modified by the exact position of neutral axis and, as such, the M- $\phi$  curve can be found as shown in Fig. 3.

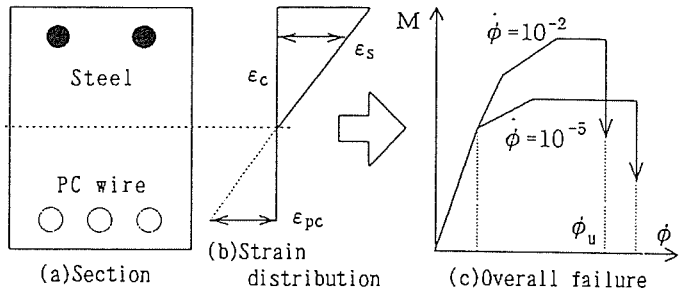


Fig.3 Dynamic bending moment and curvature relation and overall failure

2.2 Definition of Failure at Cross Section level

The failure of PC section is

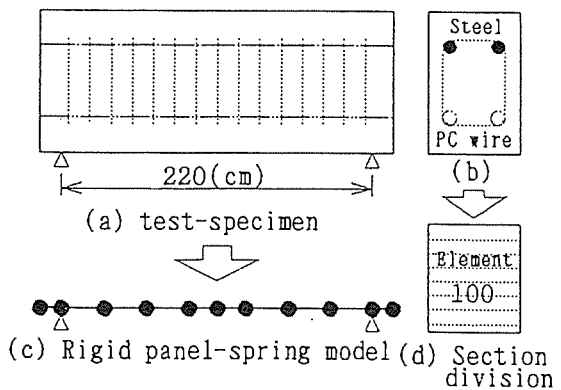


Fig.4 Model of PC beam

divided into the local failure and overall failure. The local failure is defined as the state the response strain of each material exceeds the ultimate strain considering the strain-rate effect as shown in Fig. 2.

On the other hand, the overall failure is defined as the state that the response curvature exceeds the ultimate curvature at which the equilibrium condition does not hold at section level as shown in Fig. 3.

### 3. FAILURE ANALYSIS OF PC BEAM

The failure analysis of PC beam is performed by using the rigid panel-spring model connecting with the failure analysis of the cross section of PC beam. The PC beam is firstly modelled into the rigid panel-spring elements [5] as shown in Fig. 4. The impact force is found by using the contact spring and dashpot as follows:

$$P = k_w(u_w - u_P) + C_w(\dot{u}_w - \dot{u}_P) \quad (1)$$

where,  $k_w = (EA/\ell)$ : the contact spring constant,  $C_w = (2h_w\sqrt{m_w k_w})$ : the contact damping coefficient,  $m_w (=W/g)$ : the mass of weight,  $h_w$ : the damping constant,  $u_w, u_P$ : the displacements of weight and

impact point,  $\dot{u}_w, \dot{u}_P$ : the velocities of weight and impact point,  $W$ : the weight of falling rock,  $g$ : the acceleration of gravity,  $A$ : the loading area,  $E$ : the Young's modulus of concrete,  $\ell$ : the distance between gravity points of weight and beam.

The analysis procedure is started from giving the initial velocity as the initial condition as shown in Fig. 5. The equation of motion can be solved by using the Newmark  $\beta$ -method connecting the discrete element method. The local failure and the overall failure of PC section are judged at each time step. The failure analysis is stopped at when the response curvature  $\phi_i$  reaches to the ultimate curvature  $\phi_u$  corresponding to the overall failure of the section level.

### 4. NUMERICAL CALCULATION

The numerical calculation is performed under the condition that the weight 262.4kgf is dropped at the center of the PC beam which has the length 200cm and the degrees of prestressing [6]  $q=0.189$  and  $q=0.284$  as shown in Fig. 6 at the dropping velocity  $V_0=1\text{m/sec}$  and  $V_0=8\text{m/sec}$ , respectively. This input

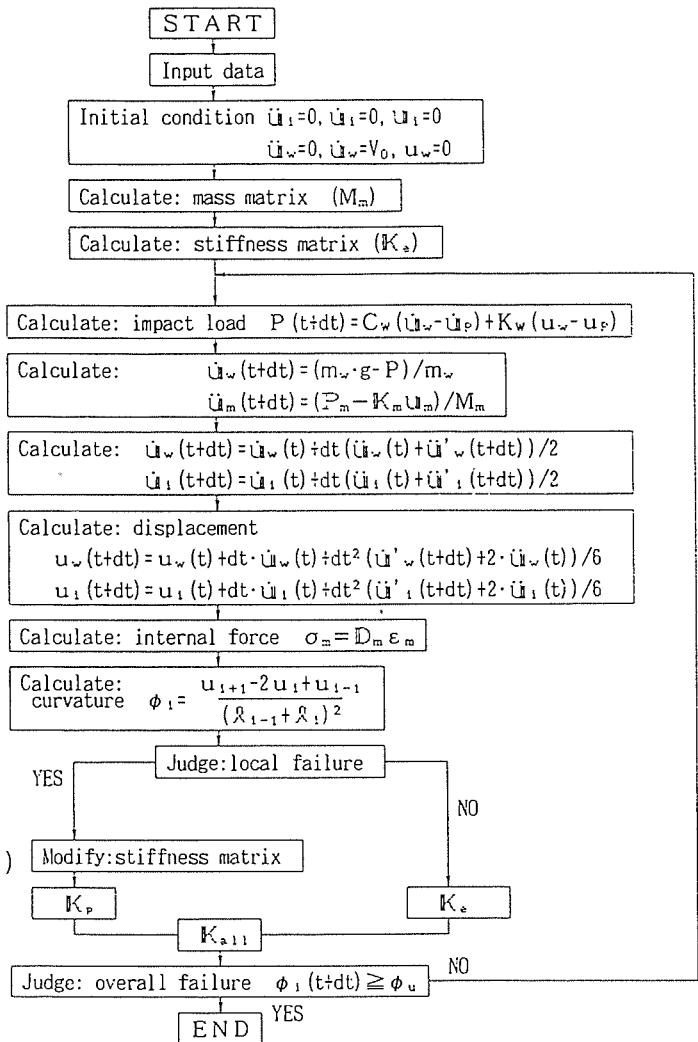


Fig. 5 Failure analysis of PC beam

data is the same as the impact test [3]. The material properties of concrete, steel and PC wire are illustrated in Table 1. Herein, the contact spring constant  $k_w=3.0 \times 10^5 \text{kgf/cm}$  ( $E=4.5 \times 10^5 \text{kgf/cm}^2$ ,  $A=20\text{cm} \times 0.5\text{cm}$ ,  $l=15\text{cm}$  are used) and the damping coefficient  $C_w=56.6 \text{kgf}\cdot\text{s/cm}$  ( $h_w=0.1$ ,  $m_w=0.267 \text{kgf}\cdot\text{s}^2/\text{cm}$  are used) are adopted in Eq. (1). The time increment is used as  $\Delta t=10^{-5}\text{sec}$  in the Newmark  $\beta$ -method.

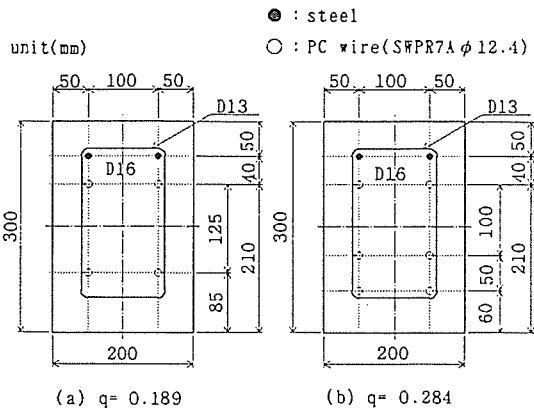


Fig. 6 PC beam section

4.1 Failure Behavior of PC Section

The dynamic bending moment-curvature relation of PC section is calculated for the degree of prestressing  $q=0.284$  by considering the strain rate  $\epsilon=10^{-2}$  in the upper extreme fibre of the PC section.

The local failure proceeds successively at the points A, B, C in Fig. 7 which are corresponding to the section failure behavior A, B, C in Fig. 8. Finally, the overall failure has occurred at the point D in Fig. 7 corresponding to the section behavior D in Fig. 8.

Table 1 Material properties of PC beam

Concrete	Water-cement ratio	0.32
	Cement (kgf/m <sup>3</sup> )	420.0
	Water (kgf/m <sup>3</sup> )	134.0
	Coarse aggregate (kgf/m <sup>3</sup> )	668.0
	Fine aggregate (kgf/m <sup>3</sup> )	1233.0
PC wire	Compressive strength (kgf/m <sup>2</sup> )	700.0
	Initial prestressig (tf/unit)	8.0

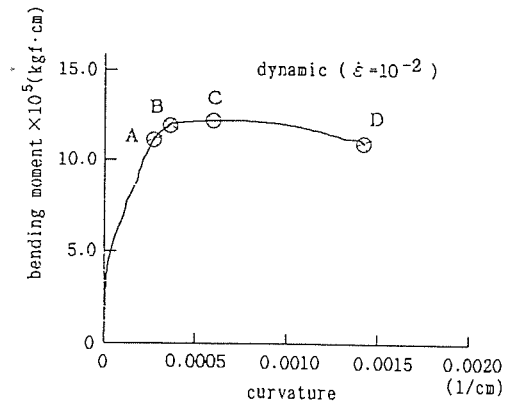


Fig. 7 Dynamic bending moment~curvature relation ( $q=0.284$ )

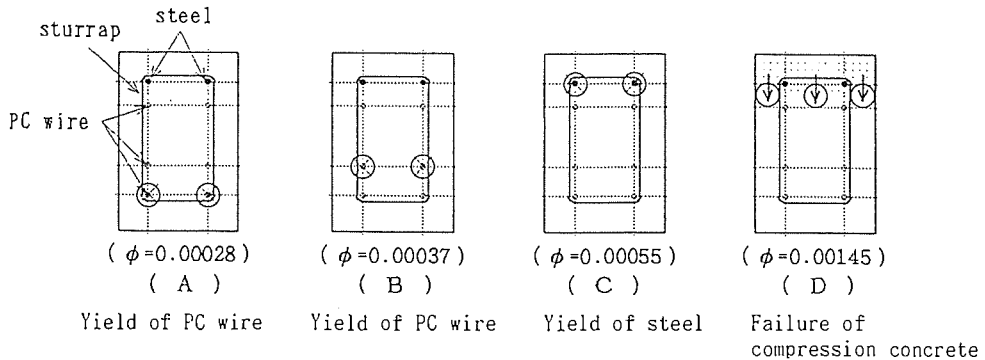


Fig. 8 Failure behavior of PC beam

4.2 Failure Behavior of PC beam

(1) Impact response of PC beam at the elastic range

The relation between the load and time of PC beam with  $q=0.284$  is compared with the experimental results at the elastic range ( $W=262.4\text{kgf}$ ,  $V_o=1.0\text{m/sec}$ ) as shown in Fig. 9. It is found that the maximum impact load obtained by this analysis agrees with the experimental one. Figure 10 shows the relationship between the displacement at the center of PC beam and time. It is also confirmed that the computational result coincides with the experimental one.

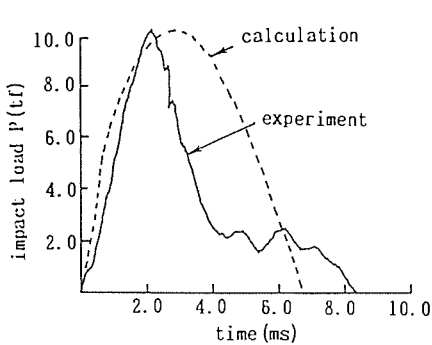


Fig.9 Impact load~time relation  
( $q=0.189$ ,  $W=262.4\text{kgf}$ ,  $V_0=1.0\text{m/s}$ )

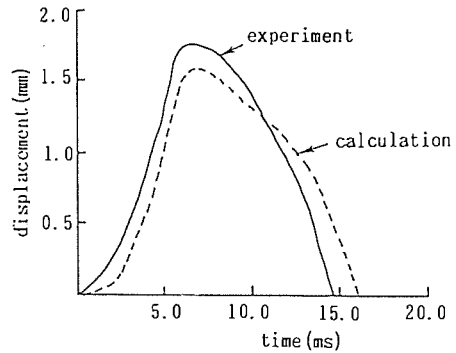


Fig.10 Displacement~time relation  
( $q=0.189$ ,  $W=262.4\text{kgf}$ ,  $V_0=1.0\text{m/s}$ )

(2) Impact response of PC beam at the ultimate level

The impact loading condition is increased until the weight  $W=400\text{kgf}$  and the velocity  $V_0=8.0\text{m/sec}$ . The two types of PC beams are used as  $q=0.189$  and  $q=0.284$ . The impact load-time relations are found as shown in Fig. 11(a),(b) and are compared with the experimental results.

The displacement-time relation are also obtained as shown in Fig. 12 and are compared with the experimental results. It is recognized that this approach can simulate well the experimental results even in the ultimate level.

(3) Failure modes of PC beams

Figure 13(a) and (b) shows the failure process of PC beams corresponding to the time history in the cases  $q=0.189$  and  $q=0.284$ , respectively. It is found from Fig. 13(a) and (b) that the failure process is changed by the

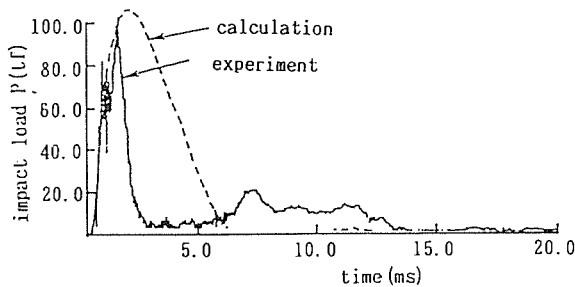


Fig.11(a) Impact load~time relation  
( $q=0.189$ ,  $W=400.0\text{kgf}$ ,  $V_0=8.0\text{m/s}$ )

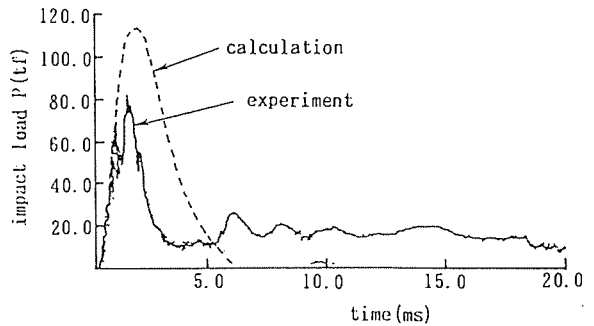


Fig.11(b) Impact load~time relation  
( $q=0.284$ ,  $W=400.0\text{kgf}$ ,  $V_0=8.0\text{m/s}$ )

degree of prestressing. In the case of  $q=0.189$ , the failure of PC beam is due to the failure of PC wire that the response strain has reached to the ultimate strain of PC wire. It is also found from Photo 1(a) that the experimental failure mode is collapsed due to the failure of the PC wire.

Then, in the case of  $q=0.284$ , the PC beam has not completely failed, although the strain of PC wire has reached to the yield strain, after the upper part of concrete has failed. It is found from Fig. 13(b) and Photo 1(b) that the analytical failure mode almost agrees with the experimental failure mode. Therefore, it is confirmed that the failure modes of the PC beam under impact loading can be simulated well by this analysis.

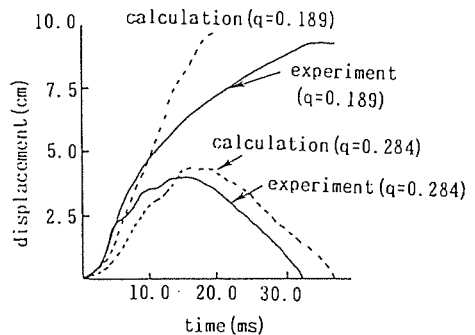


Fig.12 Displacement~time relation  
( $W=400.0\text{kgf}$ ,  $V_0=8.0\text{m/s}$ )

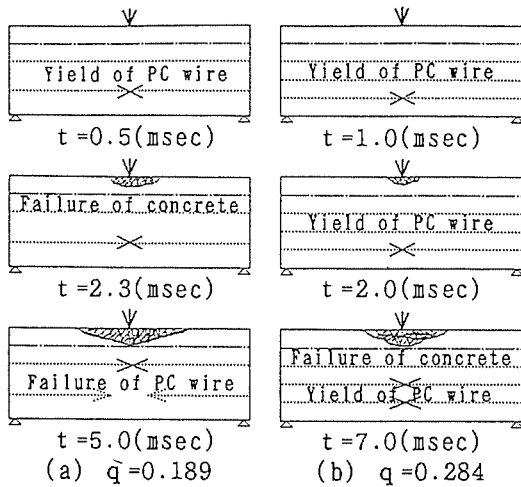
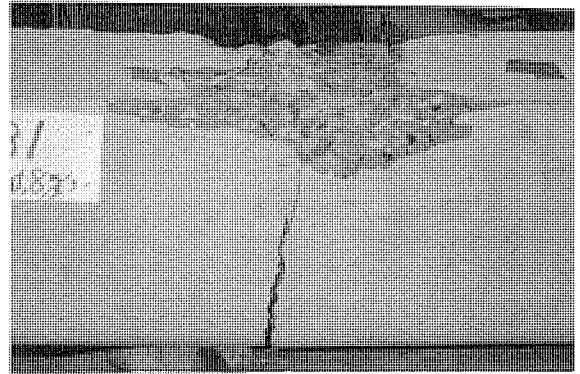
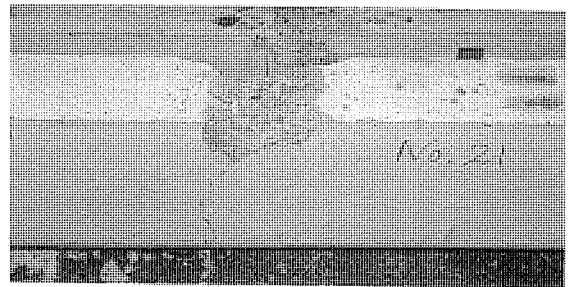


Fig.13 Failure process of PC beam  
( W=400.0kgf, V= 8.0 m/sec )



(a)  $q = 0.189$



(b)  $q = 0.284$

Photo 1 Failure mode of impact experiment

5. CONCLUSIONS

The following conclusions are drawn from this study.

- (1) The proposed analysis can find the failure behavior of the cross section of PC beam by using the discrete section element method.
- (2) In the elastic range level, this analysis has simulated well the experimental results.
- (3) In the ultimate load level, the load-time relation and the displacement-time relation by this method are almost good agreements with the experimental results.
- (4) It has been confirmed that the proposed analysis gives good agreements with the experiment results concerning the failure modes corresponding to the degree of prestressing.

Therefore, this approach is very useful for the estimation of the failure mode of the PC beam under impact loading. This method will be also applied to the safety assessment of the PC rock-shed structure under falling rocks.

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