A Stochastic Fracture Mechanical Approach to LBB Assessment for Pipings

Akira TSURUI, Takeyuki TANAKA
Kyoto University, Kyoto Japan
Hiroaki TANAKA
Hiroshima University, Hiroshima, Japan

ABSTRACT

An analytical method is proposed for probabilistic assessment of LBB in pipings based upon a stochastic fracture mechanics. First, a probability density function of random surface crack propagation is derived by utilizing a Markov approximation method. Next, discussion is made on how we apply the result to the probabilistic LBB assessment.

1. INTRODUCTION

In order to achieve a probabilistic Leak Before Break (LBB) assessment for pipings (Lo 1985) from a viewpoint of the structural reliability, we must comprehend a stochastic property of surface crack propagation considering some uncertain factors such as material inhomogeneity, random loading, and so on. Whether the LBB is realized or not depends on such a random nature with respect to crack propagation, initial crack size and shape, etc. Therefore, we need to obtain a probability distribution for the surface crack propagation by taking such uncertainties simultaneously into account.

In this paper, by utilizing a stochastic fracture mechanics, we investigate the random fatigue crack growth process and discuss a way to apply it to the probabilistic LBB assessment for pipings. Basically, we follow the assumptions used in the authors' previous studies (Tanaka *et al.* 1989a; Tanaka 1989b) such that: (i) the surface crack is always semi-elliptical, (ii) its propagation can be decomposed into surface and depth directions, (iii) the well-known Paris' law is applicable to each direction, (iv) the Newman-Raju's expression is applicable for the stress intensity factor of the surface crack.

First, we mathematically formulate the stochastic surface crack propagation and derive its approximate probability distribution by utilizing a Markov approximation method (Tsurui *et al.* 1986), under the condition that the loading process and materials' property and initial crack size are all random. Next, we propose two concepts on the probability of LBB in pipings, and discuss the way of their derivation by the use of the probability distribution of the surface crack propagation.

2. PROPAGATING EQUATION OF SURFACE CRACKS

We consider a semi-elliptical surface crack growing under uniform tensile stress in a finite plate, which is illustrated in Fig.1, in which $A_1(n)$, $A_2(n)$, B_1, B_2 represent a half surface

SMiRT 11 Transactions Vol. M (August 1991) Tokyo, Japan, © 1991

length after n cycles of loading, a crack depth after n cycles, a specimen's half width, a specimen's thickness, respectively.

As mentioned in the previous section, we assume that the propagating equation of the surface crack is of Paris' type (Newman *et al.* 1978), i.e.,

$$\frac{dA_j}{dn} = \varepsilon_{0j}(\Delta K_j)^{2(\lambda+1)} \quad (j = 1, 2), \tag{1}$$

where ΔK_1, ΔK_2 represent a stress intensity factor range at the point P_1, that at the point P_2, respectively, and ε_{01}, ε_{02}, λ are material constants. It should be noted that λ usually takes on a positive value for ordinally metallic materials.

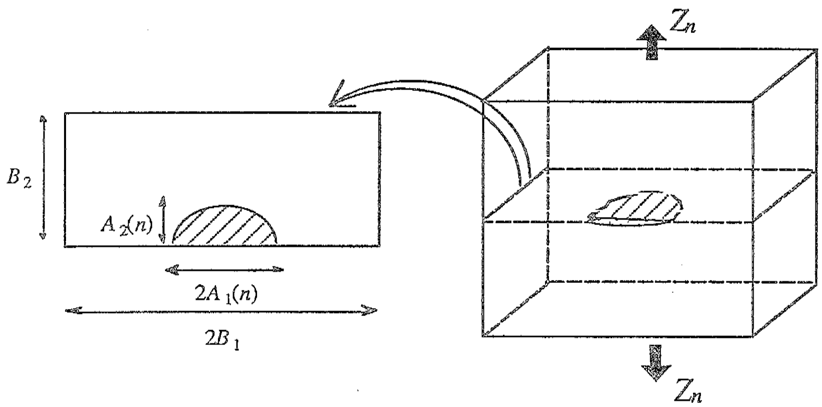

Fig.1 Semi-elliptical surface crack in a finite plate.

Applying the Newman-Raju's solution (1978), which has been obtained through a three dimensional stress analysis by the use of a finite element method, to the stress intensity factor of the surface crack, and transforming the dependent variables (A_1, A_2) into the non-dimensional ones by $X_j = A_j/B_j \quad (j = 1, 2)$, we can obtain the following simultaneous propagating equations:

$$\frac{dX_j}{dn} = \varepsilon C_{j,n} Z_n^{2(\lambda+1)} g_j(X_1, X_2) \quad (j = 1, 2), \tag{2}$$

in which Z_n represents a tensile stress amplitude at the n-th cycles of loading, $C_{j,n}$ represents a non-dimensional crack propagation resistance at the point P_j, ε is defined so as to satisfy $\varepsilon C_{j,n} = \varepsilon_{0j}\pi^{\lambda+1}B_2^{\lambda} \quad (j = 1, 2)$, and the functions $g_j \quad (j = 1, 2)$ are defined as follows:

$$g_1(X_1, X_2) = B^{*\lambda+2}\left[\frac{M(X_1, X_2)}{\sqrt{Q(X_1, X_2)}}\frac{X_2}{\sqrt{X_1}}g(X_2)f_w(X_1, X_2)\right]^{2(\lambda+1)}, \tag{3.a}$$

$$g_2(X_1, X_2) = \left[\frac{M(X_1, X_2)}{\sqrt{Q(X_1, X_2)}}\sqrt{X_2}f_w(X_1, X_2)\right]^{2(\lambda+1)}, \tag{3.b}$$

$$Q(X_1, X_2) = 1 + 1.464\{\mathrm{Min}[\frac{B^* X_2}{X_1}, \frac{X_1}{B^* X_2}]\}^{1.65}, \tag{4}$$

$$M(X_1, X_2) = \left(1.13 - 0.09B^*\frac{X_2}{X_1}\right) + \left(-0.54 + \frac{0.89}{0.2 + B^* X_2/X_1}\right)X_2^2$$

$$+\left(0.5-\frac{1}{0.65+B^{*}X_2/X_1}+14(1-B^{*}X_2/X_1)^{24}\right)X_2^4 \tag{5}$$

$$g(X_2)=1.1+0.35X_2^2, \tag{6}$$

$$f_w(X_1,X_2)=\sqrt{\sec\left(\frac{\pi}{2}X_1\sqrt{X_2}\right)}, \tag{7}$$

$$B^{*}=B_2/B_1, \tag{8}$$

It should be noted that the propagating equation (2) is available only in the domain

$$\Omega_X=\{(X_1,X_2)\mid X_1\sqrt{X_2}<1, X_1>0, X_2>0\}, \tag{9}$$

and has a singularity of $dX_j/dn\to\infty$ $(j=1,2)$ on a boundary curve $X_1\sqrt{X_2}=1$ of the domain Ω_X, which is caused by the finite width correction term given by Eq.(7). According to this singularity, if the solution process of Eq.(2) once reaches the boundary curve, it can never return to the domain Ω_X and remains to stay therein forever. That boundary curve is therefore called *death surface* according to Tsurui *et al.* (1989), which must be treated as a kind of absorbing state.

3. PROBABILITY DISTRIBUTION OF THE CRACK PROPAGATION

Let us consider the case in which both the loading and the materials' property are random. In this case, $C_{j,n}$ and Z_n must be treated as stochastic processes, and therefore Eq.(2) must be treated as a system of stochastic differential equations. To analyze it, we assume that the processes $C_{j,n}$ and Z_n are locally stationary.

Let $w(\mathbf{x},n\mid \mathbf{x}_0)$ be the conditional probability density function of the solution process of Eq.(2) under the condition $\mathbf{X}(0)=\mathbf{x}_0$. Applying the Markov approximation method (Tsurui *et al.* 1986), we can obtain the generalized Fokker-Planck equation, which is a partial differential equation to describe the temporal variation of the density w. If the crack propagating resistance does not have a random nature, we can obtain the analytical solution of the Fokker-Planck equation (Tanaka *et al.* 1990), but otherwise we can not obtain its solution analytically at the present stage. Therefore, we propose an approximate expression for the density function w.

Let us assume, as a first approximation, that each propagating resistance is set to be its expected value, then we can obtain the following "trajectory "curve from Eq.(2) by eliminating the time variable n,

$$T(\mathbf{X}:\mathbf{x}_0)\equiv\frac{1}{\lambda+2}(X_1^{\lambda+2}-x_{01}^{\lambda+2})-\frac{B^{*\lambda+2}}{\alpha}\int_{x_{02}}^{X_2}x^{\lambda+1}g(x)dx=0, \tag{10}$$

where $\alpha\equiv \mathrm{E}[C_{2,n}]/\mathrm{E}[C_{1,n}]$ represents the ratio of mean propagating resistances, and the initial condition is set as $\mathbf{X}(0)=\mathbf{x}_0$. Solving Eq.(10) with resprct to X_1 or X_2 as

$$X_1=T_1(X_2;\mathbf{x}_0),\qquad X_2=T_2(X_1;\mathbf{x}_0), \tag{11}$$

and again substituting it into Eq.(2), we can obtain the following new propagating equation for the surface crack:

$$\frac{dX_j}{dn}=\varepsilon C_{j,n}Z_n^{2(\lambda+1)}\tilde{g}_j(X_j;\mathbf{x}_0)\quad (j=1,2), \tag{12}$$

$$\tilde{g}_1(X_1)\equiv g_1(X_1,T_2(X_1;\mathbf{x}_0)),\ \tilde{g}_2(X_2)\equiv g_2(T_1(X_2;\mathbf{x}_0),X_2). \tag{13}$$

In this paper, we use Eq.(12) as the basic stochastic differential equation describing the random crack propagation.

According to Tanaka *et al.* (1989), the conditional probability density function for the solution process of Eq.(12) is given as follows:

$$w(\mathrm{x}, n \mid \mathrm{x}_0) = \frac{1}{\tilde{g}_1(x_1)\tilde{g}_2(x_2)4\pi\sqrt{G(n)}} \exp\left\{ -\frac{G_{22}x_1^{*2} - (G_{12}+G_{21})x_1^* x_2^* + G_{11}x_2^{*2}}{4G(n)} \right\}, \tag{14}$$

$$G(n) = G_{11}(n)G_{22}(n) - \{G_{12}(n) + G_{21}(n)\}^2/4, \tag{15}$$

$$G_{jk}(n) = \varepsilon^2 \int_0^n dn' \int_{-\infty}^0 \mathrm{K}[C_{j,n'}Z_{n'}^{2(\lambda+1)}, C_{k,n'+n''}Z_{n'+n''}^{2(\lambda+1)}]dn'' \qquad (j,k=1,2) \tag{16}$$

$$x_j^*(x_j, n; \mathrm{x}_0) = \int_{x_{0j}}^{x_j} \frac{dx}{\tilde{g}_j(x;\mathrm{x}_0)} - \varepsilon \mathrm{E}[C_{j,n}Z_n^{2(\lambda+1)}]n \quad (j=1,2), \tag{17}$$

in which $\mathrm{K}[\cdot,\cdot]$ denotes an operator to take covariance. We should notice that the density $w(\mathrm{x}, n \mid \mathrm{x}_0)$ is defined only in Ω_X.

If the initial crack size and shape are random, by introducing a probability density function $f_0(\mathrm{x}_0)$ for the initial state vector x_0, the probability density function after n cycles is given as follows:

$$w(\mathrm{x}, n) = \iint w(\mathrm{x}, n \mid \mathrm{x}_0) f_0(\mathrm{x}_0) d\mathrm{x}_0. \tag{18}$$

4. APPLICATION TO PROBABILISTIC LBB ASSESSMENT FOR PIPINGS

In this section, expecting that the result in the previous section is available as a model of random surface crack growth in pipings, we discuss the way of application to probabilistic LBB assessment. That is, we suppose a pipe with the internal radius B_1/π and the thickness B_2.

Let $g_L(\mathrm{X})$ and $g_B(\mathrm{X})$ be the limit state functions in the X plane for leak and for break, respectively, and we may assume that the leak of internal fluid takes place when the solution process $\mathrm{X}(n)$ arrives at the region $\{\mathrm{X} \mid g_L(\mathrm{X}) \le 0\}$ for the first time, and that the region $\{\mathrm{X} \mid g_B(\mathrm{X}) \le 0\}$ corresponds to the break of the component. For example, if we can neglect the effect of crack opening displacement, the function $g_L(\mathrm{X})$ is given as

$$g_L(\mathrm{X}) = 1 - X_2. \tag{19}$$

The domain Ω_X introduced through Eq.(9) is decomposed into the following four subdomains:

$$\begin{cases} \Omega_{\mathrm{SAFE}} = \{\mathrm{X} \mid g_L(\mathrm{X}) > 0, g_B(\mathrm{X}) > 0\} \cdots\cdots \text{Safe} \\ \Omega_{\mathrm{LBB}} = \{\mathrm{X} \mid g_L(\mathrm{X}) \le 0, g_B(\mathrm{X}) > 0\} \cdots\cdots \text{Leak Before Break} \\ \Omega_{\mathrm{BBL}} = \{\mathrm{X} \mid g_L(\mathrm{X}) > 0, g_B(\mathrm{X}) \le 0\} \cdots\cdots \text{Break Before Leak} \\ \Omega_{\mathrm{LAB}} = \{\mathrm{X} \mid g_L(\mathrm{X}) \le 0, g_B(\mathrm{X}) \le 0\} \cdots\cdots \text{Leak and Break} \end{cases}$$

which are schematically illustrated in Fig.2.

Let $p_{\mathrm{LBB}}(n)$ be the probability that the state is in LBB at the n-th cycle. If we can neglect the probability that the crack length decreases, it equals to the probability that $\mathrm{X}(n)$ lies in the domain Ω_{LBB}, which leads to

$$p_{\mathrm{LBB}}(n) = \iint_{\Omega_{\mathrm{LBB}}} w(\mathrm{x}, n) d\mathrm{x}. \tag{20}$$

where $w(\mathrm{x}, n)$ is the probability density function given by Eq.(18).

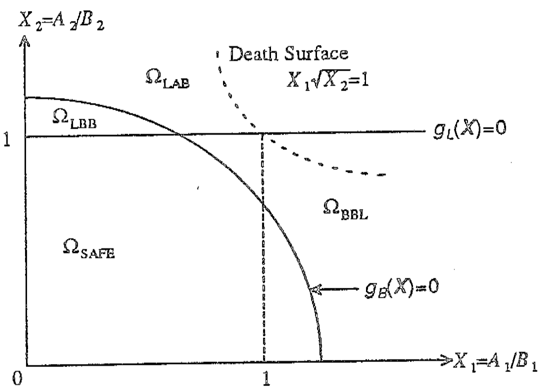

Fig.2 Schematic illustration of limit state functions.

The solution process arrives at, whether LBB is realized or not, the break zone $\Omega_{\mathrm{LAB}} \cup \Omega_{\mathrm{BBL}}$, and it is finally absorbed into the death surface. Therefore, in designing pipings, we need to obtain the knowledge on the probability that the solution process passes the domain Ω_{LBB}. We express it p^*_{LBB}, and call it *passage probability of LBB.*

To evaluate p^*_{LBB}, we introduce the following BBL occurring rate:

$$r_{\mathrm{BBL}}(n)\Delta n = \Pr[\mathrm{X} \text{ arrives at } \Omega_{\mathrm{BBL}} \text{ in } [n, n+\Delta n] \mid \mathrm{X}(n) \in \Omega_{\mathrm{SAFE}}], \tag{21}$$

which results in

$$r_{\mathrm{BBL}}(n) = \lim_{\Delta n \to 0} \frac{1}{\Delta n} \iint_{\Omega_{\mathrm{BBL}}} d\mathrm{x} \iint_{\Omega_{\mathrm{SAFE}}} w(\mathrm{x}, n+\Delta n \mid \mathrm{x}', n) w^*(\mathrm{x}', n) d\mathrm{x}', \tag{22}$$

where $w(\mathrm{x}, n+\Delta n \mid \mathrm{x}', n)$ represents the probability density function of $\mathrm{X}(n+\Delta n)$ under the condition that $\mathrm{X}(n)$ takes on x', which is easily derived by shifting the time origin in the discussion of the previous section, and $w^*(\mathrm{x}, n)$ represents the probability density function of $\mathrm{X}(n)$ under the condition that $\mathrm{X}(n)$ lies in Ω_{SAFE}. By neglecting the probability that the crack length decreases, $w^*(\mathrm{x}, n)$ is approximately given as follows:

$$w^*(\mathrm{x}, n) = \frac{w(\mathrm{x}, n)}{\iint_{\Omega_{\mathrm{SAFE}}} w(\mathrm{x}, n) d\mathrm{x}} \quad (\mathrm{x} \in \Omega_{\mathrm{SAFE}}). \tag{23}$$

Substituting Eq.(23) into Eq.(22), we can obtain the following expression for $r_{\mathrm{BBL}}(n)$:

$$r_{\mathrm{BBL}}(n) = \frac{1}{\iint_{\Omega_{\mathrm{SAFE}}} w(\mathrm{x}, n) d\mathrm{x}} \iint_{\Omega_{\mathrm{BBL}}} d\mathrm{x} \iint_{\Omega_{\mathrm{SAFE}}} w_n(\mathrm{x}, n \mid \mathrm{x}', n) w(\mathrm{x}', n) d\mathrm{x}', \tag{24}$$

where $w_n(\mathrm{x}, n \mid \mathrm{x}', n')$ denotes a partial derivative of $w(\mathrm{x}, n \mid \mathrm{x}', n')$ with respect to n.

The quantity

$$H_{\mathrm{BBL}}(n) = 1 - \exp\{-\int_0^n r_{\mathrm{BBL}}(n') dn'\}, \tag{25}$$

represents the probability that the solution process passes the domain Ω_{BBL} within n cycles of loading. Since the solution process passes either Ω_{LBB} or Ω_{BBL} with probability one, the passage probability of LBB p^*_{LBB} is finally given as

$$p^*_{\mathrm{LBB}} = 1 - \lim_{n \to \infty} H_{\mathrm{BBL}}(n) = \exp\{-\int_0^\infty r_{\mathrm{BBL}}(n') dn'\}. \tag{26}$$

5. CONCLUDING REMARKS

In this paper, by utilizing the stochastic fracture mechanics, we have derived the probability distribution of random surface crack propagation in a closed form, and with the aid of it, discussed the way to calculate the probability of LBB in pipings. Since extremely high reliability is generally required in reactor technology, it has an important meaning to obtain the probability of LBB in an analytical form.

To assess it numerically, however, we have to evaluate multi-fold integration appearing in Eq.(20) or Eq.(24) with high accuracy. Therefore, we need to develope an efficient computing routine to perform it. In addition to this problem, we have to formulate a more precise condition for LBB by taking the effect of crack opening displacement etc. into account. These are future works.

ACKNOWLEDGEMENT

This reserch was supported in part by a Scientific Grant-In-Aid 1990 (No. 02750070) from the Ministry of Education, Science and Culture, Japan.

REFERENCES

Lo, T. Y. (1985). Failure Criteria Used in a Probabilistic Fracture Mechanics Code, *Proc. of 8th SMiRT*, M1G1/3, pp.237-243.

Newman, Jr., J. C. and Raju, I. S. (1978). Analysis of Surface Cracks in Finite Plates under Tension or Bending Loads, *NASA TP-1578.*

Tanaka, H. and Tsurui, A. (1989). Random Propagation of a Semi- Elliptical Surface Crack as a Bivariate Stochastic Process, *Eng. Frac. Mech.*,**33**, No.5, pp.787-800.

Tanaka, H. (1989). Stochastic Properties of Semi-Elliptical Surface Crack Propagation Based Upon Newman-Raju's K-Expression, *Eng. Frac. Mech.*, **34**, No.1, pp.189-200.

Tanaka, H. and Tsurui, A. (1990). Probabilistic Analysis on Random Propagation of Semi-Elliptical Surface Crack in Consideration of the Specimen's Size Effect, *Proc. of 10th Symposium on Reliability Engineering in Design (JSMS)*, p.173 (in Japanese).

Tsurui, A. and Ishikawa, H. (1986). Application of the Fokker-Planck Equation to a Stochastic Fatigue Crack Growth Model, *Structural Safety*, 4, pp.15-29.

Tsurui, A., Tanaka, H. and Tanaka, T. (1989). Probabilistic Analysis of Fatigue Crack Propagation in Finite Size Specimens, *Prob. Eng. Mech.*, 4, No.3, pp.120-127.