FINITE ELEMENT SOLUTION OF NORMALLY INTERSECTING CYLINDERS

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SUMMARY

A finite element solution of the problem of stress distribution in the vicinity of thin elastic normally intersecting cylinders subjected to internal pressure will be presented in this paper.

The immediate region of the intersection curve is idealized by 12-node, three-dimensional isoparametric elements that include incompatible displacement modes (see: E.L. Wilson, R.L. Taylor, W.P. Doherty, and J. Ghabouss, "Incompatible Displacement Modes," ONR International Symposium on Numerical and Computer Methods in Structural Mechanics, Univ. of Illinois, Urbana-Illinois, U.S.A., 8-10, Sept. 1971). The positions of the shells farther away from the intersection curve are represented by superisoparametric shell elements (see: S. Ahmad, B. Irons, and O.C. Zienkiewicz, "Analysis of Thick and Thin Shell Structures by Curved Finite Elements," *International Journal for Numerical Methods in Engineering*, Vol. 2, (1970) 419-451).

These so-called Ahmad's shell elements were modified by the addition of several incompatible modes to improve their behavior for thin shell situations. The shell elements are merged to three-dimensional elements by special transition elements. The incompatible displacement modes are removed by static condensation prior to the assembly of the element stiffness matrix into the entire structure stiffness matrix. The resulting equilibrium equations are solved by Iron's frontal technique (see: B.M. Irons, "A Frontal Solution Program for Finite Element Analysis," *International Journal for Numerical Methods in Engineering*, Vol. 2, (1970), 5-32).

The results obtained from the finite element model as described above will be compared to the experimental results reported by researchers in the Oak Ridge National Laboratory (see: J.M. Corum, S.E. Bolt, W.L. Greenstreet, R.C. Gwaltney, "Theoretical and Experimental Stress Analysis of ORNL Thin-Shell Cylinder-to-Cylinder Model No. L," U.S Atomic Energy Commission Report ORNL-4553, Oak Ridge National Laboratory).

The aim of this paper is to show that a reasonable solution of the cylinder-to-cylinder intersection problem can be achieved by a realistic representation of the intersection region, i.e., three-dimensional elements where three-dimensional behavior dominates, and curved shell elements where shell behavior dominates. It will be shown in the paper that the use of curved shell elements obviates the need for using an inordinately large number of elements to obtain good solutions to the problem. The mesh is graded in such a way as to increase the number of elements in regions of sharp stress gradient. Incidentally, the finite element model employed here avoids the difficulty of defining a shell normal along the intersection curve which is encountered when flat plate or shell elements are used in the intersection region.

In addition, the paper aims to present an alternative approach to avoiding the "parasitic" shear inherent in the original Ahmad's element, through the artifice of incompatible displacement modes. It is felt that this is a more rational approach to the problem.

1. Introduction

Structures with components made up of intersecting cylinders are of common occurrence in the reactor industry. A knowledge of the state of stress, and especially the magnitude and location of maximum stresses in structures of this type, is of paramount importance in order to achieve safe and efficient designs. Unfortunately, because of the complicated geometry of the intersection region, this problem has not been amenable to the classical treatment of stress analysis. An excellent review of the state-of-the-art of theoretical analysis in regard to this problem was presented by Kekkerkerker [2]. A more attractive alternative is the use of a numerical technique, such as the finite element method.

In earlier finite element idealizations [1, 3, 4, 5, 6] flat-plate elements were used. These idealizations in effect replace the actual smooth curved shell surfaces by faceted surfaces. There is an error introduced by this geometric approximation. Such an error usually diminishes as the mesh is refined. Another difficulty arises in many shell intersections because of the necessity in shell theory of neglecting the rotation about the normal to the middle surface. For points lying along the intersection, enforcing this restriction in one shell unduly restricts bending in the other shell. The behavior in the intersection region is truly three-dimensional, and any constraints used to reduce it to two-dimensions introduce an error whose magnitude is not affected by mesh refinement.

This paper presents a method of overcoming these difficulties.

2. The Shell Element

Except for the immediate region of intersection, curved shell elements of the "superisoparametric" type were used in the idealization of the two cylinders. The element was originally formulated by Ahmed et al [7]. It was derived from the 16 node three-dimensional isoparametric element by forcing the nodes on the top and bottom faces of the element to displace in such a way that u and v vary linearly through the thickness, while w remains constant. This is achieved by defining three displacements of the middle surface and two independent rotations of the normal to the middle surface about orthogonal axes perpendicular to it.

The element as originally formulated suffered from a major defect, in that because of the relaxation of the usual Kirchoff hypothesis, the transverse shear energy in bending situations was too large compared with the correct value. The problem becomes apparent in thin shell applications where the radius to thickness ratios are large. Convergence to the true answer is so slow that it becomes uneconomical to use this element for thin shells.

To solve this problem, several approaches are available. Pawsey [8] reduced the order of numerical integration of the element stiffness matrix from 3 to 2, and Gauss points in certain directions for selected terms of the strain energy. Zienkiewicz, Too, and Taylor [9] used 2 by 2 Gaus integration points, for the ξ and η directions, the integration in the ζ direction being performed explicitly. Bakhrebah and Schnobrich [11] introduced incompatible displacement modes into the element formulation. The incompatible modes are removed by static condensation at the element level before assembly into the structure stiffness matrix.

3. The Intersection Region

As noted earlier, the behavior around the intersection region is truly three-dimensional. To represent this behavior, 12 node three-dimensional isoparametric elements were used. The response of these elements was modified by the addition of incompatible displaced modes [10, 11].

Connection between the three-dimensional and the shell elements was achieved by transforming the shell's five degrees of freedom into six translational degrees of freedom (three at the top and bottom faces of the shell).

4. Numerical Results

The Scheme outlined above was tested by solving an example for which excellent experimental data exist. Two versions of the shell element were used: one is the reduced integration element of Zienkiewicz et al, and the other is the incompatible displacement element of Bakhrebah. The results of analysis with both elements were, for all practical purposes, identical. The reduced integration element, however, was more economical to use.

The structure considered is shown in Figure 1. Because of symmetry, only 1/4 of the structure was discretized. Figs. 2 to 6 show stresses calculated at Gauss points along lines close to lines AB and BC in the cylinder and the nozzle, respectively. Comparison with the experimental results shows generally good agreement.

5. Acknowledgement

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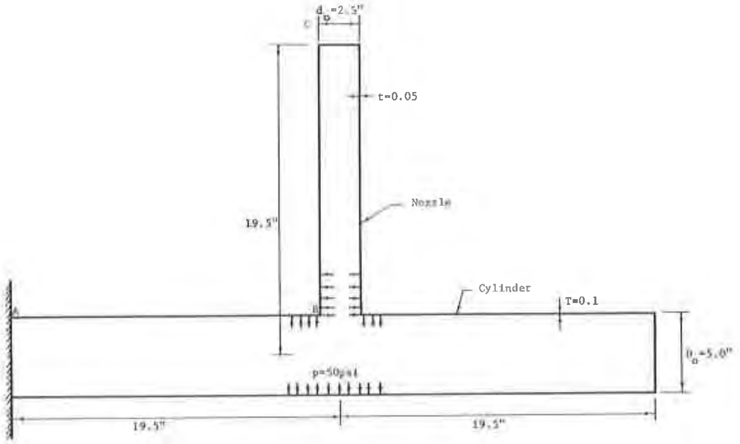

Figure 1. Test Specimen

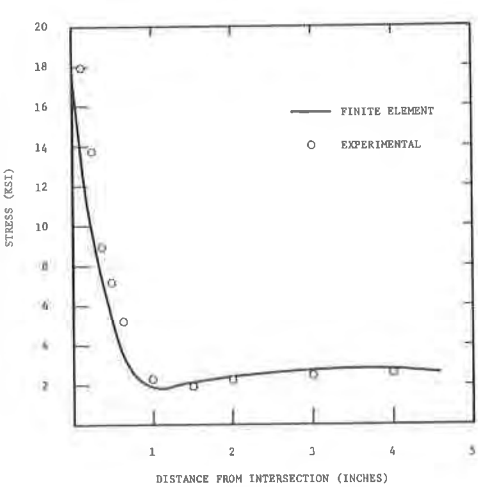

Figure 2. Hoop Stress in Main Cylinder - Outside Surface

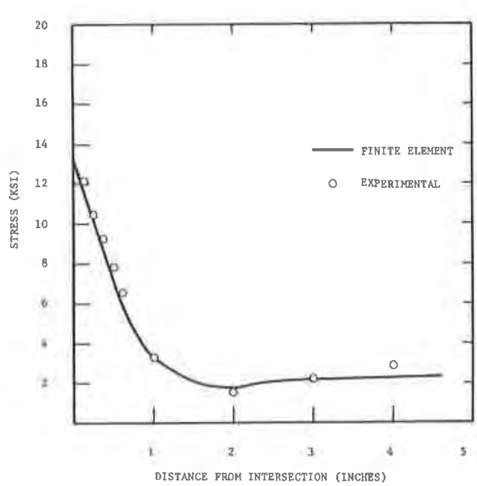

Figure 3. Hoop Stress in Main Cylinder - Inside Surface

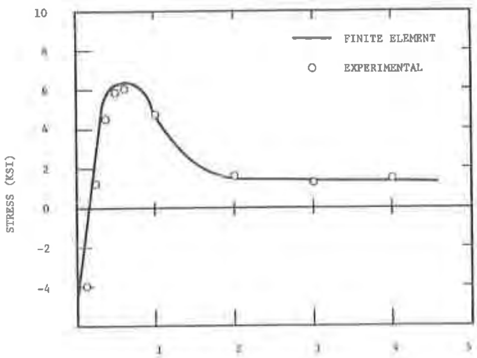

Figure 4. Axial Stress in Main Cylinder - Inside Surface

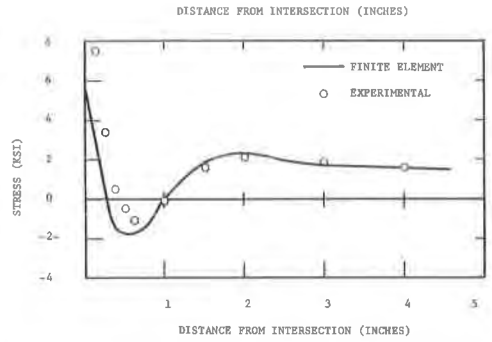

Figure 5. Axial Stress In Main Cylinder - Outside Surface

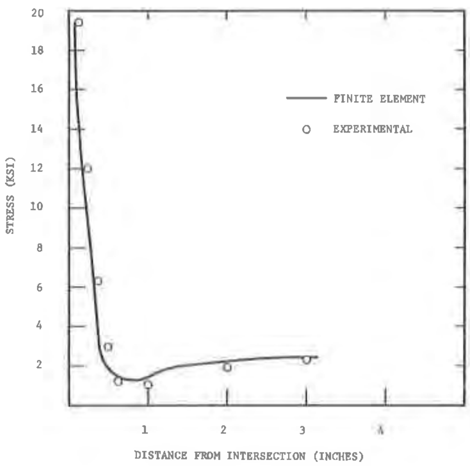

Figure 6. Hoop Stress In Nozzle - Outside Surface

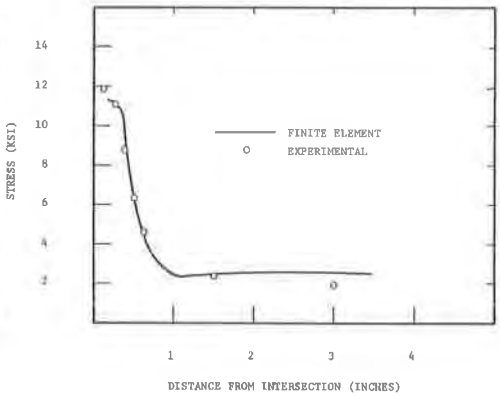

Figure 7. Hoop Stress in Nozzle - Inside Surface

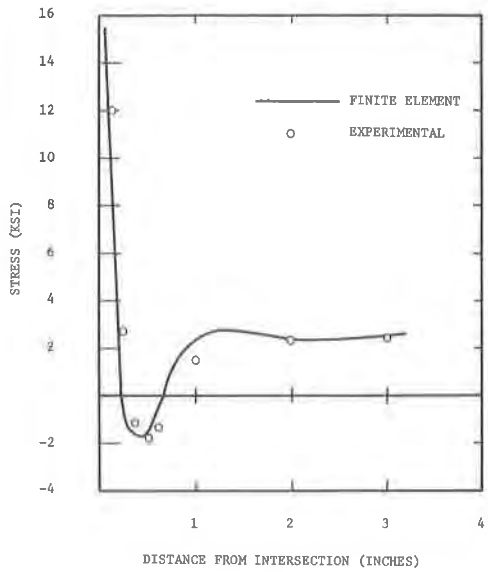

Figure 8. Axial Stress in Nozzle - Outside Surface

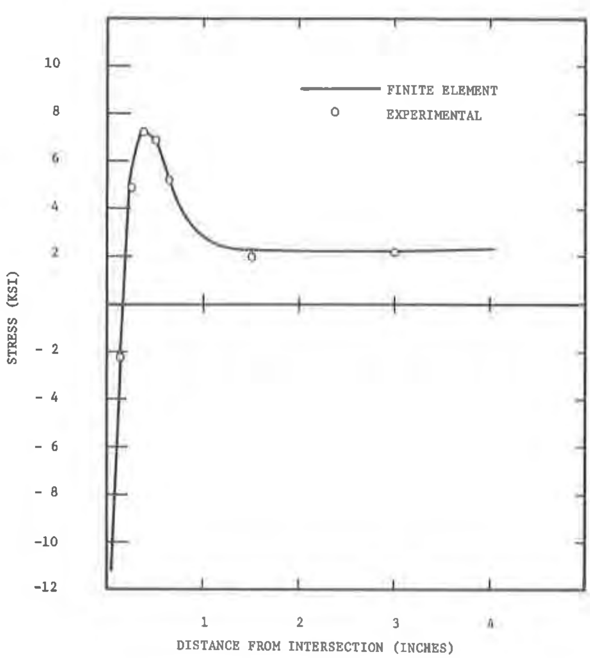

Figure 9. Axial Stress in Nozzle - Inside Surface