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Plastic Collapse and LBB Behavior of Statically Indeterminate Piping System under Static Load

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ABSTRACT

The plastic collapse and LBB behavior of statically indeterminate piping system were investigated in this study, compared with statically determinate piping system. Special attention was paid to evaluate the crack opening displacement after a crack penetrated wall thickness. The main results obtained were as follows: (1) The reduction of ultimate strength caused by a crack was relatively small in the statically indeterminate piping system. The main reason is considered that a sufficient redistribution of the bending moment occurs in this system. (2) A method to evaluate the crack opening displacement in pipe was proposed. From this method, it was known that the crack opening displacement could be evaluated by using the plastic rotation angle.

INTRODUCTION

The LBB(Leak Before Break) concept used in nuclear piping systems, high pressure vessels and LNG tanks has attracted much interest in the viewpoint of the structure integrity and the proper design of piping system. When the LBB concept is really applied to such energy-related plants, it requires not only the piping fracture analysis but also the leakage analysis in crack parts of piping system. Especially, the leakage analysis is directly related to the evaluation of COD(Crack Opening Displacement). Studies on the piping fracture and the evaluation of COD due to cracks in structure have been mainly performed on statically determinate systems[1]. As a result of that, many useful results were reflected on the standards to hold up design or inspection. However, it is essential to investigate statically indeterminate systems, considering most piping systems of energy-related plants consist of statically indeterminate ones[2].

This study describes the results of plastic collapse and LBB behavior in the statically indeterminate piping system. The entire approach was based on plastic design and LBB concept. The evaluation was mainly performed on the comparison between statically indeterminate and statically determinate piping systems with a wall thinning or a circumferential crack. Moreover, an approach to predict COD due to a crack in pipe was proposed in this study. It was known that COD in pipe with a through crack could be evaluated by using the plastic rotation angle from this approach.

THEORY

Evaluation of Plastic Collapse Load

The evaluation of plastic collapse load was based on plastic design method[3]. The selected case in the present study was the system fixed at one end and simply supported at the other. The corresponding plastic collapse model obtained from this case was illustrated in Fig.1. From Fig.1, the evaluation value of plastic collapse load(P_C) can be drawn from the following relation:

$$\frac{P_C}{2}(\upsilon_{HP}+\upsilon_{GP})=M_{AP}\phi_{AP}+M_{GP}\phi_{GP} \tag{1}$$

$$\begin{cases} \phi_{GP}=\phi_{AP}+\phi_{BP} \\ \upsilon_{HP}=a\cdot\phi_{AP} \\ \upsilon_{GP}=(L-b)\cdot\phi_{AP}=b\cdot\phi_{BP} \end{cases}$$

$$P_C=\frac{2}{a+(L-b)}\left(M_{AP}+\frac{L}{b}M_{GP}\right)=\frac{2}{b\{a+(L-b)\}}(b\cdot M_{AP}+L\cdot M_{GP}) \tag{2}$$

Where, M_P indicates plastic moment and ϕ_P or υ_P does plastic rotation angle or deflection respectively. The subscripts "A", "G" and "H" denote the location of pipe also.

Method for Predicting COD

These days, most laboratory evaluation of COD is inferred by considering the specimen halves are rigid and rotate about a hinge point. This method can be transferred to the evaluation of COD in pipe under bending moment by assuming the hinge point is consistent with neutral axis as shown in Fig.2. Referring to Fig.2, COD can be estimated by following relation:

$$COD,\delta=2R(1+\cos\beta)\cdot\sin\left(\frac{\phi_P}{2}\right) \tag{3}$$

Where, R denotes mean radius and β does the angle of neutral axis in pipe under bending load.

Net-Section Stress Approach in Pipe

The plastic collapse moment for a circumferential defected pipe is estimated by the net-section stress approach. The plastic collapse moment M_C for pipes with the defect of depth(d) and half angle(θ) such as Fig.3(b) is calculated by the net-section stress approach using the following equation[4]:

In the case of $\beta\le\pi-\theta$

$$M_C=2\sigma_f R^2 t(2\sin\beta-x\sin\theta) \tag{4}$$

$$\beta=\frac{1}{2}(\pi-x\theta)$$

In the case of $\beta > \pi - \theta$

$$M_C = 2\sigma_f R^2 t\{2(1-x)\sin\beta + x\sin\theta\} \qquad (5)$$

$$\beta = \pi + \frac{1}{1-x}\left(\frac{x\theta - \pi}{2}\right)$$

Where, σ_f denotes flow stress and x corresponds to the normalized initial notch depth(d/t).

MATERIAL AND TESTING PROCEDURE

The pipe used in this study was carbon steel of STS370, which had the following mechanical properties: yield stress 227MPa, tensile strength 406MPa and percentage of elongation 25.3%. The specimen geometry, the test conditions and so on was shown in Table 1. The initial notch of depth(d) and half angle(θ) in wall thinning or cracked pipe was located on the outer surface of the specimens using an electric discharge machine and saw. The pipe sections including an initial notch were illustrated in Fig.3. The test conditions were statically determinate and statically indeterminate systems as illustrated in Fig.4. Four-point bending test that has a load span of 150mm was adopted in this test. Actual test was carried out under displacement control of 2~5mm/min. The measurement of COD after crack penetration was done by using a clip gauge.

TEST RESULTS AND CONSIDERATIONS

Plastic Collapse and LBB Behavior

A load-displacement curve of wall thinning pipes obtained from the experiment is shown in Fig.5. It can be found that the analytical value from Eq.(1) and (2) is well compared with the experimental one. Entirely, the strength of statically indeterminate pipes was much higher than the one of statically determinate pipes and the reduction of the ultimate strength caused by defects was relatively small. The main reason for this behavior in statically indeterminate pipes is considered that a sufficient redistribution of the bending moment occurs under the condition.

Fig.6 shows a load-displacement curve of cracked pipes. The above statements may come into existence in Fig.6 also. Especially, it is possible to investigate LBB characteristic of statically indeterminate system from this figure. It has been reported that LBB behavior is easily occurred in the case that the initial crack depth is great and the initial crack angle is not extensive. In statically determinate systems of Fig.6, the pipe which had the normalized initial crack depth(d/t) of 0.85 showed LBB behavior. However, in the pipe that had d/t of 0.60, load decreases continuously after penetration. This case generally belongs to NO LBB behavior. In statically indeterminate systems, any pipe that had d/t of 0.85 or 0.60 showed LBB behavior in the present tests. From these results of Fig.6, it is considered that LBB behavior of statically indeterminate pipes is related to the redistribution of the bending moment founded on Eq.(1) and (2).

Evaluation of COD

Tada-Paris method to evaluate COA(Crack Opening Area) for pipe with a through crack is based on

LEFM(Linear Elastic Fracture Mechanics). When COA is obtained from this method, COD can be evaluated by assuming that a crack is ellipsoidal in shape. A comparison between the calculated COD from Tada-Paris method and the experimental one is illustrated in Fig.7. The result of Fig.7 indicates that Tada-Paris method may give a conservative value under the basis of jet force.

A correlation between the experimental COD and the plastic rotation angle is shown in Fig.8. At this, the plastic rotation angle(ϕ_p) was obtained from the cracked part. From this figure, it is found that the experimental COD could be arranged with the plastic rotation angle of the cracked part regardless of initial crack depth and test condition.

With this concept for evaluating COD, a model such as Fig.2 was devised by assuming the hinge point is consistent with neutral axis in the cracked pipe under bending moment. Fig.9 shows a comparison between the calculated COD from Eq.(3) and the experimental one. The result of the calculated COD shows an excellent agreement with the experimental one.

General Yielding in Pipe Section

The piping system should have a significant deformation capacity until the plastic collapse occurs, in the viewpoint of structure integrity, even if a pipe element contains the defect such as a local wall thinning and a crack. To that purpose, the acceptable defect size considering the deformation of a pipe was estimated in this study by comparing the plastic moment at the defective part and the general yielding moment at the gross section.

General yielding moment(M_Y) at the gross section of pipe is evaluated by the following equation:

$$M_Y = 4\sigma_Y R^2 t \qquad (6)$$

Where, σ_Y denotes yield stress of a material.

From Eq.(4), (5) and (6), the limit value of a defect size to allow general yielding to occur at the gross section can be predicted. The analytical result for the present pipes was plotted in Fig.10. From this figure, it is found that general yielding does not occur in pipes of which the defect depth is great and the angle is extensive. Moreover, it is considered that if the defect depth and the angle are relatively small the pipe undergoes general yielding and large deformation at the gross section.

CONCLUSION

This study describes the results of plastic collapse and LBB behavior for pipes with a wall thinning or a circumferential crack through the comparison between statically indeterminate and statically determinate systems. The main results obtained in this study were as follows:

(1) LBB behavior is easily apt to occur in statically indeterminate systems. The main reason for this behavior is considered that a sufficient redistribution of the bending moment occurs under statically indeterminate systems.

(2) A method to predict COD was proposed in this study. From this method, it was known that COD resulted from a through crack could be evaluated by using the plastic rotation angle.

(3) The acceptable defect size considering the deformation of a pipe was estimated by comparing the plastic moment at the defective part and the general yielding moment at the gross section.

REFERENCES

1. Matsumoto, K., Nakamura, S. and Gotoh, N. et al, "Study on Crack Opening Area and Coolant Leak Rates on Pipe Cracks," Int. J. Pres. Ves. & Piping. 46, 1991, pp. 35-50.
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4. Hasegawa, K., Sakata, S., Simizu, T. and Shida, S., "Prediction of Fracture Tolerances for Stainless Steel Pipes with Circumferential Cracks," ASME Int. J. Press. Ves. and Piping. 95, 1983, pp. 65-78.

Table 1. Specimen geometry and test conditions

No.	Mean Radius, R (mm)	Wall Thickness, t (mm)	Depth of Initial Defect, d (mm)	Angle of Initial Defect, 2θ (deg.)	d / t	S.D. or S.I†
Wall Thinning Pipe						
PD-1	22.9	5.2	4.3	67.5	0.83	S.D.
PD-2	22.9	5.2	3.8	63.4	0.73	S.D.
PD-3	22.9	5.2	3.2	58.0	0.62	S.D.
PI-1	22.9	5.2	4.3	67.5	0.83	S.I.
PI-2	22.9	5.2	3.8	63.4	0.73	S.I.
PI-3	22.9	5.2	3.2	58.0	0.62	S.I.
Cracked Pipe						
SD-1	22.9	5.2	4.42	90.0	0.85	S.D.
SD-2	22.9	5.2	3.12	90.0	0.60	S.D.
SI-1	22.9	5.2	4.42	90.0	0.85	S.I.
SI-2	22.9	5.2	3.12	90.0	0.60	S.I.

† S.D. : Statically Determinate
S.I. : Statically Indeterminate

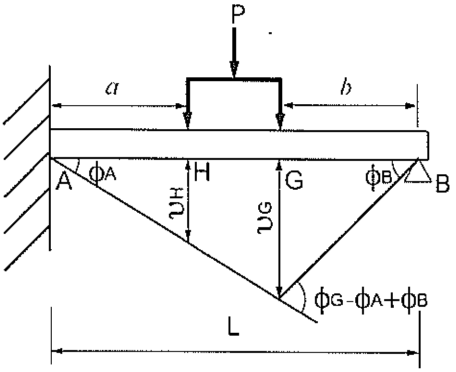


Fig.1 Plastic collapse model of the system fixed at one end and simply supported at the other

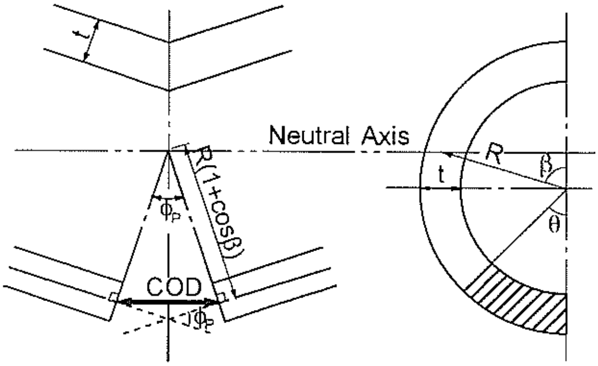
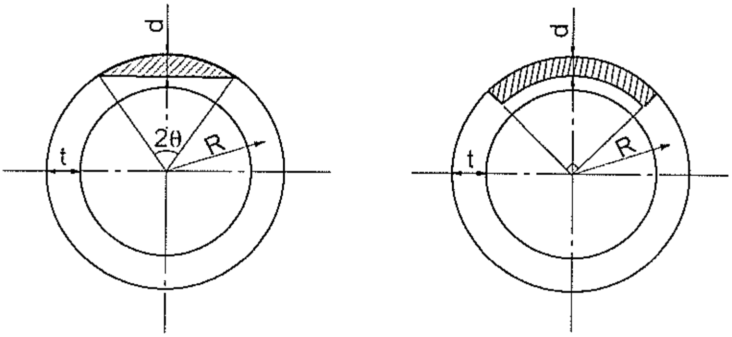


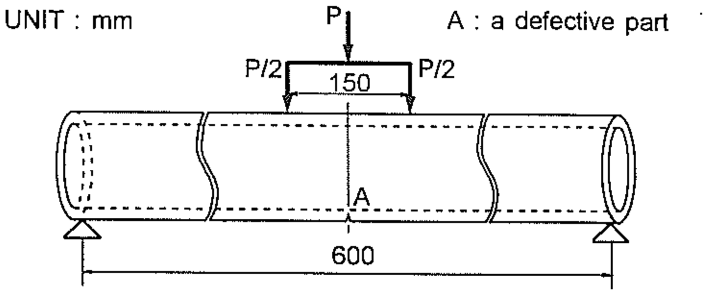
Fig.2 A model to evaluate COD in pipe under bending moment



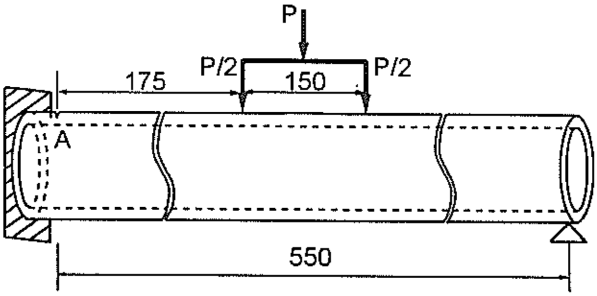
(a) Wall thinning pipe

(b) Cracked pipe

Fig.3 Pipe geometry in sections including an initial defect



(a) Statically determinate piping system



(b) Statically indeterminate piping system

Fig.4 Four-point bending test

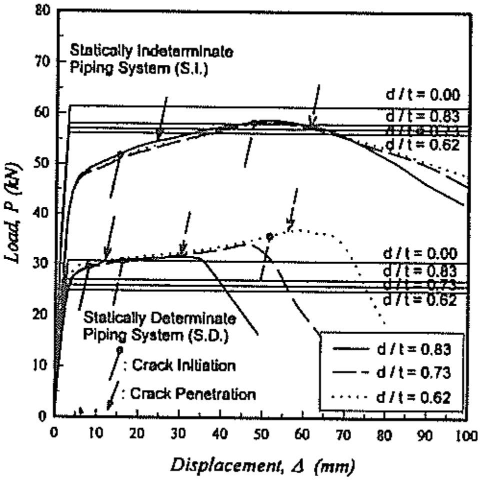


Fig.5 Load-displacement curve of wall thinning pipes

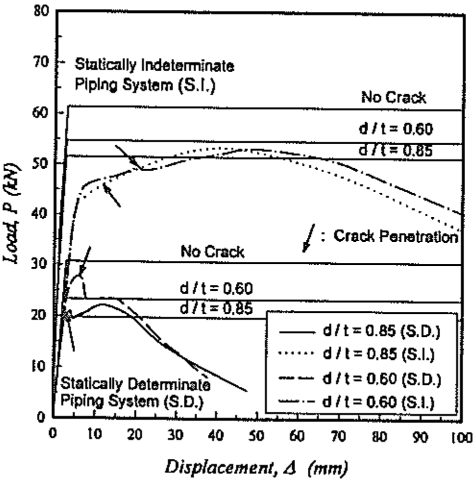


Fig.6 Load-displacement curve of cracked pipes

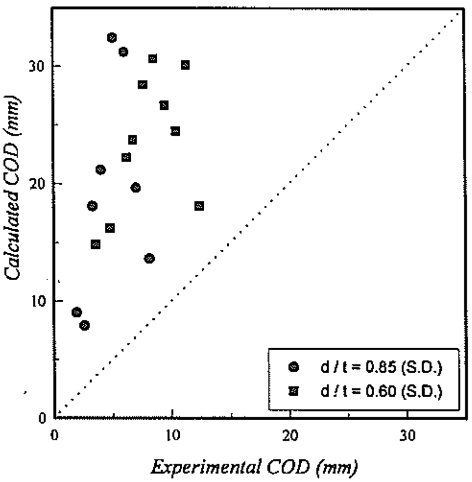


Fig.7 Comparison between the calculated COD and the experimental one (Tada-Paris method)

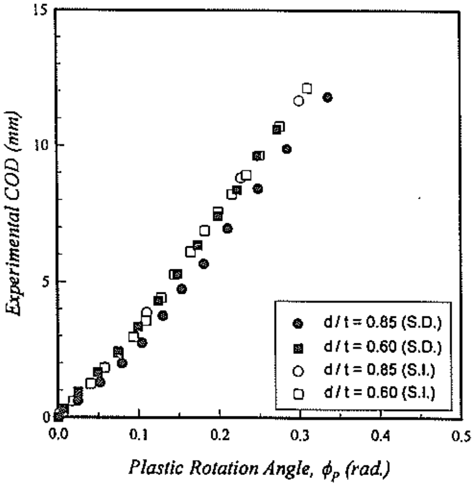


Fig.8 Correlation between the experimental COD and the plastic rotation angle

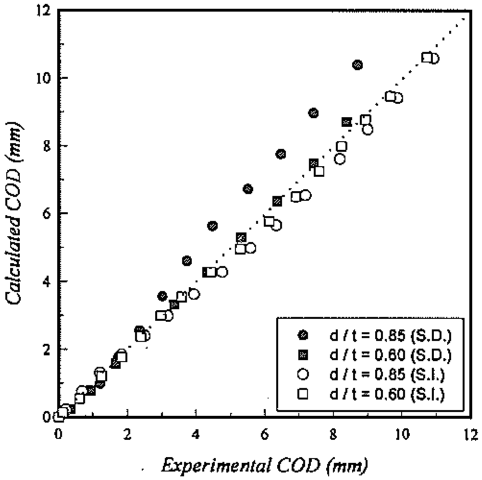


Fig.9 Comparison between the calculated COD and the experimental one (Evaluation method of Fig.2)

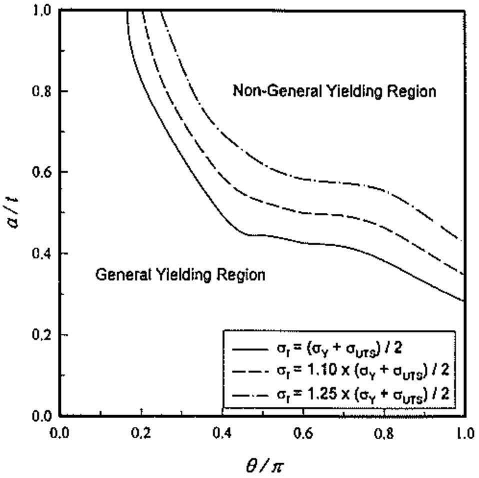


Fig.10 Limit value of a defect size to allow general yielding to occur at the gross section of a pipe