Numerical Simulation of Stick-Slip Motion

Felipe Gustavo de S. PEÑALOZA[1], José Luís V. de BRITO[1] and Graciela N. Doz[1]

1) Department of Civil and Environmental Engineering – University of Brasília, Brazil

ABSTRACT

Earthquakes are sudden movements of the earth's surface that have a short duration and release great amount of energy. Such movements represent a threat to human beings, not only due to the number of victims registered after every event, but also because of the great economical and social devastation that falls upon the affected area. The nature of the physical phenomena related to earthquakes are complex and still not well understood. The most important aspects of seismic excitation may be represented by a stick-slip motion, considered as the most likely source mechanism for shallow earthquakes. On this work, we used a numerical model which has proved able to initially simulate the stick-slip motion of a homogeneous block resting on a rigid surface. Subsequently, the non-homogeneity of the medium on that model was taken into consideration. The results obtained so far are satisfactory and encourage us to do further simulations on larger scale. Doing that we could, for instance, obtain data in order to generate attenuation curves of acceleration as a function of epicentral distance, and also try to reproduce artificial accelerograms.

INTRODUCTION

Earthquakes are among the most destructive natural phenomena affecting human beings, not only due to the number of lives lost, but also because of the huge social and economic losses brought to the affected area. Despite the increasing development of seismology since the beginning of the twentieth century, the physics of tectonic movements still remains not fully understood. Multidisciplinary studies need to be carried out in order to give us a better comprehension of earthquakes.

Brace & Byerlee [1], showed that previously fractured rock surfaces might present unstable movements when under shear stress. This kind of movement was named stick-slip motion, and is nowadays considered as the most likely source mechanism of shallow earthquakes. Burridge & Knopoff [2] developed the first numerical model to simulate the stick-slip motion. Such model consists in making an unidimensional representation of the fault with a discrete mass-spring system where the contact surface of mass elements obeys a specific friction law. Brune *et al.* [3] used foam rubber blocks in order to show that the stick-slip motion occurs at the same time as important normal vibrations on the fault plane, which could play a crucial role in the source mechanism of earthquakes. Mora & Place [4] used a numerical solid lattice model in order to simulate the colapse process on a rocky material, as well as the effects of roughness on the contact surfaces. The stick-slip motion observed by them occurred spontaneously, without the necessity of imposing a friction law to the interface. Furthermore, the results obtained for particle trajectories were consistent with the results reached by Brune *et al.* [5].

Aki [6] carried out a discussion towards asperity and barrier models. In this model, we assume the existence of regions with different strengths along the fault surface (asperities and barriers). Higher spot of resistance would be responsible for the detention of the rupture process. Rimal [7] corroborates Aki's idea when he states that, the physics of earthquakes is essentially the physics of the collapse of rocks, and that the heterogeneity of the medium is the reason for the deterrence of the rupture process. Another way to model stick-slip motion is through laws according to which we consider velocity-weakening or displacement-weakening, both of which might include a state variable for the contact surface. For more details see Dieterich [8] and Ruina [9].

Doz [10] did an extensive review on the rupture phenomena in solids that led to the conclusion that fracture propagation due to shear stress in crystalline rocks is not feasible. Therefore, earthquakes might occur due to the sliding of adjacent sides of a pre-existing fault, whereas microcracks may give rise to unstable fracture propagation due to tensile stress. Under that assumption and using the discrete elements method, Doz [10] simulated the seismic phenomenon with the stick-slip motions of a homogeneous block resting on a rigid surface. The model was able to represent important aspects of the seismic phenomenon, even if considering a simple constitutive law based on the Coulomb's friction.

METHOD DESCRIPTION

The formulation employed herein to simulate the stick-slip motions is based on the study carried out by Doz [10]. The model consists in the utilization of an array of uniaxial elements in order to represent the continuous medium, called Discrete Element Methods (DEM). The equivalent relations between properties of continuous media and cubic bar arrangements, were originally developed by Nayfeh & Hefzy [11]. Fig. 1 shows details of the cubic module's geometry and an example of the utilized arrangements of such module.

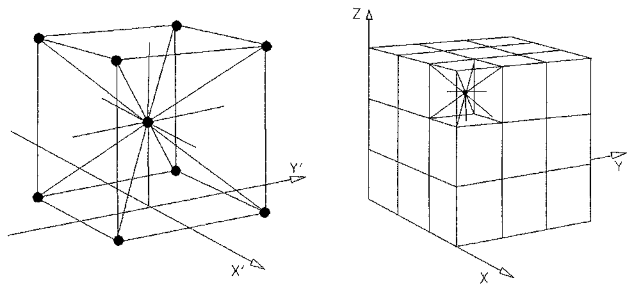

Fig. 1: (a) cubic module. (b) generation of a prismatic body using cubic module.

It is possible to distinguish two types of bars: diagonal, whose length is $L_c\sqrt{3}/2$ and normal bars, with length L_c . Assuming the hipotesis of linear elastic and isotropic material, and knowing Young's module (E) as well as Poisson's rate (ν), it is possible to obtain the equivalent stiffness of diagonal and normal bars according to Eq. (1) and Eq. (2), respectively:

$$E_d = \frac{2\,\delta}{\sqrt{3}}\,\alpha\,E\,L_c^{\,2} \tag{1}$$

$$E_n = \alpha\,E\,L_c^{\,2} \tag{2}$$

where α and δ bellow are parameters of cubic truss model:

$$\alpha = \frac{(9+8\delta)}{(18+24\delta)} \quad \text{and} \quad \delta = \frac{9\,\nu}{(4-8\nu)} \tag{3}$$

On the cubic arrangement, masses are lumped on the nodes. The position occupied on every instant by each node is determined according to the movement law expressed by Eq. (4). The utilization of an explicit scheme of integration, based on central finite differences, enables us to simulate the medium's behavior from continuous to discontinuous states as a consequence of the rupture process.

$$m\frac{d^2 u_i}{d\,t^2} + c\frac{d\,u_i}{d\,t} = f_i \tag{4}$$

On the Eq. (4) m means the nodal mass, c is the damping constant, f_i are components of resultant forces at the i node and u_i are components of nodal vector coordinates. Strictly, any other integration method might be used, nevertheless, the central finite difference method was used due to the fact that, among other advantages, it presents the highest critical step of integration, expressed on Eq. (5):

$$\Delta t \leq 0.6\frac{L_c}{C_o} \tag{5}$$

where C_o represents the velocity of P-wave propagation and L_c is the bar's critical length.

CONSIDERATION OF THE NON-HOMOGENEITY OF THE MEDIUM

The consideration of the non-homogeneity of the medium is a significant feature since it enables the adjustment of the model to a more realistic physical situation, represented, for instance, by the heterogeneity of bedrock as well as of fault planes. Its importance becomes more evident when it is necessary to know, for example, the acceleration spectrum on a specific interest point enclosed on the bedrock. The consideration of the non-homogeneity of the medium also allows us to take into account the micro cracking which arises from the interior of the bedrock, whose distribution and orientation depend on material properties, generally assigned random variables. Rimal [7] asserts that medium heterogeneity is responsible for the retention of rupture front, without which a failure could be diffused endlessly. Therefore, the occurrence of new crackings might be associated with the non-homogeneity of the material and with the induced vibrations during fault slipping. Thus, differently from Doz [10], who considered the medium homogeneous, we will assume on this paper that the medium has random properties, in particular, referring to then fact that Young's module may vary. This allows for the representation of less-resistant regions on the interior of the bedrock which would be subject to collapse due to induced vibrations. It also allows us to obtain different spectra of accelerations from point to point.

The formulation used in order to deal with non-homogeneity was developed by Rocha [12] who has studied the rupture and size scale phenomena on non-homogeneous materials. Such formulation consists in carefully choosing which material property must be dealt as a random variable, and attributing to it Weibull's distribution law with two parameters – according to Eq. (6). This sort of distribution is usually employed in order to model the reliability of systems and their components.

$$F(x) = 1 - e^{-(x/\beta)^{\gamma}} \tag{6}$$

where x represents the random variable, β and γ are, respectively, the scale and shape parameters of the function. Mean value of x and its variance are represented by Eq.(7) and Eq. (8).

$$\overline{x} = \beta \left[\Gamma\left(1+\frac{1}{\gamma}\right) \right] \tag{7}$$

$$\sigma^2 = \beta^2 \left[\Gamma\left(1+\frac{2}{\gamma}\right) - \Gamma^2\left(1+\frac{1}{\gamma}\right) \right] \tag{8}$$

where $\Gamma(x)$ is the Gamma function.

Since the distribution function is defined, equivalent stiffness values x are randomly generated making use of Eq. (9), in which U means a pseudo aleatory number with uniform density probability between 0 and 1.

$$\overline{x} = \beta \left[-\ln(1-U) \right]^{\frac{1}{\gamma}} \tag{9}$$

Rocha [12] highlighted the convenience of representing random variable by means of a function of its mean value times one parameter of randomness Φ, according Eq. (10).

$$x = \Phi\,\overline{x} \tag{10}$$

where:

$$\Phi = \frac{\left[-\ln(1-U)\right]^{\frac{1}{\gamma}}}{\Gamma\left(1+\frac{1}{\gamma}\right)} \tag{11}$$

On Eq. (11) Φ represents a random number with Weibull's distribution and average 1. The form of parameter γ defines the variation coefficient.

NUMERICAL SIMULATION

The numerical simulation refers to a prismatic block based on a rigid surface and subjected, primarily, to a load resulting from the action of its own weight. This stage aims to representing the compression forces applied to the bedrock. On a second stage, a gradual velocity gradient is imposed to the block, so that a state of shear stress is produced, leading to unstable block movement. The imposition of velocity is done to a lateral block node, as indicated on Fig. 2 and it obeys the exponential law shown on Eq. (12).

$$\dot{u}(t) = \dot{u}_f \left[1 - e^{-\left(\frac{t}{t_0}\right)} \right] \tag{12}$$

where: $\dot{u}(t)$ is the instantaneous velocity, $\dot{u}_f$ is the final displacement velocity and t_0 represents the moment when velocity reaches 63% of its maximum assintotic value.

Each bar in the truss-like model obeys an elementary constitutive relation. Fig 2 shows the discretization of block into 50 cubic module, resulting in 182 nodes and 781 bars. Control nodes were used to obtain acceleration responses at the desired point. Such nodes are indicated by numbers on Fig 2. The material features are: modulus of elasticity: $E=2.0 \times 10^{10}$ N/m^2, specific mass: $\rho = 2.4 \times 10^{3}$ Kg/m^3, Poisson's ratio: $\nu = 0.2$ and friction coefficient $\mu = 5.0$.

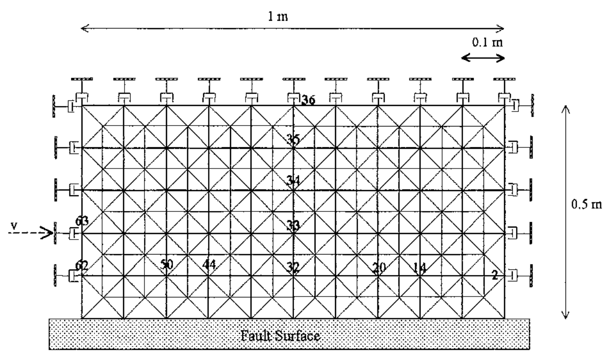

Fig. 2 Discretization of prismatic block

For reduced shear stress level, we verify a linear shear tension behavior in relation to time. Whenever shear stress along any fault point exceeds the local sliding strength ($\tau = \mu \times \sigma$) there is a sliding of the point that eventually might be propagated thought the surrounding areas causing a sudden stress drop, which occurs at the same time as important displacements and induced vibrations.

Fig 3 to Fig 7 respectively represent the obtained responses to stress drop, normal stress, mean displacement of contact surface and normal acceleration at node 36. Young's module was considered as a random variable, and its coefficient of variation was set at 5 %. For each simulation, it is necessary to set an initial seed, whose purpose is to generate a random number that will be used on the definition of the equivalent stiffness values of bars.

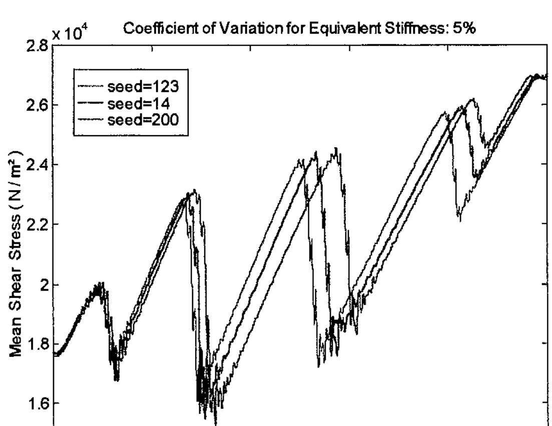

Fig. 3 Shear stress time history

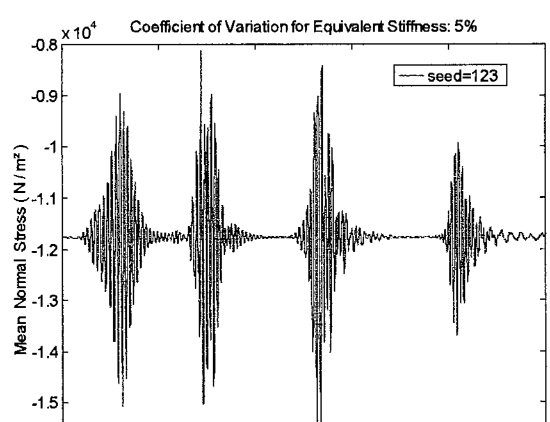

Fig. 4 Normal stress time history

Fig. 3 shows the typical behavior of shear stress during the stick-slip motion. One feature of such kind of motion is the occurrence of a cycle of increase in shear stress, followed by a sudden stress drop. Such stress drop represents the main shock in an earthquake. It is possible to verify that the main event is preceded by small vibrations, which could be associated with foreshock events. In the same way, the vibrations that occur right after the main stress drop may be associated with aftershocks. Different seeds have been used in order to generate random property for bars, producing different shear stress time-histories. All of them, however, showed in common the occurrence of four main shocks (S1, S2, S3 e S4). The analysis of these events indicates that they are all, indeed, formed by shorter stress drops which occur delayed for a very reduced time, suggesting a segmentation of the rupture process along the fault. Through the consideration of non-homogeneity, it is possible to define a range of likely response values. Although we used only three seeds on this example, a more accurate analysis would demand a greater number of seeds and consequently more simulations.

Another feature of stick-slip motion is the normal oscillation which occurs at the same time as the main stress drop. Fig. 4, corresponding to the simulation in which the seed is 123, shows that normal stress oscillation may reach 30% from the block's own weight. This indicates that the normal stress response is quite sensitive in relation to stress drop. Brune *et al.* [5] have suggested that fault slip occurs when contact surfaces are unlocked. If that is correct, normal vibrations could play a significant role in this kind of motion and be one of the most important factors controlling seismic source. Fig. 5 shows that the main events are also associated to an important tangential displacement of block.

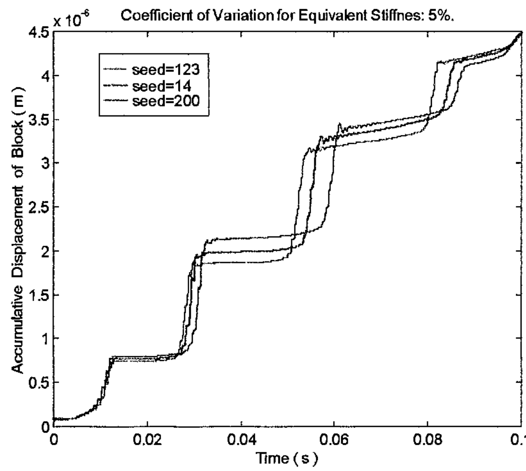

Fig. 5 Accumulative displacement of block

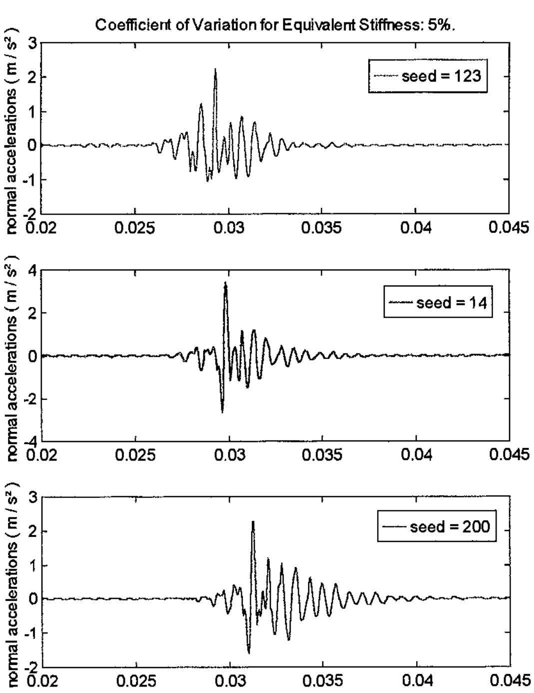

Fig. 6 Acceleration time history for node 36 (S2 event)

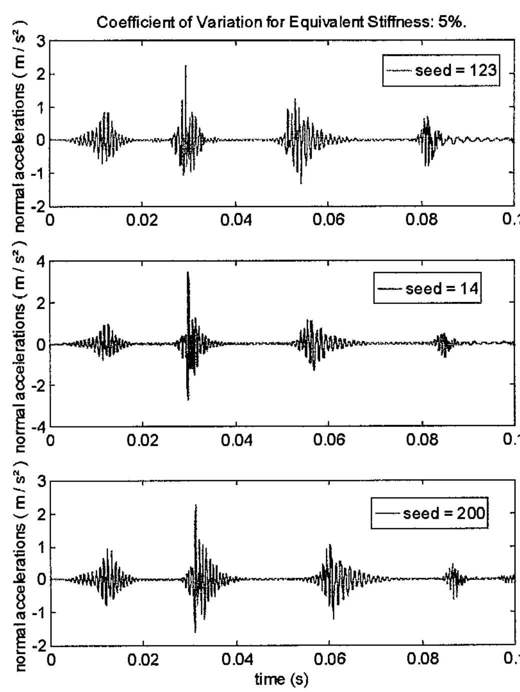

Fig. 7 Acceleration time history for node 36 (S1 to S4 events)

Fig. 6 shows induced accelerations on node 36 during event S2. The resultant effect of the consideration of non-homogeneity becomes clear when we analyze the local responses from node to node. It is possible to verify trough Fig. 6 that the use of different seeds has produced accelerograms equally distinct for the attenuation form, and for the maximum peaks of acceleration. This is a specially useful feature when it is desired to generate artificial accelerograms in a point of specific interest. On Fig. 7, it is fully observed the acceleration time history of node 36 obtained with the use of the different seeds.

SIMILARITY ANALYSIS

Dimensional analysis and similarity theory can help on the validation of the model employed to simulate stick-slip motions by means of comparing the responses obtained with data from a real earthquake. In fact, we should be cautious when doing any overstatement towards the model, since the responses obtained up to this moment are valid only in the case of a non-homogeneous prismatic block that has a constitutive relation of interface based on Coulomb's friction law. Besides the aspects related to seismic phenomena, many of the complexities concerning the accurate study of friction laws were not considered in this model. One of the main advantage of this model is the capacity of describing the stick-slip motion in a simple and satisfactory way.

Assuming that the properties of the material employed on simulations are identical to the ones found in the bedrock, we can state that the rupture velocity and wave propagation will be the same either on the model or on the prototype. In case of geometrical similarity, only one non-dimensional parameter λ_1, defined as the ratio between the fault lengths of the prototype and of the model, would be used on the solution of the problem. Time scale on prototype is expressed by Eq. (13).

$$t_p = \lambda_1 \, t_m \tag{13}$$

where t_p represents time on the prototype, t_m represents time on the model and λ_1 is the non-dimensional parameter.

Assuming a prototype whose fault is 10 km long, and taking the stress drop for the S2 event, previously shown on Fig. 3 (seed 200), we can determine that: a) Mean stress drop - $\Delta\sigma$ = 7600 N/m²; b) Event duration S2 - t_m = 0.0021 s.; c) Recurrence period between S1 and S2 events - $(\Delta t \, between 2 shocks)_m$ = 0.0258 s. The non-dimensional parameter λ_1 will be equal to 10^4, and the duration of the event S2 on prototype will be t_p = 21 s.

On the other hand, the recurrence interval between consecutive seismic events must consider the ratio between the velocity applied to the model $\dot{u}_m$ and the strain gradient on the prototype $\dot{u}_p$, according to Eq. (14).

$$(\Delta t \, between \, 2 \, shocks)_p = (\Delta t \, between \, 2 \, shocks)_m \quad \lambda_1 \quad (\dot{u}_m \, / \, \dot{u}_p) \tag{14}$$

in which: $(\Delta t \, between \, 2 \, shocks)_p$ represents recurrence interval between seismic events on prototype, $(\Delta t \, between \, 2 shocks)_m$ represents the recurrence interval between seismic events on model, $\dot{u}_m$ is the final velocity applied to the model and $\dot{u}_p$ is the strain ratio of the prototype. A close value for $\dot{u}_p$ is 1 cm/year, or 3.17 x 10^{-10} m/s. Substituting such value on Eq. (14) we obtain: $(\Delta t \, between 2 \, shocks)_p \approx 2.83 \, years$.

It is also possible to determine the magnitude of the S2 event on the prototype but, initially, we need to compute the seismic moment as follows:

$$M_{0 \, prototype} = \Delta\sigma \, L \, A_r \, \lambda_1 \quad \text{(dyne x cm)} \tag{15}$$

where: $\Delta\sigma$ is the mean stress drop, L is the fault's length and A_r represent the rupture area.

Substituting the values we have $M_0 = 7.6 \times 10^{25}$ (dyne x cm). The magnitude can be obtained from Eq. (16), suggested by Riera *et al.* [13]:

$$M = 0.6536 (Log M_0 - 15.51 - 0.483 \, X) \tag{16}$$

where: X = 0 and X = 1 for, respectively, interplate and intraplate earthquakes. Substituting the values on Eq.(16) we find the magnitude of the S2 event on the prototype (interplate earthquake):

$$M = 6.77$$

CONCLUSIONS

It is possible to verify that the consideration of the non-homogeneity of the medium in the model that Doz [10] previously studied, works satisfactorily. The choice of Young's Module as a random property has permitted the obtainment of different records of acceleration for the same point, simply by changing the seed employed for generating random numbers. The model may become potentially useful for the analysis of seismic effects on a specific area. We can alternatively represent the non-homogeneity of the contact surface by setting the coefficient of friction as a random variable.

The stress drop results, as well as the mean displacement of the block obtained after the implementation of the non-homogeneity the of medium, suggest that main shocks are indeed formed by smaller events that occur on a quite reduced time interval.

The similarity theory indicates that the truss-like model used here has the capacity of describing important seismic aspects through the use of parameters such as stress drop and rupture area.

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