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LIFE PREDICTION OF WELDED TUBES UNDERGOING INTERNAL PRESSURE

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Introduction

In power and nuclear engineering problem of creep of welded tubes with internal hot liquid medium is often undertaken. The rupture properties of welded joints under creep conditions, particularly during strong concentration of stresses, determine the critical life time of the constructions [1,2,8,9]. The influence of rheological effects on the life time of welded joints should be analyzed in the process of design of elements subjected to high temperatures. For this purpose the equations of creep-damage relations are introduced. The creep rupture properties of weldments depend on the stress distribution as a function of time and the damage accumulation at critical locations. The state of stress at these locations result from the external loading, the geometry of the weld, and the dissimilar material properties of the base metal, heat affected zone and weld metal. The complex stress fields in weldments under a given loading can be described by the thermo-elastic-plastic finite element model. The practical problem of creep in welded zones is solved basing on the mathematical model.

Equation of nonlinear thermomechanics

In this paper the more classical approach of separating the non-elastic strains into two parts is used, one time dependent (creep) and the other time independent (plasticity) [7],[10]. Making this assumption, in compliance with small-strain theory, we get

(1) $$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{p} + d\varepsilon_{ij}^{c} + d\varepsilon_{ij}^{T}$$

where: $d\varepsilon_{ij}, d\varepsilon_{ij}^{e}, d\varepsilon_{ij}^{p}, d\varepsilon_{ij}^{c}$ and $d\varepsilon_{ij}^{T}$ are the changes in total, elastic, plastic, creep and thermal strain tensors, respectively.In the isothermal plasticity theory Hill's yield criterion for orthotropic materials, which reduces to the von Mises yield criterion for isotropic materials, is used to predict initial yield and the subsequent loading surface

(2) $$f(\sigma_{ij} - \alpha_{ij}) - K(\bar{\varepsilon}^{p}, T) = 0,$$

where α_{ij} represents the translation of the loading surface in stress space $\sigma_{ij}, \bar{\varepsilon}^p$ corresponds to the effective plastic strain:

$$\bar{\varepsilon}^p = \sqrt{\frac{2}{3}\varepsilon_{ij}^p \varepsilon_{ij}^p}, \tag{3}$$

and T corresponds to tempearture; the Prandtl-Reuss asociated flow rule:

$$d\varepsilon_{ij}^p = \lambda \frac{\partial f}{\partial \sigma_{ij}} \tag{4}$$

in which the change in the plastic strain tensor is normal to the loading surface; and a hardening law based on the Prager-Ziegler kinematic hardening theory:

$$d\alpha_{ij} = \mu(\sigma_{ij} - \alpha_{ij})$$

$$d\alpha_{ij} \frac{\partial f}{\partial \alpha_{ij}} = c(\bar{\varepsilon}^p, T) d\varepsilon_{ij}^p \frac{\partial f}{\partial \sigma_{ij}} \tag{5}$$

Differentiating equation (2) and assuming that :

$$dK = g(\beta_p)\left(\frac{2}{3} d\varepsilon_{ij}^p d\varepsilon_{ij}^p\right)^{\frac{1}{2}} \tag{6}$$

where β_p is the accumulated plastic strain, results in:

$$\frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \alpha_{ij}} d\alpha_{ij} - g d\bar{\varepsilon}^p = 0 \tag{7}$$

Material creep law under uni- and multi axial stresses

The constitutive laws for the material are functional representations of the relation of strain as a function of time for various stresses. The stress analysis of solids involving creep requires a similar constitutive law to describe the material behavior under loads. Such a constitutive law is often called the "creep law". A general form of function describing a typical creep strain curve for a solid subjected to constant uniaxial load can be expressed as follows:

$$\varepsilon^c = f(\sigma, t, T) = f_1(\sigma) f_2(t) f_3(T) \tag{10}$$

where σ,t and T are: the applied stress, time and temperature, respectively.

There are, of course, various forms of the functionals f_1 (s), f_2(t), and f_3 (T) proposed by researchers.

For most engineering application Norton's law is used:

$$f_1(\sigma) = K\sigma^n \tag{11}$$

where K, n are material constants.

A common form of f_3 (T) is:

$$f_3(T) = A\exp(-Q/RT) \tag{12}$$

where Q = activation energy, Q = 21660 cal/mol,
R = Boltzman's constant, R = 1,986 cal/mol
T = absolute temperature.
Referring to (10), (11) and (12), the typical form of creep law becomes:

(13) $$\varepsilon^c = K_c \sigma^n t \exp(-Q / RT)$$

where K_c and n are material constants.
Constitutive laws which correlate the stresses and the corresponding strains in the material as the functions of time and temperature can be derived from the results of various types of uniaxial creep tests. In reality, however, engineers are likely to deal with structures subjected to multi axial stress states. It is logical to expect that relevant constitutive laws can be derived from multidimensional loadings. A common practice is therefore simply to express these constitutive laws in terms of effective stress and strain for multidimensional stress cases. The general form of Norton's law in (12) and (13) for multidimensional analysis can thus be expressed in the form :

(14) $$\bar{\varepsilon}^c = K_c \bar{\sigma}^n t \exp(-Q / RT)$$

or

(15) $$\dot{\bar{\varepsilon}}^c = K_c \bar{\sigma}^n \exp(-Q / RT)$$

where $\bar{\varepsilon}^c$ is the effective creep strain and $\bar{\sigma}$ is the effective creep stress.

Flow rule

Similar to the Prandtl-Reuss relation, incremental creep strain can be expressed in terms of a creep potential function:

(16) $$\dot{\varepsilon}_{ij}^c = \dot{\beta} \frac{\partial \phi(\sigma_{ij})}{\partial \sigma_{ij}}$$

where $\dot{\beta}$ is a positive parameter depending on the loading history and f(σ_{ij}) is the creep potential function similar to the plastic potential function. If the material is assumed to be homogeneous, initially isotropic, incompressible and obeys von Mises yield criterion, then

(17) $$\sigma_{ij}^{'} = \frac{\partial \phi(\sigma_{ij})}{\partial \sigma_{ij}} = \frac{\partial J_2}{\partial \sigma_{ij}}$$

deviatoric stress invariant. Since the effective creep strain can be expressed to bewhere $\sigma_{ij}^{'}$ is the deviatoric stress components, J_2 is the second :

(18) $$\dot{\bar{\varepsilon}}^c = \left(\frac{2}{3} \dot{\varepsilon}_{ij}^c \dot{\varepsilon}_{ij}^c \right)^{\frac{1}{2}}$$

and by combining (16) and (18), the incremental creep strain rate and strain can be computed from the expressions:

$$\dot{\beta} = \frac{3}{2}\frac{\dot{\bar{\varepsilon}}}{\bar{\sigma}}$$

and

(19) $$\dot{\varepsilon}_{ij}^{c} = \frac{3}{2}\frac{\dot{\bar{\varepsilon}}^{c}}{\bar{\sigma}}\sigma_{ij}^{'}$$

or

(20) $$d\varepsilon_{ij}^{c} = \frac{3}{2}\left(d\dot{\bar{\varepsilon}}^{c} / \bar{\sigma}\right)\sigma_{ij}^{'}$$

in which the numerical values of $\dot{\bar{\varepsilon}}$ are determined from the experimentally derived creep law as shown in (11).

Finite element formulation of thermoelastic-plastic creep stress analysis

The constitutive law may be applied to the conservation of momentum via an appropriate variational principle. First consider the virtual work expression within a total Lagrangian description:

(21) $$\int_{V_o} \sigma_{ij}^{t+\Delta t}\delta \in_{ij}^{t+\Delta t} dV = \int_{A} T_K \delta u_K dA + \int_{V_o} \rho_o F_K \delta u_K dV$$

where $\sigma_{ij}^{t+\Delta t}$ is the second Piola-Kirhoff stress tensor at the time t+Δt referred to initial configuration at time t=0, $\delta \in_{ij}^{t+\Delta t}$ is the variation in the Green-Lagrange strains at t+Δt referred to initial configuration V_o, T_k are the surface tractions at time t+Δt referred to the surface of the configuration A, δu_k is the variation in the displacements, r_o is the local density in the initial configuration and F_k is the body force per unit mass at time t+Δt, referred to the initial configuration V_o . The resulting equations of motion are:

(22) $$\left([K_L]^t + [K_{NL}]^t\right)\{\Delta u\} = \{R\}^{t+\Delta t} - \{F\}^t$$

where

(23) $$[K_L] = \int_{Vo} [B_L]^T [D][B_L] dV$$

(24) $$[K_{NL}] = \int_{Vo} [B_{NL}]\left([\sigma] - [D][\Delta \in^c] - [D][\Delta \in^T]\right)[B_{NL}] dV$$

$[B_L]$ and $[B_{NL}]$ are the linear strain displacement transformation matrix and the nonlinear strain displacement matrix, respectively.

Integration schemes and solution algorithm

The numerical algorithm assumed for the analyzed problem requires the choice of proper integration scheme, that guaranties to achieve both convergence and stability of the solution. From all integration schemes available for this purpose we adopt the Taylor scheme, because of its easy implementation for the analyzed problem and good computational efficiency. The Taylor series scheme can be outlined as follows:

(25) $$d\bar{\varepsilon}_c = \dot{\bar{\varepsilon}}^c dt \,/ \left(1 - \frac{\Delta t}{2}\frac{\partial \bar{\varepsilon}^c}{\partial \bar{\varepsilon}_c}\right)$$

For materials which obey Norton's law:

$$\dot{\bar{\varepsilon}}^c = K_L \bar{\sigma}^n f(T)$$

The maximum time increment allowed in the computation becomes

(26) $$\Delta t_m = (2/B) K_c f(T) n \bar{\sigma}^{(n-1)}$$

where $B = d\bar{\sigma} / d\bar{\varepsilon}_c > 0$.

There is, of course, another limit imposed on the size of time steps which governs the computational stability. A limit which applies to von Mises type viscoplastic material is adopted:

(27) $$\Delta t_c = 4(1+\nu)/3EK_c f(T) n \bar{\sigma}^{(n-1)}$$

where E, υ are the modulus of elasticity and Poisson's ratio.

The expressions (26) and (27) were applied to elaborate the automatic time step generation scheme in numerical algorithm. This algorithm performs the following operations:

(A).Initialize $\{\varepsilon\}$ and $\{\dot{\varepsilon}\}$ for all elements in the model.

(B).Compute $\{T\},\{u\},\{\varepsilon\},\{\sigma\}$ as a function of initial loads (i.e., before creep deformation begins) by the usual thermoelastic analysis.

(C).Set the time increment Δt according to (26).

(D).Determine $\{\Delta\varepsilon_c\}$ from (20) where $d\bar{\varepsilon}_c$ is computed from:

(28) $$d\bar{\varepsilon}_c = \dot{\bar{\varepsilon}}^c \Delta t\left(1 - \frac{\Delta t}{2}\frac{\Delta\dot{\bar{\varepsilon}}^c}{\Delta\bar{\varepsilon}^c}\right)$$

(E).Assemble the overall stiffness matrix [K] and construct the overall structural equilibrium equations:

$$[K]\{\Delta u\} = \{\Delta u\}$$

(F).Solve for {Δu} by Gaussian elimination technique and then:

$$\{u\}^{t+\Delta t} = \{u\}^t + \{\Delta u\}$$

(G).Compute $\{\Delta\varepsilon\},\{\Delta\sigma\}$ and then compute $\{\sigma\},\{\varepsilon\}$:

(H).Update nodal coordinates

(I).Repeat Step (C) to find next Δt.

(J).Repeat the same procedure from Step (D).

Creep of welded tube

Consider an example of creep in welded tube undergoing internal pressure. The metallurgical zones in a V-groove weld are illustrated in Fig.1.Geometrical characteristics of the weld are assumed on the basis of their technological properties. Particularly it concerns the heat affected zone and the magnitude of the root. The thickness of the analyzed tube is equal to 8 mm and its diameter 50 mm. Tubes are welded with root gap 1-3 mm. It is assumed, that at 600 C the creep is characterized by Norton constants. We assume that the internal pressure in the tube is equal to 60 MPa. The initial temperature of the tube 20 C. On initial loading, the stresses are elastic and uniform in all three directions. However, the nonuniform creep in the zones of the weld causes the stress redistribution. The distribution of stresses on the line of the weld symmetry is presented in Fig.2

Final remarks

Weldments strength is a function of the strengths and ductilities of the metallurgical zones of the weld geometry. The weld metal is more anisotropic, and its strength and viscoplastic properties are lower at lower strain rates or high temperatures. The strength of weld metal can be higher than base metal for low temperatures and short time of exploitation, but lower for high temperatures and long exploitation. The method presented can be a source of information as to the preparation of the possibly best welded joints. It deepens the knowledge on physical and geometrical parameters leading to changes in strength of welded joints during its work.

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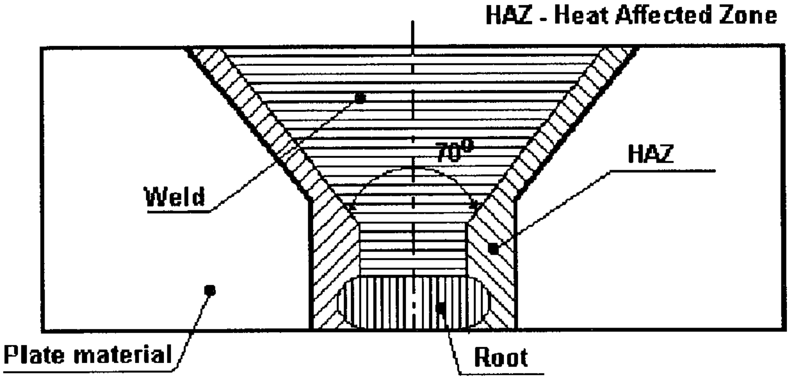

Fig. 1. Scheme of the V-weld

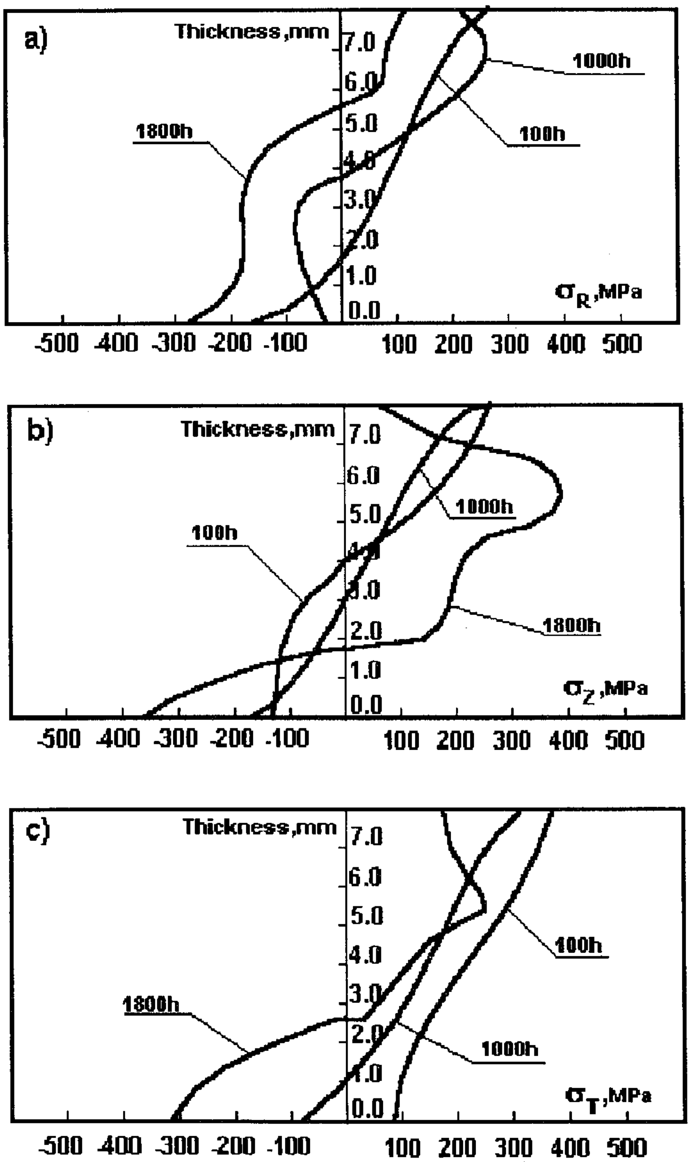

Fig. 2. Distribution of stresses on the line of the weld symetry: a) radial, b) axial, c) hoop