# ANALYSIS OF ELASTICALLY COUPLED SHELLS ON ELASTIC FOUNDATION BY HYBRID METHOD

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Brief discussion of analysis methods for shells and bodies of revolution is given. Summary of derivations of governing shell equations and pertinent body of revolution equations is included. Theory associated with the elastic coupling of shells is fully developed and resulting equations brought into the form suitable for computer programming.

Constraint equations for coupling shells and bodies of revolution as well as the equations governing bolt closures are developed. Finite element heat transfer equations are developed and presented in the form suitable to the coupling to stress analysis. Automatic finite element grid generator for the structures under consideration is described.

Computer program description and non-trivial sample problem solutions are given.

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DISCUSSION

**Q** A. KALNINS, U. S. A.

I would like to compliment Dr. Zudans for his excellent lecture. I see a great future in his approach. I think that for too long the finite element and continuum analysts have worked in two different camps, with as little interaction as possible. What we can learn from Dr. Zudans' lecture is that each has its useful application, that finite elements ought to be used only when needed. I would like to ask Dr. Zudans two questions.

1. Do you find the linear displacement distribution at the shell-ring joint acceptable ?
2. Could you explain the concept of the bolt element ?

**A** Z. ZUDANS, U. S. A.

I thank Prof. Kalnins for his kind comment.

1. The joint shell-ring linear displacement approximation is in the spirit of the Kirchhoff's approximation. As less restrictive theories are developed for shells the interface conditions can be modified accordingly. In order to have a fair representation, the interface must be moved into the region where shell approximations are valid.
2. Bolt is taken as a discrete element assemblage smeared out around circumference. Number of bolts must be large enough so that such an approximation has validity.

**Q** D. FISCHER, Austria

How do you take into account the weakening by bolt-holes in the flange ?

**A** Z. ZUDANS, U. S. A.

By finite element modelling of a flange one makes the layout of elements in such a manner that the bolt holes are covered by one set of elements and the remaining part by others. The elements covering the hole section are then specific with reduced and/or modified orthotropic properties. This provides for an approximate consideration of bolt hole influence.

**Q** J. E. GOLDBERG, U. S. A.

Inadvertently, Dr. Zudans stated that the system of equations for the thin shell portion were "reduced" to the first-order form. This is not quite correct. The basic equation of thin shell theory are, in fact, derived in a first-order form; and the method of numerical integration which was first developed by Goldberg and Bogdanoff makes use of the fact that these equations involve no derivatives of the material or shell properties and contain, as intrinsic variables, all of the displacements, stress-resultants and stress-couples which are exposed at the boundaries of a thin shell of revolution.

I would like to ask one question: Due to ring rotation, there may be an unsymmetrical pressure between the head of the bolt and the flange which, under certain circumstances, may introduce a nonlinear element to the problem. I would like to ask Dr. Zudans if he has observed this effect in connection with rings of practical dimensions ?

**A** Z. ZUDANS, U. S. A.

The first paragraph of Dr. Goldberg's question explains details of the method by which the required shell theory was derived, his comment is naturally correct and well taken.

Relative to the second paragraph, the method described here does not consider any nonlinearities. Extension of this work into areas of buckling and non-linear problems as well as the dynamics is contemplated for the future.