A Random Probabilistic Approach to Seismic Nuclear Power Plant Analysis

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Abstract

A probabilistic method for the seismic analysis of structures which takes into account the random nature of earthquakes and of the soil parameter uncertainties is presented in this paper. The method was developed combining elements of the Theory of Perturbations, the Random Vibration Theory and the Complex Response Method. The probabilistic method is evaluated by comparing the responses of a single degree of freedom system computed with this approach and the Monte Carlo Method.

1. Introduction

It is widely acknowledged that seismic events are random by nature and that the analyses based on one sample can yield misleading results. Henceforth, in the seismic design of significant structures such as nuclear power plants it is of prime importance to take into account the randomness of the earthquake loading by defining the input motion in terms of a stochastic process.

Similarly,there exists an increasing volume of field and laboratory evidence clearly showing that the commonly accepted deterministic assumption for characterization of soil properties is not longer tenable. Therefore, a better modelling of the dynamic behavior of the foundation geotechnical materials could be achieved considering the dynamic soil parameters as random variables.

Consequently, it would be convenient to have available a procedure which includes these two probabilistic aspects. In this paper the general formulation and the theoretical development of a random probabilistic method, which takes into account both the randomness of the excitation and the uncertainties in the soil material characteristics is presented. The validity of the proposed method is shown through a comparison between the responses of a single degree of freedom system calculated by the probabilistic and the Monte Carlo Methods.

2. Problem Statement

The seismic response of a structure depends on the behavior of the constituent materials and of the input motion characteristics. The dynamic properties of the foundation soils are at the most qualitatively known due to a number of factors that vary from geological aspects to sampling and testing procedures. Similarly, although to a lesser degree, the mechanical parameters of structural elements also show random variations. Henceforth, in order to model adequately the prototype it is required to consider the dynamic material properties as random variables. (Particularly, the shear modulus, G , and the damping ratio,β, are assumed to be random variables with normal distribution). Likewise, the seismic environment should be considered random. In this study it is represented by a ergodic, stationary Gaussian process with zero mean, and characterized by a mean power spectral density and uncertainty bands.

Consideration of these aspects in the seismic response problem results in a problem formulation in probabilistic terms. The solution to this problem involves a number of steps (see Fig. 1) including the definition of the mean input power spectral density and the corresponding uncertainty bands, generation of random complex transfer functions, computation of the probabilistic response in terms of a mean power spectral density and uncertainty levels, and finally, evaluation of mean extreme values of accelerations, stress, displacements, etc., and mean response spectra and their corresponding uncertainty levels.

3. Mathematical Model

The equation of motion in the frequency domain for the system shown in Fig. 1 is

$$(K - \omega_r^2 M)\{U\}_r = - \{m\} \{Y\}_r \tag{1}$$

where K and M are the stiffness and mass matrices, respectively; $\{U\}_r$ are the displacements of the system relative to the rigid base of the model, for each frequency $\omega_r (r = 0,1,2, \ldots , N)$ in the excitation $\{Y\}_r$; $\{m\}$ is a vector related to M and the direction of the input motion. The damping material, β, is introduced using a complex shear modulus: $G^* = G \exp (2i\beta)$, where G is material shear modulus; and $i = \sqrt{-1}$.

Statistical studies on the results of laboratory dynamic tests clearly show that the shear modulus and the damping ratio must be considered as random variables. Similarly, field testing results evidence the random spatial variation of the material properties. Therefore, a truly probabilistic procedure should take into account both types of uncertainty sources. This problem has been studied in the past and some approximations to the problem of random fields have been proposed [5] and implemented in finite element computer programs [6]. However, to properly define the random field of a given site it would be required an overly detailed field testing investigation. This necessarily would unduly fall upon the overall cost of the project and hence would be unacceptable in most practical cases. Moreover, for earth structures as embankments or soil sites where free field conditions are modified by the presence of massive structures like nuclear power plants the random fields must be obtained a posteriori. On the basis of this brief discussion it appears that definition of reliable random fields for most engineering problems involving soil materials is not feasible.

Accordingly, an alternate procedure which captures the key aspects of the randomness of the spatial variation of the soil properties has been recently developed [1] and then applied to evaluate the seismic response of massive structures [2,3,4] . The method is based on the Theory of Perturbations and makes use of the finite element method to solve the resulting probabilistic equation of motion. The random characteristics of the shear modulus, G, and of the damping ratio, β , are accounted for by assuming their random spatial variation as uncorrelated stochastic processses. Thus, the random field within each element of the finite element mesh is considered to be a white noise process, i.e. $G = \overline{G} (1+ \delta)$, where $\overline{G}$ is the mean value of the shear modulus within the element and δ is a random variable with normal probability density function N(0,1).

As mentioned, using perturbation techniques the following probabilistic equation is obtained [1]:

$$\{U\}_r = (-I + L_r^{-1} Q)\, L_r^{-1} \{m\} \{Y_r\} \tag{2}$$

where I is the unit matrix; $L_r^{-1} = (K - \omega_r^2 M)^{-1}$; and Q is the random stiffness matrix. This relationship allows computation of a sample of responses, $\{U\}_r$, throughout simulations of Q. Statistical analysis of this sample yields the seismic response in terms of mean values and confidence levels. The accurancy of this approach has been demostrated elsewhere |3,4|.

In eq. 2 the input motion, $\{Y\}_r$, is considered deterministic. As discussed previously, the seismic environment should be considered probabilistic. In this paper the excitation is represented by a zero-mean stationary Gaussian process characterized by a mean power spectral density function and confidence limits. This problem is formulated using a random vibration approach. Thus, writing eq. 2 as

$$\{U\}_r = \{H\}_r \{Y\}_r \tag{3}$$

where $\{H\}_r = (-I + L_r^{-1} Q)\, L_r^{-1} \{m\}$. Using random vibration theory it is easy to show that eq. 3 is transformed to

$$\{P_U\}_r = |H|_r^2 \{P_Y\}_r \tag{4}$$

where $\{P_U\}_r$ and $\{P_Y\}_r$ are the power spectral densities of $\{U\}_r$ and $\{Y\}_r$, respectively; and $|H|_r$ is the complex absolute value of $\{H\}_r$.

Now, assuming that the spectral amplitudes of $\{P_Y\}_r$ are random (i.e. vary around the mean value in accordance with a normal probability density function and that they are statistically independent), expanding eq. 4 in Taylor's series around the mean values of $\{H\}_r$ and $\{P_Y\}_r$ the following relationship is obtained:

$$\{P_U\}_r = |\bar{H}|_r^2 \{\bar{P}_Y\}_r + 2|H|_r |\bar{H}|_r (\{P_Y\}_r - \{\bar{P}_Y\}_r) + |H|_r^2 \{\bar{P}_Y\}_r - |\bar{H}|_r^2 \{P_Y\}_r \tag{5}$$

where the bar above the letters indicates mean value and the remaining are the random variations around the mean value. This probabilistic equation can be used to compute a sample of responses $\{P_U\}_r$ from simulations of the complex transfer function, $\{H\}_r$, and of the input power spectral density $\{P_Y\}_r$. Statistical analysis of this sample yields the seismic response in terms of mean power spectral densities and confidence levels.

The general procedure for generation of a response sample, $\{P_U\}_r$, includes the following steps: a) solve the equation of motion (eq. 1) using mean values for soil parameters and store the unitary solution L_r^{-1}; b) simulate N stiffness matrices, Q , and compute the corresponding complex transfer functions, $\{H\}_r$; c) simulate N input power spectral densities, $\{P_Y\}_r$; d) using the probabilistic equation of motion (eq. 5) and the N simulated values of Q and $\{P_Y\}_r$ compute a sample of N components of the response power spectral density, $\{P_U\}_r$; e) perform the statistical analysis of the response sample, $\{P_U\}_r$, and determine mean values and confidence levels; and f) estimate corresponding mean extreme values of accelerations, stresses, etc.

It should be emphasized that in the above procedure the equation of motion is solved only once. The additional operations involved in the numerical procedure are random simulations of the stiffness matrix, Q , and of the input power spectral density, $\{P_Y\}_r$, and matrix multiplications. It is evident that use of this probabilistic method of analysis results much more economic than the Monte Carlo Method.

4. Probabilistic Response of a Simple Damped Oscillator

With the purpose of evaluating the probabilistic method of analysis presented in the previous section, in what follows it is presented a comparison between the responses of a single degree of freedom system computed by eq. 5 and the Monte Carlo Method.

The complex transfer function for relative displacements of a simple damped oscillator is

$$H = \frac{-1}{(\omega_o^2 - \omega^2) + 2i\,\beta\,\omega_o\,\omega} \tag{6}$$

where ω_o is the natural frequency of the system. Consider that the mean input power spectral density is given by

$$\overline{P}_Y = \frac{b}{(4.0 - \omega^2)^2 + 0.01\,\omega^2} \tag{7}$$

where b is an intensity parameter.

Let us assume that in eqs. 6 and 7 ω_o and b are random variables with normal distribution. Consider that their range of variation is given in the table below

mean value	variation range
$\omega_o = 1.0$ Hz	$0.8 < \omega_o < 1.2$
$b = 0.1$	$0.08 < b < 0.12$

The mean values of $|H|^2$ and P_Y as well as the upper and lower levels are shown in Figs 2 and 3.

Now, using eqs 5-7 coupled with a simulation process a sample of 20 responses, P_U, was generated. The results of the corresponding statistical study is included in Figs 4 and 5. Similarly, using the Monte Carlo Method and eqs. 4,6 and 7, a sample of 20 responses, P_U, was obtained. The corresponding mean and mean plus one standard deviation are presented in Figs. 4 and 5.

The mean response spectra, P_U, shown in Fig. 4, computed by both procedures clearly indicate the accuracy of the proposed method as compared to the Monte Carlo approach. In this figure it is also included the deterministic response power spectrum, P_U, obtained using eqs. 4, 6 and 7 with $\omega_o = 1.0$ and $b = 0.1$. It is seen that there exist some differences between the mean power spectra and the deterministic power spectrum. These are more significant in the lower frequency range. It is interesting to note that while the deterministic response power spectrum shows distinctly two peaks (one of the input and other of the mean complex transfer function) the mean response power spectrum shows only one peak (corresponding to the input). The other peak is smoothed by the effect of having a band of transfer functions with varying predominat frequencies.

The mean plus one standard deviation response power spectra evaluated by both procedures are plotted in Fig. 5. The agreement is excellent. Finally, the mean and the mean plus one standard deviation response power spectra computed by the proposed probabilistic method are shown in Fig. 6.

Once the response power spectra are known, the variances corresponding to mean and lower and upper levels can be computed easily using the following relation:

$$U = \int_{o}^{\omega_n} P_U(\omega)\, d\omega \qquad (8)$$

where ω_n is the maximum frequency in the input power spectrum. Mean extreme values of the response can be computed using available solutions of the first passage problem. Thus, for example, maximum displacements can be estimated using the following equation

$$U_m = \delta \sqrt{U} \qquad (9)$$

where δ is a peak factor which depends on the uncertainty level and frequency content and frequency distribution of the response power spectrum, P_U. Expressions to obtain δ may be found in a number of papers published elsewhere |i.e. 7|.

5. Conclusions

In this paper it is presented a method of seismic analysis which takes into account the randomness of both the input motion and soil material properties. The probabilistic method, which is based on Theory of Perturbations and Random Vibration Theory, is compared to the Monte Carlo Method for the case of a simple damped oscillator. The results show and excellent agreement, thus the proposed method may be considered reliable.

From the comparison between the deterministic and the mean responses (Fig. 4), it may be concluded that seismic design of structures based on deterministic analyses may be hazardous. Accordingly, for designing important structures such as nuclear power plants in seismic zones a probabilistic approach should be used as a rule.

6. References

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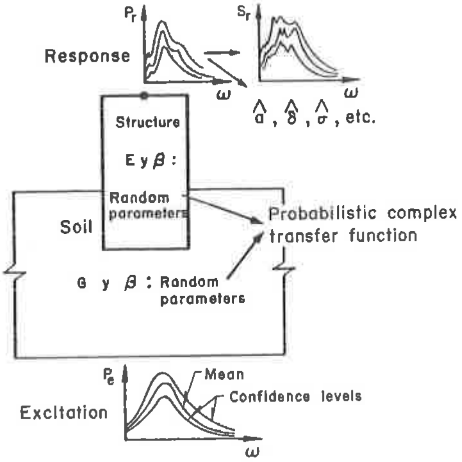

Fig 1 Method of analysis

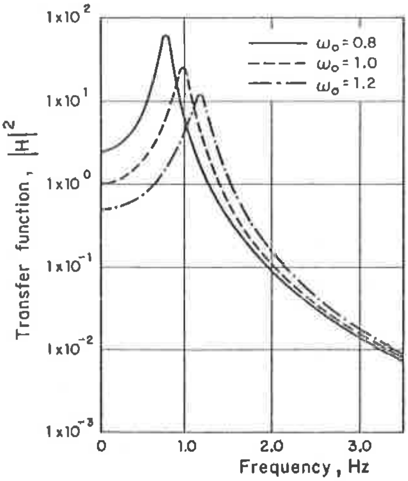

Fig 2 Variation range for complex transfer function

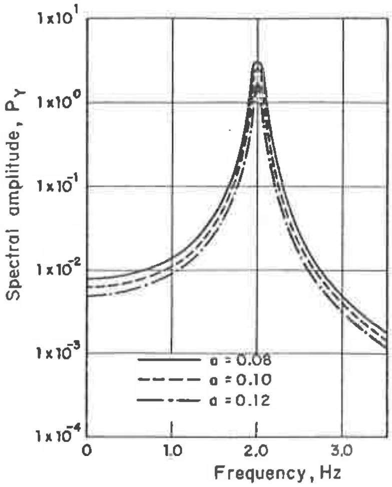

Fig 3 Variation range for input power spectral density

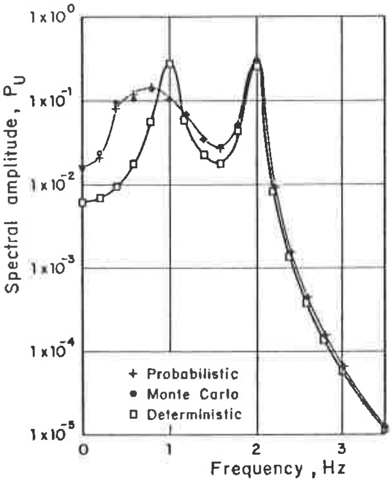

Fig 4 Response power spectra.
Mean values and deterministic

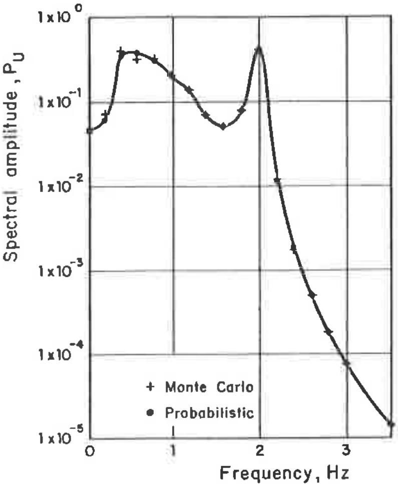

Fig 5 Response power spectra.
Mean plus standard deviation

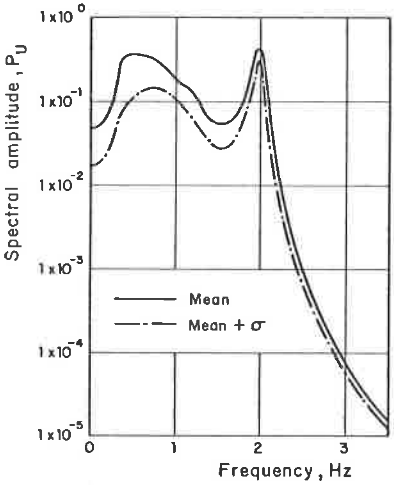

Fig 6 Mean and mean plus standard deviation response power spectra