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ANALYTICAL APPROACH ON PRESSURE DROP SOURCES AND WAVE REFLECTIONS IN A NUCLEAR REACTOR DUE TO TRANSIENTS

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Abstract

Nuclear reactor circuits generally consist of a tubular network where obstacles to flow, usually named singularities, take place. In the presence of a transient it's necessary to know how the pressure waves propagate and what is the damping mechanism associeted with the pressure loss due to the singularities.

This paper intents to establish the role of flat singularities and how they influence the pressure waves in a transitional flow. It also demostrate some simplified analytical expressions which enable closed solutions to be obtained, providing basic elements for the understanding of the phenomenon, as well as demonstrating the effects of the refletions on the responses, comparing them with experimental results.

1 - REFLECTION AND PRESSURE LOSS DURING A SUDDEN CLOSURE OF VALVE

1.1 - Effect of the sudden interruption of flow

In this case, the pressure wave due to the rough obstruction of a flow will be studied when it takes to a singularity.

Simplifying hypotheses:

- The pipe is rigid and without friction
- The lower pressures in the pipe are always higher than the water steam pressure.
- The water is incompressible, and the flow is unidimensional

The figure below shows the problem:

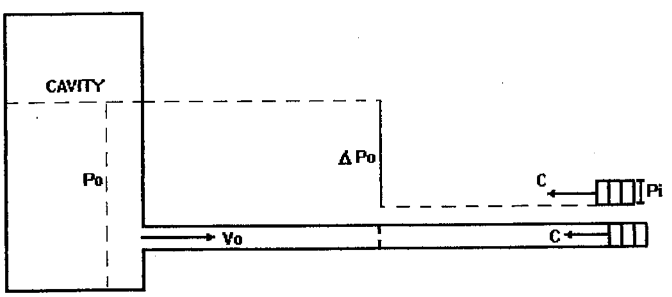

figure 1: Transient due to the rough closure of a valve.

The equations that rule the phenomena are:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{-1}{\rho}\frac{\partial p}{\partial x} \qquad \text{(motion quantity)} \tag{1}$$

The term $u\frac{\partial u}{\partial t}$ is small compared to $\frac{\partial u}{\partial t}$ so: $\frac{\partial u}{\partial t} = \frac{1}{\rho}\frac{\partial p}{\partial x}$ (2)

and $\frac{\partial(\rho \cdot u)}{\partial x} = 0$ (continuity) (3)

The solution of equation (2) can be obtained using the auxiliary unknown (θ):

$$\theta = t \pm \frac{x}{c} \tag{4}$$

from that $\pm c\frac{du}{d\theta} = \frac{-1}{\rho}\frac{\partial p}{\partial\theta}$ (5)

The integration of the equation (5) results in $\frac{Pb - Pa}{Ub - Ua} = \pm\rho c$ (6)

($\Delta P/\Delta U$) is positive when the wave front dislocates in the (x) positive direction; so:

$$\tilde{pi} = \rho\, c\, Vo\, Y(t) \tag{7}$$

Supposing that at t = 0 the wave takes to the singularity:

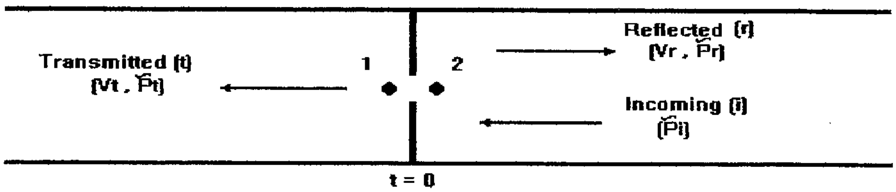

Figure 2: Aspect of the wave at the singularity.

The magnitude and direction of acoustics waves are given by the following equations:

- motion quantity (equation (6)):

$$\tilde{pr} = \rho\, c\, Vr \tag{8}$$

$$\tilde{pt} = -\rho\, c\,(Vt - Vo) \tag{9}$$

- continuity (velocities)

$$Vt = Vi + Vr \tag{10}$$

$$Vi = Vo - Vo\, Y(t) \tag{11}$$

Considering Y(t) as the unitary step function, it gives:

$$Vt = [1 - Y(t)]\, Vo + Vr = \frac{s}{S}\, V \tag{12}$$

V = velocity at the singularity, s =area of the singularity orifice and S =area of the cross section of the pipe. From equations (7) and (11) it takes to:

$$\tilde{pi} = \tilde{pr} + \tilde{pt} \tag{13}$$

The pressure loss ($\Delta p'$) due to the singularity during the transient is:

$$\Delta p' = p1' - p2' = \rho \ell \frac{dV}{dt} + K\rho \frac{V^2}{2} \tag{14}$$

K = singular pressure loss coefficient relative to the velocity (V) in the orifice.

$\ell = leq + e$ (effective length of the singularity)

p1' and p2' are the total pressures before and after the obstacle, during the transient.

so $p1' = po + \tilde{p}t$ (15)

and $p2' = (po - \Delta po) + \tilde{p}i + \tilde{p}r$ (16)

po = static pressure that causes the flow

$\Delta po = K\,(\frac{s}{S})\,\rho^2\,(\frac{Vo^2}{2})$ pressure loss in the steady state. (17)

From equations (13), (14), (15) and (16), it gives: $\Delta po - \Delta p' = 2\,\tilde{p}r$ (18)

Substituting equations (8) and (11) in (18), it takes to:

$$\rho \ell \frac{dV}{dt} + \frac{1}{2} K \rho\, V^2 - \frac{1}{2} K \rho\, V o^2 \,(\frac{s}{S})^2 = 2\,\rho\, c\,(\frac{1}{S}\, V - [1 - Y(t)]\, Vo) \tag{19}$$

That gives: $A\dot{x} + Bx^2 + 2x = B + 2\,[1 - y(t)]$ (20)

Where $A = (\frac{S}{s})\frac{\ell}{c}$; $B = \frac{1}{2} K\,(\frac{Vo}{c})\,(\frac{S}{s})^2$ and $x = (\frac{s}{S})\,(\frac{V}{Vo})$

1.2 - Solution of the equation (20):

let: c = 2 ; B+2[1-Y(t)] = D and X = z + λ => $\dot{X}$ = z

so $A\dot{z} + B(z^2 + 2\,\lambda\, z + \lambda^2) + c\,(z + \lambda) = D$ (21)

and $A\dot{z} + B\,z^2 + c'z = 0$ (Bernoulli's equation) (22)

where $c' = 2\,\lambda\, B + C$ and $B\,\lambda^2 + c\,\lambda = D$ (23)

Performing the following unknowns change:

$U = \frac{1}{z}$; $z = \frac{1}{u}$ and $\dot{z} = -\frac{1}{u^2}\,\dot{u}$

it gives the equation: A$\dot{U}$ - c'U - B = 0 (24)

which general solution is:

$U = H\ e^{(c't/A)} - (\frac{B}{c'})$ or (25)

$$X = \frac{(e^{-(c't/A)})}{(H - (\frac{B}{c'})\ e^{(-c't/A)})} + \lambda \tag{26}$$

where H is determined from the initial condition: $X(t)_{t=0} = 1$, so:

$$X = \frac{(1 - \lambda)\, e^{-\delta t}}{1 + \gamma\,(1 - \lambda)\,[1 - e^{-\delta t}]} + \lambda \qquad \text{where } \delta = \frac{c'}{A} \text{ and } \gamma = \frac{B}{c'} \tag{27}$$

From the equation (23) we have $B\lambda^2 + c\lambda - D = 0$ (28)

For t = 0; Y(t) = 0 and λ= 1, c'= 2 (B+1) and γ = B / [2 (B + 1)]

For t > 0; Y(t) = 1 and λ= B/2; c'= 2 + B^2 and γ = B / (2 + B^2)

Analyzing the solution of the equation (27), it gives:

For t = 0; λ=1 ; X=1

For $t \to \infty$; X= λ= B / 2

For t > 0 it gives:

$$X = \frac{(1 - B/2)\ e^{-(2+B^2/A)t}}{1 + (B/2 + B^2)(1 - B/2)[1 - e^{-(2+B^2/A)t}]} + \frac{B}{2} \tag{29}$$

The factor (A/2+ B^2) = τs is the time constant of the singularity, and gives the inertia effects of the orifice due to the transient.

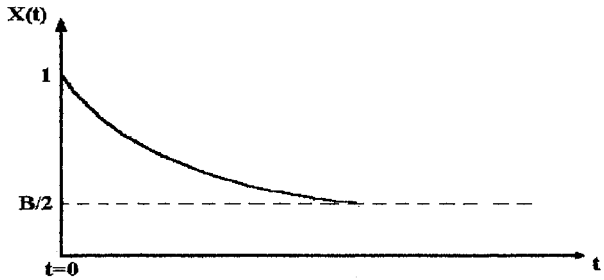

Figure 3: Variation of (x) in function of (t)

1.3 - Calculation of the pressures:

Before the obstacle: $p2' = \rho\ c\ V2' + (po - \Delta po) + \tilde{p}i$ (30)

Where $V2' = Vr = -[1 - Y(t)]\ Vo + XVo$ (equation 11)

$P2 = \tilde{p}i[((1 - Y(t)) - X) + 1] + (po - \Delta po)$

from that:

$$\frac{p2' - [po - \Delta po]}{\tilde{p}i} = [-1 + Y(t)] + (1 + X) \tag{31}$$

The equation (31) gives the relative values of the pressure fluctuations due to the wave reflection ,in relation to the initial wave.

The figure (4) shows a typical response of a point before the singularity under the effect of the reflection.

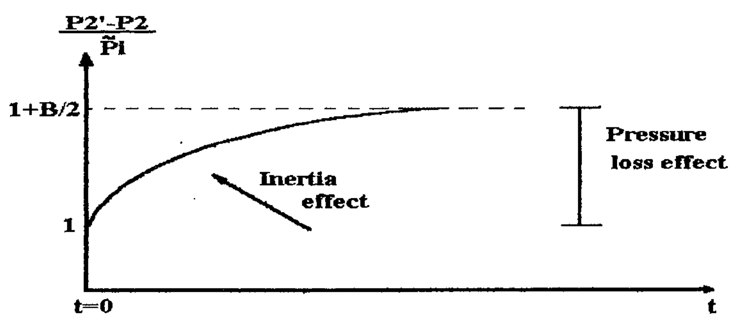

figure 4 - Effect of a reflected wave.

2 - COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS.

The figure 5 shows the experimental apparatus [4] which sign is received by the gauge (B) as shown in figure 6.

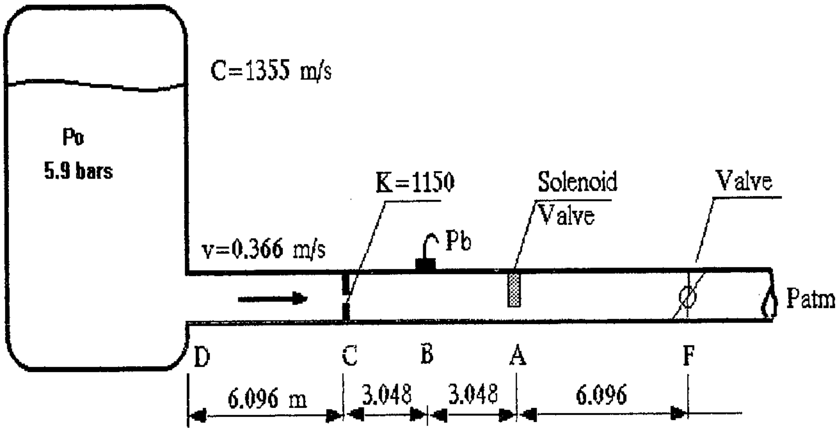

Figure 5: Experimental apparatus [4]

To illustrate the phenomenon, the figure (6) shows the response of the pressure gauge (B), for the first cycle of phenomenon (turbulent flow); and the results calculated by the use of the already developed expressions [1].

It can be noticed that the singular losses due to the transient by itself are not significant: ($\Delta p' \ll \Delta po$) and the reflected wave is calculated by:

$\tilde{pr} = \frac{\Delta po}{2}$ (equation 18). It can be checked in figure 6 and table 1.

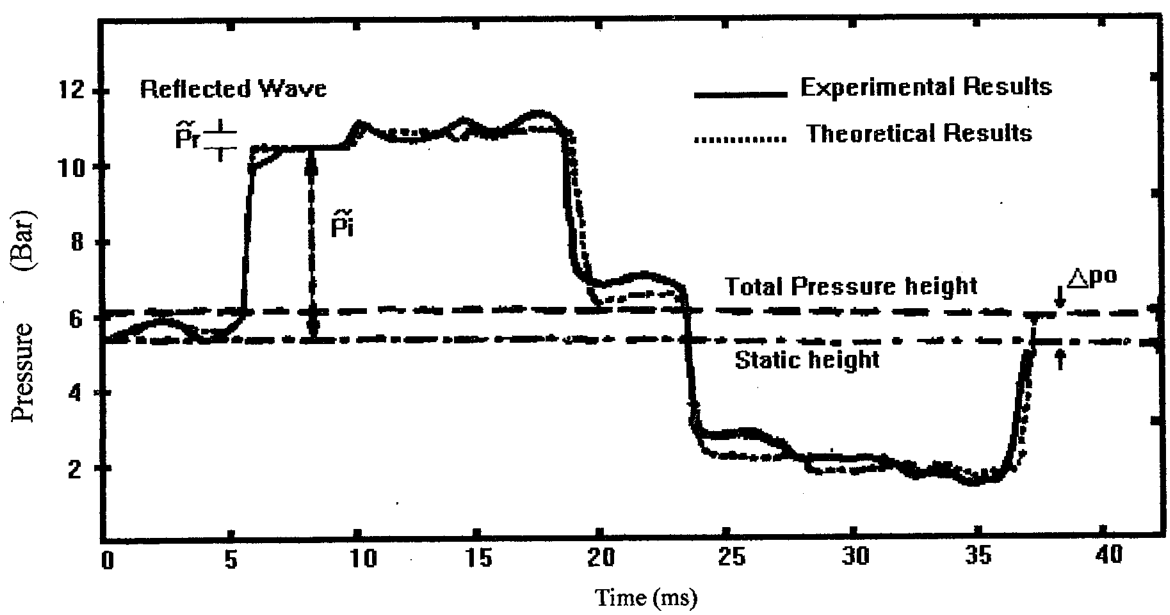

Figure 6: Effect of the reflection of a wave at a singularity: comparison between the essay and calculus [4]; and the response of the model shown in this paper (pressure gauge at B).

Table 1: Comparison between experimental results and this paper results.

	$\tilde{P}i$	ΔPo	$B/2$	$\tilde{P}r$
Experimental	4.9998	0.76924	——	0.41665
This paper model	4.9593	0.77025	0.07766	0.38514

Refering to the singular losses due to a transient (wave attenuation), it is shown [1] that these effects are associeted to the way the waves reflect at a singularity. The reflected waves have an intensity proportional to the steady state losses, which magnitude is reached exponentially with a time constant characteristic of the inertia effect of the singularity.

Besides the importance of the singular loss coefficient (k=1150), the effects of singular localized losses due to a transient doesn't show do be very important, and the damping of the waves is very weak ($\varepsilon > 3.2\%$). This is attributed to the fact that the velocities of the flow are small. As a consequence, in the case shown, the pressure fluctuations are very important, and this is not applicable to the flow fluctuations.

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