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Influence of Boundary Approximations on Soil-Structure Interaction Response

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Abstract

Finite element or finite difference numerical solutions of soil-structure interaction introduce artificial exterior boundaries to truncate the physical model. Absorbing boundary conditions are sometimes used to minimize errors associated with these artificial boundaries. This paper investigates the accuracy of the simplest absorbing boundary algorithm applicable to time domain analyses, by comparing the computed response for a rigid, massless strip foundation with its analytical solution.

1. Introduction

Discrete solution techniques based on finite elements or finite differences are sometimes used to perform soil-structure interaction (SSI) analyses. These approaches are necessary when complex geometry, embedment, site geology or nonlinearities preclude the use of simplified analytical methods. However, discrete approaches often suffer from the influence of grid boundaries on structural response.

A problem with these models is that energy radiated away from the structure (radiation damping) is reflected by the grid boundaries, thereby trapping the energy within the model. The trapped energy can excite various response modes of the bounded continuum. When grid boundaries are placed far enough from the structure to avoid interaction between boundaries and structure, Day and Frazier [1] have shown that time domain, finite element methods can accurately compute complex impedance functions for an embedded foundation. However, often the cost of solving a model in which grid boundaries are placed sufficiently far away is prohibitively expensive.

Several studies have demonstrated fundamental differences between the response of an infinite halfspace and that of a continuum region bounded by fixed boundaries. Westermo and Wong [2] show analytically that the resonant behavior of the bounded continuum cause a structure's response to differ significantly from its response on an infinite continuum. Luco et al. [3] investigated modeling of an infinite halfspace with finite elements. The computational grids used had the bottom boundary fixed. The side boundaries had either the horizontal or vertical degrees-of-freedom fixed, depending on the type of response being considered. No attempt was made to incorporate absorbing boundaries to allow radiated energy to pass out of the model. The study considered the response of a rigid strip foundation (see Fig. 1) placed on a homogeneous site using different grid sizes. Results were compared with analytic results for vertical, horizontal and rocking response. Luco et al. concluded that to adequately approximate the frequency-dependent horizontal and rocking stiffness of an infinite halfspace requires a model with grid boundaries located at least 12a from the structure, where a denotes the half width of the foundation; while a much larger model would be required to approximate vertical stiffness coefficients. The Luco study also concluded that the finite element model was incapable of adequately representing the radiation damping of the unbounded continuum. Even the use of material damping to represent the influence of radiation damping on structural response was found ineffective due to fundamental differences in the nature of each of these energy dissipation mechanisms.

It is clear from these studies that absorbing or transparent boundary conditions must be used to simulate the radiation of energy into the continuum. Although hybrid techniques have been developed which combine analytical or boundary integral solutions for the far-field with discretized solutions for the structure and near-field, these techniques are usually only applicable to frequency domain analysis. Solutions performed in the frequency domain are limited to linear models and may be very expensive for detailed three-dimensional analyses of SSI.

Discrete solvers which perform explicit integration of the equations of motion in the time domain may be used to address two- and three-dimensional, linear and nonlinear SSI problems, assuming an adequate absorbing boundary approximation is available. One of the simplest approximations is the impedance boundary of Lysmer and Kuhlemeyer [4], which assumes normal incidence of plane dilational and shear waves at the grid boundaries. For wave fields which satisfy this assumption, the boundaries of the discretized mesh are transparent, hence no boundary reflections. For most real problems, however, this normality assumption is only approximate and errors associated with it should be identified and quantified in order to evaluate the acceptability of the resulting solution. This study evaluates the effectiveness of the impedance absorbing boundary condition.

2. Computational Approach

Solutions for a rigid, massless strip foundation on an elastic halfspace were computed using the FLEX [5] finite element code. Recently developed, FLEX is designed to take advantage of the latest generation of vector supercomputers to minimize the cost of solving two- and three-dimensional SSI problems.

The finite element models used are illustrated in Fig. 2. Both welded and frictionless interface conditions beneath the foundation were considered. Discretization is uniform throughout the continuum, with the element size chosen to provide 12 elements beneath the foundation (see Fig. 2a). This grid provides good resolution of the stress distribution beneath the foundation and accurately propagates seismic waves associated with the highest frequencies considered in the study. Three different grid sizes were selected, as shown in Fig. 2b. Steady-state results for these three models were computed at 15 discrete frequencies in the range $.05 \leq a_0 \leq 1.5$, where $a_0 = a\omega / c_s$ is the reduced frequency, ω is the frequency of harmonic excitation and c_s is the shear wave speed for the continuum. All computations were performed for a Poisson's ratio of 1/3. No artificial or material damping is included in the computational model.

An absorbing boundary based on [4] is implemented in FLEX and is applied on all subsurface boundaries of the computational model. As formulated, the normal and tangential velocity of the i^{th} node on the boundary is defined by

$$v_i^n = \sigma_i / (\rho c_p) \qquad (2\text{-}1)$$

and

$$v_i^T = \tau_i / (\rho c_s) \qquad (2\text{-}2)$$

respectively, where σ_i is the normal stress, τ_i is the shear stress and ρ and c_p are the mass density and dilatational wavespeed of the continuum.

Response of the rigid foundation for sinusoidal excitation was determined for vertical, horizontal and rocking modes. Response quantities which are output by FLEX include velocity-time histories of the foundation. From these, steady-state amplitude, D and phase angle, δ are determined. Steady-state conditions are typically observed in 6 to 10 cycles of response.

3. Discussion of Results

Analytical results developed by Karasudhi et al. [6] were used as the basis for evaluating adequacy of the finite element models. Their solutions assume a frictionless interface between foundation and continuum. Steady-state amplitude and phase angle are converted to Karasudhi's stiffness and damping parameters from the relations,

$$b_1 = \frac{P_0}{\pi G D} \cos \delta \qquad (2\text{-}3)$$

$$b_2 = \frac{-P_0}{\pi G D} \sin \delta \qquad (2\text{-}4)$$

where P_0 is the applied force per unit length of the foundation and $G = \rho c_s^2$. The stiffness term b_1 is effectively the spring coefficient and b_2 the damping coefficient of the single-degree-of-freedom soil-structure system. Comparisons of numerical and analytical values of b_1 and b_2 are presented for vertical, horizontal and rocking response in Figs. 3, 4 and 5, respectively.

Results for vertical and horizontal motion with a welded interface condition are qualitatively similar. All models produce results which have the proper trend, as a_0 increases. The finite element results tend to oscillate about the analytical solution over the frequency range considered. The magnitude of oscillation depends on the size of the model. The largest grid produces the smallest oscillations. The values of b_1 and b_2

are both represented with equal accuracy by the computational model. The influence of welded vs. frictionless interface conditions was investigated using the finite element element model. Results indicate less than a 2 percent difference in response due to the difference in foundation interface conditions.

Results for rocking (Fig. 5) show significantly less oscillation than the vertical and horizontal response modes and are presented for a welded interface condition in Fig. 5a. In this case, $D = a^2 \psi$ where ψ is the foundation rotation. The difference between results for 4a and 16a grids is relatively minor. The impedance boundary accurately represents the values of b_2, indicating that radiation damping is well simulated by the model. The value of b_1 appears to be about 10 percent above the analytic solution. The influence of welded vs. frictionless interface conditions was also considered for rocking response. Results shown in Fig. 5b indicate that for rocking, the effect of a frictionless interface is to reduce the effective spring stiffness of the continuum by about 5 percent, with only a slight modification of b_2. The slightly higher value of b_1 computed by the finite element model relative to the analytic result for the frictionless interface condition is attributed to discretization error at the edge of the foundation.

The following conclusions may be drawn from these results:

a. Use of the impedance absorbing boundary condition significantly improves the accuracy of structural response simulations relative to a model with fixed boundaries.

b. Structural response errors attributed to the absorbing boundary approximation decrease as the boundaries are placed further from the structure.

c. The rocking mode of response is the least affected by the boundary approximation, as would be expected for a mode with relatively little radiation damping. Even the 4a grid is adequate for simulating rocking response.

d. Vertical and horizontal response for the 4a grid deviates significantly from the analytic results. Results for the 8a model are adequate for most uses. The improvement in accuracy in going to a 16a model may not be cost-effective, considering uncertainties associated with input excitation, site properties, and local geology.

4. Conclusions

The response of a rigid, massless strip foundation has been solved using an explicit integration, time domain finite element approach. A simple absorbing boundary condition is used at the grid boundaries to simulate an infinite halfspace. Comparisons with analytic solutions for vertical, horizontal and rocking response indicate the absorbing boundary is adequate in many applications for finite element grids whose boundaries are on the order of eight structural half-widths from the structure. The results presented herein demonstrate that errors associated with the influence of artificial boundaries on structural response can be overcome if absorbing boundaries are used. More sophisticated absorbing boundaries should be investigated to determine if they are cost-effective and result in significant improvements.

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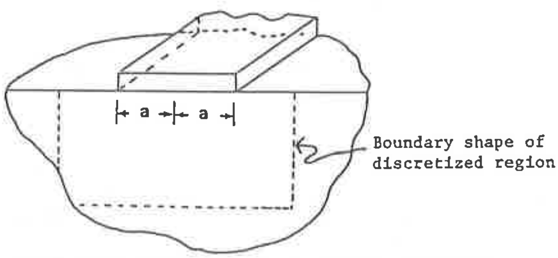
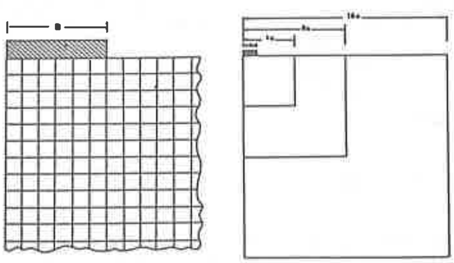


Figure 1. Rigid strip foundation on an elastic halfspace.



(a) (b)

Figure 2. Finite element models used to compute strip foundation response
(a) Discretization of finite element model
(b) Three grid sizes considered

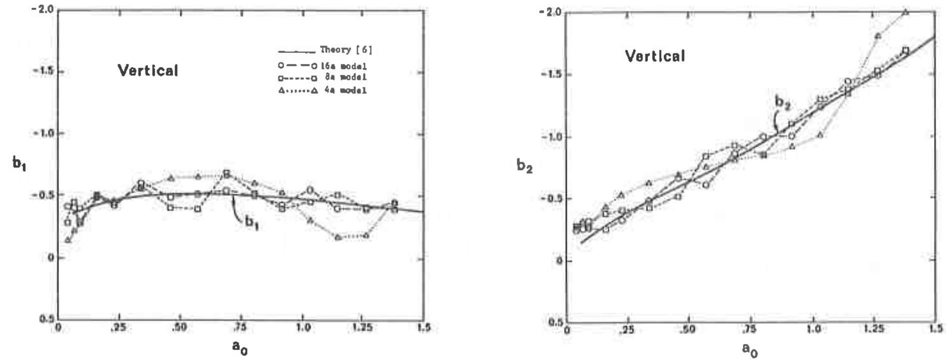


Figure 3. Vertical response comparisons.

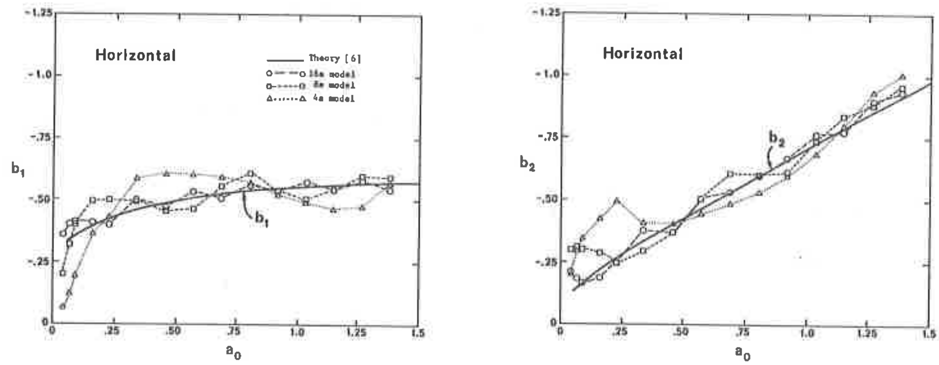


Figure 4. Horizontal response comparisons.

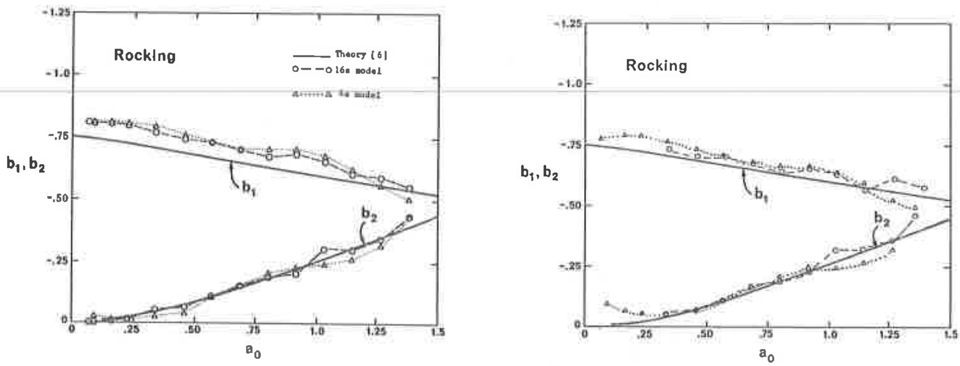


Figure 5. Rocking response comparisons. Welded finite element interface comparison on left and frictionless on right.