ALPHA-STABLE DISTRIBUTIONS FOR PREDICTING EXTREME VALUES OF SURFACE STRAINS IN REINFORCED CONCRETE FLEXURAL BEAMS

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ABSTRACT

The use of alpha-stable distribution for modeling the variations in strain at the level of reinforcement in reinforced concrete flexural beams is studied using the strains measured from an experimental investigation obtained from literature. From this study it is found that: i) alpha-stable distribution performs better than normal distribution for modeling the measured surface strains at a given stage of loading, and, ii) there is a good agreement between the measured maximum crackwidths and those estimated using 95% fractiles of strain, assuming strain follows alpha-stable distribution. These findings suggest that alpha-stable distribution is a suitable distribution to characterize the variations in surface strain at the level of reinforcement in reinforced concrete flexural beams.

INTRODUCTION

One of the important serviceability limit states to be considered in the design of reinforced concrete (RC) beams is crackwidth. It is known that the crackwidth in flexural beams, under a specified loading, can be computed by multiplying the strain in steel with the crack spacing. Hence, an important input required for crackwidth estimation is strain. Also, strains have an important role in the context of condition assessment of existing RC structures [1].

It has been reported that the experimentally measured strains in RC flexural beams at any stage of loading exhibit large scatter. The observed scatter is due to variations in material properties, cross-sectional dimensions, and, the inherent variability in the mechanism of cracking [2, 3]. To take into account this scatter, the strain at a given position on the surface of beam, at a given cross-section, and at a given loading stage should be considered as a random variable. While some attempts have been made in the literature to characterise the statistical properties of strains, to the authors' knowledge, studies on determining an appropriate probability distribution for modelling the variations in strain are scanty. The use of alpha-stable distribution for modelling the variations in surface strains in RC flexural beams is explored in the present study.

DETAILS OF EXPERIMENTAL INVESTIGATIONS [4, 5]

The data on measured surface strains at the level of reinforcement in RC beams used in this study is taken from the results of experimental investigations reported by Prakash Desayi and Balaji Rao [4,5]. This data is used in the present study, since strain measurements over the entire constant bending moment region along the span at eight different positions for different loading stages (upto ultimate) for three RC beams have been taken and reported, which will be useful for studying the usefulness of alpha-stable distribution for modeling the variations in measured strain. Availability of such extensive data is scanty in literature. Salient information regarding the experimental investigations is given below.

Three beams of similar cross-sectional dimensions of 250mm x 350mm and 4.8m long were cast and tested in two-point bending over an effective span of 4.2m. Stirrups of 6mm diameter were provided in the combined bending and shear zone to avoid shear failure, and no stirrups were provided in the constant bending moment zone. Details of the beams are given in Table 1.

Table 1: Details of beams [4, 5]

Beam	Effective depth (d) (mm)	A_{st} (mm^2)	150 mm concrete cube compressive strength (MPa)	Modulus of rupture (MPa)*	Cracking load (kN)*	Ultimate load (kN)*
KB1	311.0	402.123	33.078	4.036	23.549	95.389
KB2	305.4	437.929	40.417	3.578	14.014	104.653
KB3	303.5	529.327	22.508	2.950	8.899	84.291

(Note: * - obtained from experimental investigations)

The constant bending moment zone of the beams (1.4m) was divided into eight sections on either face, with each section having a gauge length of 200 mm (see Fig. 1). In each section, demec points were fixed at eight different positions on both faces of the beam. Position 7 in case of beam KB1 and positions 7 and 8 in case of beams KB2 and KB3 correspond to position of steel bars (Fig. 1). To measure the surface strains, a demec gauge with least count 1×10^{-5} and gauge length of 200.1 mm was used. The loads applied on the beams at different loading stages are given in Table 2.

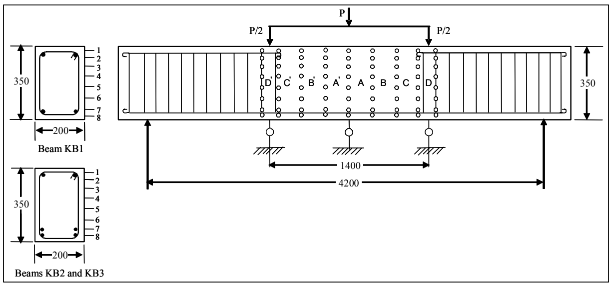

Fig. 1 Schematic representation of test program [4, 5] (dimensions in mm)

Table 2: Loads applied on the beams at different stages of loading [4, 5]

Loading Stage		1	2	3	4	5	6	7	8	9
Load Applied (kN)	KB1	10.52	16.61	28.99	38.28	48.95	57.37	65.66	-	-
	KB2	4.48	8.9	14.24	17.8	22.25	26.67	35.42	44.16	57.44
	KB3	8.9	17.8	26.67	39.78	52.97	70.9	-	-	-

STATISTICAL ANALYSIS OF STRAIN

In the present study, the focus is on statistical modelling of strains. Only the strains measured (over a gauge length of 200.1 mm) at the level of bottom-most tension reinforcement are considered further in the analysis. The applicability of alpha-stable distributions for modelling the variations in strains is explored in this paper. The justification for using alpha-stable distribution is given below.

Justification for using Alpha-Stable Distributions

Based on phenomenological considerations:

Consideration based on fractal nature of cracking: Surface strain measured (using demec gauge) in reinforced concrete is a macroscopic quantity affected by a number of parameters such as bond-slip between steel and concrete, total perimeter of tension reinforcement, tensile- and bond- strength of concrete, and, micro-fracturing and cracking in concrete. Therefore, the strain is a resultant of several microscopic phenomena, and hence, a normal distribution will be a preferred choice for modelling the random strain, the use of which may be justified using central limit theorem. However, the central limit theorem is applicable only if the all the microscopic phenomena, contributing to the strain developed, have finite mean and variance. Also, the cracking in concrete has a significant contribution to the strain development in damaged concrete. Recent developments in NDE techniques (viz. AE, GPR and other sensors) have made it possible to probe the cracking phenomenon at micro-scale levels. It has been shown by different researchers that the random variables associated with micro-cracks (size as well as geometry) formed in quasi-brittle materials like concrete have power-law distribution [6, 7]. Colombo *et al.* [8, 9], with a view to identify the damage due to cracking using acoustic emission (AE) technique, conducted experimental investigations on both in-situ bridge girder and RC beam subjected to cyclic loading. They were able to identify both micro-cracking and macro-cracking and their location based on the intensity of AE signals. They suggested that the *b*-value (the negative gradient of the log-linear AE frequency/amplitude plot) can be used for characterizing the fracture/damage process in RC beams; the minimum *b*-value trend (1.0 < *b*-value < 1.2) indicating that the micro-cracks have formed, while the maximum *b*-value trend (*b*-value > 1.7) indicating the growth of micro-cracks. These studies thus help in locating- and assessment (qualitative) of- the damage. Carpinteri *et al.* [10], based on both in-situ field test on RC member and laboratory tests on concrete specimens, have shown that the *b*-value can be linked to the value of exponent of the power law distribution of the crack size, and that the value of exponent of the power law can be interpreted as the fractal dimension of the damage domain. It may be noted that, according to Carpinteri *et al.* [10], the form of probability distribution over the entire range of crack sizes need not be a power law form;

only the tail portion of the distribution need to have the power law form. Therefore, the random variables associated with crack size in RC flexural beams should have a form consistent with the power law distribution (thus, may not have finite moments).By viewing the surface strains as a result of the indicated microscopic phenomena (such as bond-slip between steel and concrete, micro-cracking in concrete), the limiting distribution (attractor) is to be an alpha-stable distribution, knowing that the microscopic components may have power law distributions.

Thermodynamics considerations: One of the outstanding features of the behaviour of reinforced concrete flexural beams under the external loading is the emergent structure‡ at different stages of loading. It is known that when the principles of thermodynamics are applied to describe the cracking phenomenon of RC beams, the stage at which emergent structure forms corresponds to a transient non-equilibrium condition and subsequent formation of a meta-stable state(s) [3]. The loading drops at the incipience of an emergent structure marking the non-equilibrium thermodynamic state. This is transient non-equilibrium thermodynamic state because, further increase in load (from the load level to which it has reduced) can be achieved only with the increase in deflection of the beam. Once a particular stabilised crack pattern has formed, the beam will take further loading, which defines local equilibrium state with respect to cracking. This crack pattern will correspond to a meta-stable equilibrium state. The response evolution at and around the point of instability is governed more by the fluctuations than the mean. Hence, mean field theory cannot be used to predict the behaviour around this point. However, beyond the unstable point, the response evolution can be predicted using mean field theory till another non-equilibrium point is reached (if at all possible). Also, large fluctuations in strain are expected for an observed crackwidth. Balaji Rao [3] pointed out that the application of thermodynamic principles to irreversible processes (which is typically the case of cracking in reinforced concrete beams) would result in large fluctuation at the points of emergent structure. Hence, to characterise and quantify the fluctuations at these points, a distribution with heavy tails is to be used.

Based on statistical arguments:

For normal distribution 99.74% of total probability content is contained within three times standard deviation about the mean, thus giving low values of probability to the tail regions. The symmetric nature of normal distribution also restricts its applicability to phenomena exhibiting small skewness. Hence, there is a need to use distributions with heavy tails to model the strains showing large fluctuations and to estimate the extreme values. While there are different heavy-tailed alternatives to normal distribution, like alpha-stable distribution, Student's t-distribution, hyperbolic distribution, the use of alpha-stable distribution is supported by the generalized central limit theorem [12, 13]. The use of alpha-stable distribution over normal distribution has found applications in different areas (see [13, 14]). The use of alpha-stable distribution for modeling the strain in RC flexural beams at a given loading stage is explored in the present study to account for the large fluctuations.

From the above, it is clear that to predict the extreme value of strains developed in RC flexural beams, at any stage of loading, a probability distribution with power-law tails should be used.

Alpha-Stable Distribution

The alpha-stable distribution (for which an explicit expression for probability density function generally does not exist) is described by its characteristic function, given by:

$$L_{\alpha,\beta}(t) = E[\exp(itX)] = \begin{cases} \exp\left\{-c^{\alpha}|t|^{\alpha}\left[1 - i\beta\, sign(t)\tan\left(\dfrac{\pi\alpha}{2}\right)\right] + i\delta t\right\}; \text{ for } \alpha \neq 1 \\ \exp\left\{-c|t|\left[1 + i\beta\, sign(t)\dfrac{2}{\pi}\ln|t|\right] + i\delta t\right\}; \text{ for } \alpha = 1 \end{cases} \tag{1}$$

where X is the random variable, i is the imaginary unit, t is the argument of the characteristic function ($t \in \Re$), $E[\exp(itX)]$ denotes the expected value of $\exp(itX)$, α is an index of stability or characteristic exponent ($\alpha \in (0,2]$) and decides the order of moments available for a random variable, β is the skewness parameter

‡ According to Balaji Rao [3], emergent (dissipative) structure refers to formation of new cracks and/or widening and lengthening of the existing cracks on the surface of the flexural member, in the constant bending moment zone, as the loading is increased monotonically. At a given stage of loading, the emergent structure is characterized by the crack length, crack spacing, crack width, and these quantities depend on cross-section dimensions of the beam, the material strengths and the reinforcement details.

($\beta \in [-1,1]$), *c* is a scale parameter ($c > 0$), δ is a location parameter ($\delta \in \Re$), *ln* denotes the natural logarithm and *sign*(*t*) is a logical function which takes values -1, 0, 1 for t < 0, t = 0 and t >0, respectively. As α approaches 2, β loses its effect and the distribution approaches the normal distribution regardless of β [11]. It may be noted that the skewness parameter β is not the same as the classical skewness parameter [13], since for non-Gaussian stable distributions, the moments do not exist. In the case of the alpha-stable distribution, the values of β indicate whether the distribution is right-skewed ($\beta > 0$), left-skewed ($\beta < 0$) or symmetric ($\beta = 0$). A stable probability density function (pdf) is symmetrical when β =0. The distribution is called standard stable when *c* =1 and δ =0.

Generalised Central Limit Theorem

The central limit theorem states that the sum of independent, identical random variables with finite means and variances (typical of pdfs having exponential tails) converges to a normal distribution. This suggests that as long as a macroscopic phenomenon is infinitely divisible into microscopic phenomena, which exhibits finite variance with exponential tails, the random variable associated with the macroscopic phenomenon can be represented using a normal distribution. However, a wide variety of microscopic phenomena associated with strain measured, in the present context, can exhibit statistics that need to be characterised with algebraic tailed distributions. An example is the random variables associated with micro-cracks (size as well as geometry) in concrete which exhibit a power-law distribution [15], and hence may not have a finite mean and/or variance. In such cases, the random variable associated with the macroscopic phenomenon may not have a finite mean and/or variance (see Eq. 3), and normal distribution will not be the limiting distribution. Thus, a generalized central limit theorem is required to determine the limiting distribution.

The generalised central limit theorem states that the sum of a number of random variables with power-law tail distributions, decreasing as $1/|x|^{\alpha+1}$ where $0 < \alpha < 2$ (and therefore having infinite variance), will tend to a stable distribution as the number of variables grows. Longer power-law tails will lead to divergence of even lower order moments. This should not be treated as a limitation, since, in some of the physical systems, the pdf of response quantities can have power-law tails.

Statistical Modeling of Strains

At any given loading stage, sixteen strain readings (eight on the west face and eight on the east face) in the constant bending moment region are available for the beams KB1, KB2 and KB3. These values are further processed for modeling the random variations in strain in concrete at the level of reinforcement (position 7 in case of beam KB1 and position 8 in case of beams KB2 and KB3).

RESULTS AND DISCUSSION

The statistical properties (namely, mean, standard deviation, skewness and kurtosis) of the strain in concrete at the level of reinforcement have been computed based on the measured strain values and are presented in Table 3. An alpha-stable distribution, S(α, β, c, δ) is fitted to the strains in concrete at the level of reinforcement at each loading stage for the beams considered. The parameters α, β, c and δ of the alpha-stable distribution (Eq. 1) are estimated using an optimization procedure by minimizing the sum of squares of the difference between the cumulative distribution function of the measured strains (empirical distribution function) and the cumulative distribution function (CDF) of the alpha-stable distribution. The parameters of the alpha-stable distribution of strain so estimated for the beams KB1, KB2 and KB3, are also provided in Table 3. The experimental CDF, alpha-stable CDF and the normal CDF, typically for the beam KB1, for an applied load of 57.37 kN, are shown in Fig. 2. From Fig. 2, it is noted that CDF corresponding to the alpha-stable distribution compares with the experimental CDF better than the normal CDF.

The results of the K-S test, carried out to check the goodness-of-fit of the hypothesized alpha-stable- and normal- distributions to the experimental CDF, are given in Table 4. From the results, it is noted that both alpha-stable distribution and normal distribution cannot be rejected at 5% significance level; however, the alpha-stable distribution have lower values of K-S test statistic suggesting that this distribution is a better fit to the measured strain readings (especially in the tail regions which is of interest in estimating extreme values of strains).

From the variations in α with applied load (see Fig. 3) for the beams considered, it is noted that in general, α becomes close to 1.0 at loads corresponding to approximately the working loads (working loads are taken as loads corresponding to $M/M_u = 0.67$, i.e., load factor of 1.5). It is noted from Table 3 that the trends of skewness of the fitted alpha-stable distribution are in agreement with the trends of skewness shown by the measured strain readings. This observation also suggests the ability of the fitted alpha-stable distribution to represent the measured variations in strain. From the variation of β with applied load (Fig. 3) for the beams considered, it is noted that in general, the strain distribution becomes symmetrical (β = 0.0) at loads corresponding to approximately the working loads. (It is noted from Fig. 3 that for the beam KB3, the value of α approaches 2.0 as the loading increases. Since the beam KB3 has more number of smaller diameter bars (i.e., 5 nos. of 10mm dia. Bars) compared to the beams KB2 and KB3, the cracks will be more evenly distributed in the constant bending moment zone for the beam KB3, and hence the variability in strains are less, leading to exponential form of tails than power-law form. Therefore, the probability distribution of the surface strain will tend towards the normal distribution, i.e., α = 2.0).

Table 3: Statistical properties of measured strains (based on tests conducted) and parameters of alpha-stable distribution of strain

Beam	Applied Load (kN)	Statistical properties of experimentally measured strains				Parameters of α-stable distribution for strain			
		Mean	SD	Skew	Kurt	α	β	c	δ
KB1	10.52	0.000057	0.000017	0.7689	0.4232	1.2440	0.5040	0.000010	0.000065
	28.99	0.000499	0.000205	0.0401	-0.053	1.4181	0.0687	0.000146	0.000505
	48.95	0.001252	0.000478	-0.0720	0.5030	1.1038	0.1880	0.000282	0.001536
KB2	4.48	0.000036	0.000018	1.8463	4.6546	1.4839	0.999	0.000010	0.000039
	14.24	0.000131	0.000033	-0.1281	1.2193	1.3705	-0.425	0.000020	0.000123
	22.25	0.000440	0.000108	0.1806	-0.922	2.000	0.1107	0.000091	0.000438
	57.44	0.002693	0.000929	0.4189	-0.56	0.9001	0.0446	0.000319	0.001775
KB3	8.90	0.000079	2.74E-05	-0.8468	0.4996	1.1794	-0.221	0.000017	0.000071
	26.67	0.000741	0.000249	0.3524	-1.073	1.7428	0.999	0.000206	0.000776
	52.97	0.002069	0.000644	-0.3139	-1.15	1.828	-0.999	0.000555	0.002014

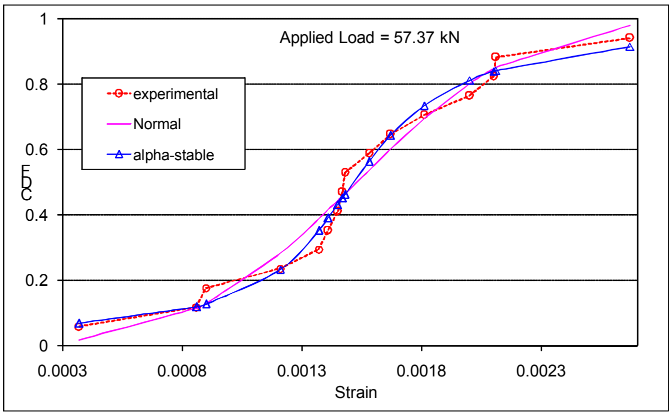

Fig. 2: Comparison of CDFs of strain for beam KB1 (applied load = 57.37 kN)

Table 4: K-S test statistics of strain

Loading Stage	K-S test statistic					
	KB1		KB2		KB3	
	Normal	alpha-stable	Normal	alpha-stable	Normal	alpha-stable
1	0.190	0.148	0.155	0.118	0.122	0.101
3	0.103	0.097	0.074	0.052	0.117	0.074
5	0.097	0.085	0.089	0.048	0.130	0.099
7	0.091	0.054	0.069	0.045	-	-
h9	-	-	0.177	0.095	-	-

The values of mean and standard deviation of measured strain readings are compared with the values of δ and c√2 (parameters of the corresponding alpha-stable distribution) for the applied loads considered (see Fig. 4). From these figures, it is noted that the values of mean and standard deviation of measured strain are comparable with values of δ and c√2.

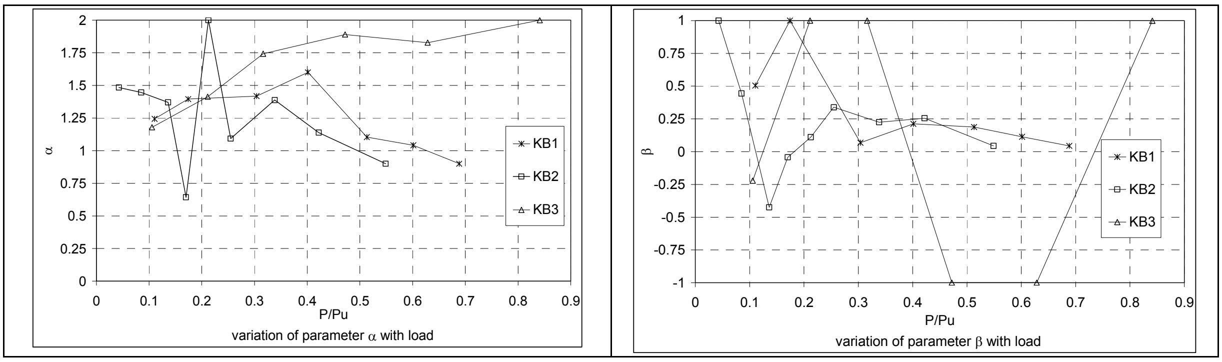

Fig. 3: Variation in parameters α and β with applied load

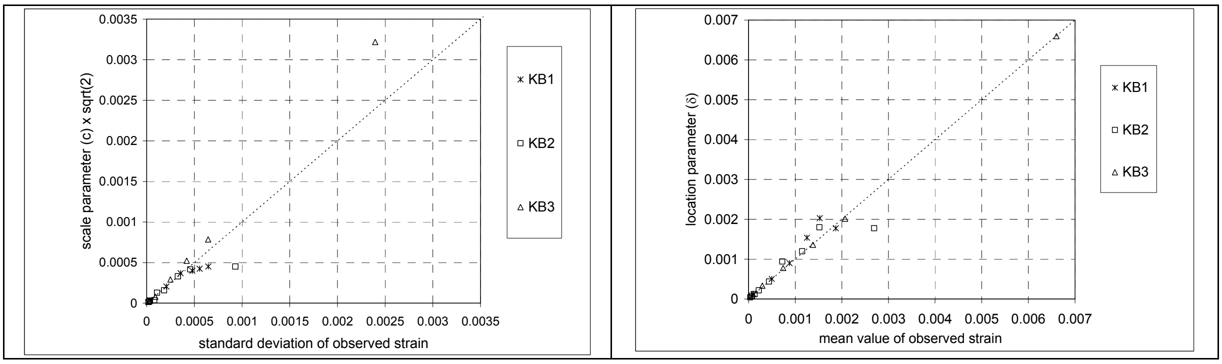

Fig. 4: Comparison of *c* and δ with the mean and standard deviation of measured strain values

From the results and discussions presented above, it is noted that the alpha-stable distributions are a better alternative to model the random variations of strain in concrete at the level of reinforcement. The above discussion also indicate that it is possible to define an alpha-stable distribution for describing the variations in strain in concrete at the level of reinforcement in RC flexural beams subjected to loads corresponding to approximately the working loads, if information regarding the mean and standard deviation of strain are available (by taking the parameters of alpha-stable distribution as $\alpha = 1.0$, $\beta = 0.0$, c = standard deviation of strain/√2 and, δ = mean strain). It is noted that when $\alpha = 1.0$ and $\beta = 0.0$, the distribution becomes a special case of alpha-stable distribution, namely the Cauchy distribution (Nolan, 2009). A Cauchy distribution is symmetric and bell-shaped similar to a normal distribution, with the main distinction being that a Cauchy distribution has much heavier tails [13]. This representation will be useful in determining the characteristic strain values for crackwidth computations, as explained in the next section.

DETERMINATION OF CRACKWIDTH

A procedure is proposed for determining the crackwidth corresponding to characteristic strain for RC flexural beams under service loads [16]. In this procedure, depth of neutral axis, gross moment of inertia of the gross section, moment of inertia of the cracked section and modulus of elasticity of concrete are considered as random variables, with statistical properties as given in Table 5. First order approximation is used for determining the mean ($\langle \varepsilon_s \rangle$) and standard deviation (σ_{ε_s}) of strain at the level of reinforcement, and using this information, an alpha-stable distribution of strain is defined ($\alpha = 1.0$, $\beta = 0.0$, $c = \sigma_{\varepsilon_s}/\sqrt{2}$ and, $\delta = \langle \varepsilon_s \rangle$). The 95% fractile is more commonly used as the characteristic value, and hence, in this study, strain corresponding to 95% fractile is considered as the characteristic strain.

The crackwidth corresponding to characteristic strain ($\varepsilon_{s_{ch}}$) at any stage of loading is computed using the relation [17]

$$w_{as} = a_c\, \varepsilon_{s_{ch}} = \frac{k_t\, f_{ct}\, A_{ct}}{k_b \left(M/M_u\right)^{\gamma} f_{bu} \sum \pi\phi}\, \varepsilon_{s_{ch}}\,. \qquad (2)$$

where a_c is the average crack spacing, k_t and k_b are factors giving average bond strength (taken as 2/3) and average tensile strength (taken as 2/3), respectively, f_{ct} is the tensile strength of concrete, A_{ct} is the effective concrete area in tension, M is the moment applied, M_u is the ultimate moment of resistance of the section, γ is a constant (taken as 0.33), f_{bu} is the ultimate bond strength, and, $\sum \pi\phi$ is the total perimeter of bars in tension. The value of characteristic strain is obtained from the Cauchy distribution, which describes the variations in measured strains at the level of reinforcement in reinforced concrete beams, at a given stage of loading. Thus, the effects of variations in crack spacing have already been taken into consideration while modeling the variations in (measured) strain. Therefore, in the present study, a_c is considered as deterministic, and is computed using the mean values of f_{ct}, M_u, A_{ct}, f_{bu} and $\sum \pi\phi$.

Table 5: Statistical properties of random variables considered in the probabilistic analysis

Random Variable	Mean	coefficient of variation
Depth of neutral axis (X)	from analysis	0.10
Gross moment of inertia of the gross section (I_G) in mm^4	$bD^3/12$	0.05
Moment of inertia of the cracked section (I_{CR}) in mm^4	$\frac{bX^3}{3}+\frac{E_s}{E_C}A_{st}(d-X)^2$	0.15
Modulus of elasticity of concrete (E_C) in MPa	$5015\left(f_c'\right)^{1/2}$	0.15

(Note: f_c' - the compression strength of concrete cylinder in MPa)

The crackwidths determined using the characteristic strain values for different loading stages are compared with the measured maximum crackwidths in Fig. 5. From this figure, it is noted that the crackwidths determined using 95% fractile strain values are conservative (a tenet required in the design). This observation supports the use of 95% fractile ($\varepsilon_{s_{0.95}}$) as the characteristic value. The 95% fractile can be obtained as (Fama and Roll, 1968)

$$\begin{aligned}\varepsilon_{s_{0.95}} &= \delta + 6.314c \\ &= \langle\varepsilon_s\rangle + 4.465\sigma_{\varepsilon_s};\ \text{since } \delta = \langle\varepsilon_s\rangle \text{ and } c = \sigma_{\varepsilon_s}\big/\sqrt{2}\end{aligned} \tag{3}$$

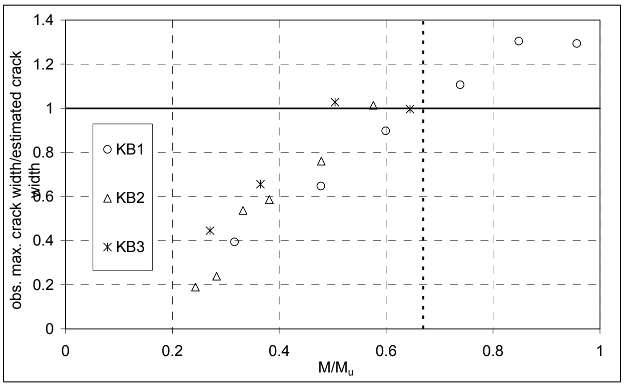

Fig. 5: Comparison of measured maximum crackwidth with the crackwidth estimated using 95% fractile strain

From the discussions presented above, it is clear that alpha-stable distribution (namely, the Cauchy distribution in the present study) is able to capture the variations in strain, and can be used for predicting the maximum crackwidths for loads corresponding to approximately the working loads. However, as is the common practice, if a normal distribution is used to represent the variations in strain, the maximum crackwidth would have been under-predicted. This is because the Cauchy distribution has heavier tails compared to the normal distribution, and hence the characteristic values corresponding to 95% fractile will be higher in the case of Cauchy distribution. This is evident from the use of factor of 4.465 in Eq. 3, which would have been 1.645 if the considered distribution is normal. This observation is significant since rational prediction of crackwidth is important with respect to the durability and service life aspects. Thus, there is a need to represent the variations in strain using alpha-stable distribution while carrying out the durability-based service life design of reinforced concrete flexural beams.

CONCLUSION

From the studies carried out, it is noted that alpha-stable distribution is a better fit to the measured strain readings in RC flexural beams at a given stage of loading, than the normal distribution, especially in the tail regions which is of interest in estimating extreme values of strains. The performance of the alpha-stable distribution to characterize the variations in surface strain at the level of reinforcement in reinforced concrete flexural beams is also evident from the good agreement obtained between the measured maximum crackwidths and those estimated using 95% fractiles of strain. It is also found that the characteristic crackwidths estimated using alpha-stable distribution for strain are conservative while those estimated using normal distribution for strain can be unconservative (an undesirable characteristic with respect to design of RC structures). Based on the studies carried out, a relation for determining the characteristic values strain, using first order approximations of mean- and standard deviation- of strain, is proposed, which would be useful in the durability-based service life design of RC flexural beams.

ACKNOWLEDGEMENT

This paper is being published with the kind permission of the Director, CSIR-SERC, Chennai, India.

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