A STUDY ON THE MULTI-DIMENSIONAL SPECTRAL ANALYSIS FOR RESPONSE OF A PIPING MODEL WITH TWO-SEISMIC INPUTS

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SUMMARY

The power and the cross power spectrum analysis by which the vibration characteristic of structures, such as natural frequency, mode of vibration and damping ratio, can be identified would be effective for the confirmation of the characteristics after the construction is completed by using the response for small earthquakes or the micro-tremor under the operating condition. This method of analysis seems to have been utilized only from the view point of systems with single input so far; it is extensively applied for the analysis of a medium scale model of a piping system subjected to two seismic inputs in this paper.

The data for which the method is applied was allowed to use from the experiment carried out in a project supported by the Japan Electric Society. The piping system attached to a three storied concrete structure model which is constructed on a shaking table was excited due to earthquake motions. The inputs to the piping system were recorded at the second floor and the ceiling of the third floor where the system was attached to. The output, the response of the piping system, was instrumented at a middle point on the system.

General formulation of the analysis of the multi-dimensional spectrum analysis is described at first and it is reduced to the description for the care of the system with two inputs. Emphasis is put on that coherency which gives a measure of reliability of the evaluated gain and phase characteristic should be obtained in the multiple and the partial one. The former is expressed in terms of both inputs involved and the latter is with regard to the relation between the output and the respective input.

The natural frequency of the concrete structure is given as 3.83 Hz, and the first and the second natural frequency of the piping is 3.83 and 11.70 Hz. The first one is made equal to that of the structure. The power spectrum of the respective signal, and the gain and the phase characteristics of the output to the respective input are obtained at first. This makes it obvious how the respective input gives the influence on the transfer characteristic and the results as for the coherency shows the quantitative extent of the effect, that is, the partial coherency suggests that the input at the second floor has stronger effects for the gain of the first natural frequency and the other input affects more strongly the characteristic of the second natural frequency. Thus the gain characteristic is identified taking the effect of the respective input into consideration. This was impossible by the conventional method utilized so far.

As the result, the paper presents that the multi-dimensional power spectrum analysis is effective for a more reliable identification of the vibration characteristics of the multi-input structure system.

This report concerns estimation of the dynamic response characteristics of the mechanical system subjected to multiple seismic input such as piping system using the multi-dimensional spectral analysis technique.

As a numerical example of the application, the frequency response function for a piping model to two input excitations is estimated using experimental data of a vibration test.

A multiple coherence function and a partial coherence function are introduced in determining confidence limits for this analysis. Influence of the respective input on the total response of the system is estimated by computing the ratio of the partial coherence function to the multiple coherence function in the regarding frequency range.

1. Preliminary Formulation of the Multi-Input Problem[1],[2]

A linear system responding to multiple input signals $y_1(t),\ y_2(t),\ \cdots\cdots,\ y_n(t)$ and a single response output process $x(t)$ as shown in Fig. 1 is now considered. In the following discussion these n input processes are assumed to be stationary random signal, their mean values to be zero and their auto covariance function is supposed to belong to range $L_1(-\infty,\infty)$, i.e.,

$$\left.\begin{aligned} &E\{y_i(t)\} = E\{x(t)\} = 0 \quad (i=1,2,\cdots,n) \\ &\int_{-\infty}^{\infty}|R(\tau)|\,d\tau < \infty \end{aligned}\right\} \tag{1}$$

It is assumed that the output process $x(t)$ may be considered as the sum of n output-component processes $x_i(t)$ for i=1,2,$\cdots$,n, that is,

$$x(t) = \sum_{i=1}^{n} x_i(t) \tag{2}$$

where $x_i(t)$ is defined as the component of $x(t)$ and is the output for that $y_i(t)$ is made only input to the system.

When an impulse response function $h_i(\tau)$ which is associated with the input process $y_i(t)$ and the output component $x_i(t)$ are determined for a system, the following relation is given

$$x_i(t) = \int_0^{\infty} h_i(\tau)\,y_i(t-\tau)\,d\tau \tag{3}$$

Hence the total output process is represented by

$$x(t) = \sum_{i=1}^{n}\int_0^{\infty} h_i(\tau)\,y_i(t-\tau)\,d\tau \tag{4}$$

or by its Fourier transforms

$$X(f) = \sum_{i=1}^{n} H_i(f)\,Y_i(f) \tag{5}$$

where $H_i(f)$ means a frequency response function related with $Y_i(f)$ and $X_i(f)$ which represent the Fourier transforms of $y_i(t)$ and $x_i(t)$ respectively.

Equation (4) or (5) is basic relation for the analysis of the multiple input system. Generally it is convenient for multi-dimensional analysis to describe the formulae by the matrix notation.

First let us define the n-dimensional input matrix, $\mathbb{y}(t)$ and n-dimensional frequency response function matrix $\mathbb{H}(f)$, i.e.,

$$\mathbb{y}(t) = [\, y_1(t),\ y_2(t), \cdots\cdots, y_n(t)\,] \tag{6}$$

$$\mathbb{H}(f) = [\, H_1(f),\ H_2(f), \cdots\cdots, H_n(f)\,] \tag{7}$$

Next, the definition of an n-dimensional cross-power spectrum matrix of the output process $x(t)$ associated with n-input processes $y_i(t)$ for i=1,2,⋯,n, and an $n \times n$ matrix of cross power spectra for all the input processes would be given as follows,

$$\mathbb{S}_{yx}(f) = [\, S_{1x}(f), S_{2x}(f), \cdots\cdots, S_{nx}(f)\,] \tag{8}$$

and

$$\mathbb{S}_{yy}(f) = \begin{bmatrix} S_{11}(f), S_{12}(f), \cdots\cdots, S_{1n}(f) \\ S_{21}(f), S_{22}(f), \qquad \vdots \\ \vdots \quad \vdots \quad \ddots \quad \vdots \\ S_{n1}(f), S_{n2}(f), \cdots\cdots, S_{nn}(f) \end{bmatrix} \tag{9}$$

In the above representations $S_{ix}(f)$ is equal to $S_{y_i x}(f)$ and denotes the cross power spectrum of the output $x(t)$ with one of the input processes $y_i(t)$, and $S_{ij}(f)$ is abbreviated from $S_{y_i y_j}(f)$ and means the cross power spectrum between an input process $y_i(t)$ and another one $y_j(t)$. Diagonal element $S_{ii}(f)$ in the matrix of equation (9) means power spectrum of the respective input $y_i(t)$ and is given as a scalar. Using the relation (4) or (5) the power spectrum of the output response process can be obtained

$$\begin{aligned} S_{xx}(f) &= \sum_{i=1}^{n}\sum_{\ell=1}^{n} \int_0^\infty h_i(\tau_1) e^{j2\pi f\tau_1} d\tau_1 \int_0^\infty R_{i\ell}(t) e^{-j2\pi ft} dt \int_0^\infty h_\ell(\tau_2) e^{-j2\pi f\tau_2} d\tau_2 \\ &= \sum_{i=1}^{n}\sum_{\ell=1}^{n} H_i(f)\, S_{i\ell}(f)\, \bar{H}_\ell(f) \\ &= \mathbb{H}(f)\, \mathbb{S}_{yy}(f)\, \bar{\mathbb{H}}'(f) \end{aligned} \tag{10}$$

where $R_{i\ell}(t)$ is a cross-correlation function which is concerned with mutually correlated two input processes $y_i(t)$ and $y_\ell(t)$. $\bar{H}_\ell(f)$ indicates the complex conjugate of $H_\ell(f)$ and $\bar{\mathbb{H}}'(f)$ does the complex conjugate transposed matrix of $\mathbb{H}(f)$.

Since cross power spectrum between the input process and the output process can be written as

$$\begin{aligned} S_{ix}(f) &= \int_{-\infty}^{\infty} R_{ix}(\tau) e^{-j2\pi f\tau} d\tau \\ &= \sum_{\ell=1}^{n} H_\ell(f)\, S_{i\ell}(f) \end{aligned} \tag{11}$$

the system of equations for i=1,2,⋯,n can be represented by the following matrix notation

$$\mathbb{S}'_{yx}(f) = \mathbb{S}_{yy}(f)\, \mathbb{H}'(f) \tag{12}$$

where $\mathbb{S}'_{yx}(f)$ and $\mathbb{H}'(f)$ are the transposed column matrices of the row matrices $\mathbb{S}_{yx}(f)$ and $\mathbb{H}(f)$ respectively. Equation (12) can be solved as for the transposed row matrix $\mathbb{H}'(f)$ if $\mathbb{S}_{yx}(f)$ and $\mathbb{S}_{yy}(f)$ are known. That is

$$\mathbb{H}'(f) = \mathbb{S}_{yy}^{-1}(f)\, \mathbb{S}'_{yx}(f) \tag{13}$$

This can also be written in the following form

$$\begin{bmatrix} H_1(f) \\ H_2(f) \\ \vdots \\ H_n(f) \end{bmatrix} = \begin{bmatrix} S_{11}(f), S_{12}(f), \cdots, S_{1n}(f) \\ S_{21}(f), S_{22}(f), \cdots\cdots \\ \vdots \quad \ddots \quad \vdots \\ S_{n1}(f), \cdots\cdots\ S_{nn}(f) \end{bmatrix}^{-1} \begin{bmatrix} S_{1x}(f) \\ S_{2x}(f) \\ \vdots \\ S_{nx}(f) \end{bmatrix} \tag{14}$$

The response characteristics of the system which is to subjected multiple random input excitations can be effectively estimated by using eq. (14).

2. Identification of the Vibration Characteristics of the Two Inputs Piping Model

The analysis technique aforementioned is extensively applied to estimate the vibration characteristics of a piping model system with two seismic inputs. This method has an advantage to be able to identify the dynamic characteristics after the construction is completed by using the data of response for small earthquakes or the micro tremor under the operating condition.[3] The data of time history in the analysis are allowed to use from those obtained by the project managed by the Japan Electric Society.[4]

In Fig. 2 a schematic model system of this experiment is shown. This model which is a three story reinforced concrete building accomodated with some equipments and piping models is constructed on a shaking table the maximum capacity of which is 120 ton. This table is operated by hydraulic actuators (total vector force is 60 ton) taking displacement motion of Matsushiro swarm earthquake as the input signal. Displacement signal can be obtained from the original acceleration through the integral operation. It is carried out by hybrid computer which is attached to the shaking table as its control system.

A flat steel bar with 9 kg weight mass at the middle, that is, a piping model which restricts the motion in one direction is taken as the objective of the analysis in this study. A higher end of this model is fixed on the ceiling of the third floor of the concrete structure and another lower end is fixed on the second floor. Hence this model is excited by two input motions from both ends while the table is shaken. Table 1 shows a natural frequency of this two inputs piping model for two cases: the case that an oil damper is attached to the weight mass and the case without it. The lowest natural frequency 3.83 Hz is adjusted so that it may be equal to that of the concrete structure.

In Fig. 3 some examples of the response output wave forms obtained by the shaking test are shown. INPUT No. 1 and INPUT No. 2 are the response wave form recorded at the ceiling of the third floor and at the second floor corresponding to ① and ② in Fig. 2 respectively in Fig. 2. These are regarded as the inputs to the piping model. RESPONSE denotes the acceleration response at the weight mass of this model which is shown ③ in Fig. 2. Strictly speaking these seismic response wave forms are nonstationary random vibration, but in this study these are supposed to be narrow band stationary signal. In order to examine the frequency characteristics of these signals the power spectra of the two inputs at ① and ② , and the response output at ③ are computed. Figure 4 shows some computed results of the power spectra as for these data.

Values of parameters necessary for the numerical computation of the spectrum are taken as below,

(1) total number of the respective signal; $n = 1300$

(2) time interval between adjacent data values; $\Delta t = 0.01$ s

(3) number of lags of correlation; $h = 100$

(4) lag window for the smoothing of power spectra; $a_0 = 0.64$, $a_1 = a_{-1} = 0.24$, $a_2 = a_{-2} = -0.06$ (proposed by Akaike)[5]

Power spectra of two input motions $S_{11}(f)$ and $S_{22}(f)$ have three peaks. The peak at about 4 Hz shows the first natural frequency of this concrete structure model. The other two peaks in comparatively higher frequency range, those at about 8 Hz and 15 Hz, mean predominant frequency components which are contained in the original earthquake motions added to the shaking table. Power spectrum of the output response motion at the weight mass $S_{xx}(f)$

has two sharp peaks at about 3.8 Hz and 11.5 Hz. The peak at the lower frequency means the first natural frequency of this piping model which is taken equal to the natural frequency of the concrete structure. On the other hand small peak at about 11.5 Hz means the second natural frequency of this piping model as shown in Tab. 1.

Next the frequency response function is estimated by using the method of multi-dimensional spectral analysis discussed in the last section. In this computation following characteristics which have been obtained from the previous study [6] for multi-input system are introduced.

(1) If the mass of an appendage system or a secondary system such as piping and equipment is small compared with that of a building structure or a primary system, the effect of the reaction from the appendage to the building can be negligible.

(2) For the response analysis of the multi-input system it is usually necessary to take not only the acceleration but also the velocity and the displacement into account as input signal.

When the system can be considered to have similar stiffness and damping characteristic as for the respective input-end, only the acceleration signal can be taken as the input.

So that this two inputs system can be simplified as depicted in Fig. 5. For this case

$$\mathbb{H}(f) = [H_1(f), H_2(f)] \quad (15)$$

$$S_{yx}(f) = [S_{1x}(f), S_{2x}(f)] \quad (16)$$

$$S_{yy}(f) = \begin{bmatrix} S_{11}(f), & S_{12}(f) \\ S_{21}(f), & S_{22}(f) \end{bmatrix} \quad (17)$$

can be obtained from eqs. (7), (8) and (9). The response characteristics can be solved through the relations

$$\left.\begin{aligned} H_1(f) &= \{S_{11}(f)S_{1x}(f) - S_{12}(f)S_{2x}(f)\}/\Delta(f) \\ H_2(f) &= \{S_{11}(f)S_{2x}(f) - S_{21}(f)S_{1x}(f)\}/\Delta(f) \end{aligned}\right\} \quad (18)$$

which are derived from eq. (14), where

$$\Delta(f) = S_{11}(f)S_{22}(f) - |S_{12}(f)|^2 \quad (19)$$

Hence the quantities of $H_1(f)$ and $H_2(f)$ are computed using the quantities of power spectra of input signals $S_{11}(f)$ and $S_{22}(f)$ and the cross power spectra between the respective input and output $S_{1x}(f)$, $S_{2x}(f)$ and those between two input, $S_{12}(f)$.

Some examples of numerical computation of frequency response functions for this system applying the equations for the data from shaking test are shown in Fig. 6 and Fig. 7. The amount of shift of the data window for computation of the cross power spectrum K is taken 30. From Fig. 3 it is expected that two-input signals $y_1(t)$ and $y_2(t)$ are highly correlated. Hence as is seen in Fig. 6 the gains $|H_1(f)|$ and $|H_2(f)|$ have similar characteristics. Observing these gain curves more precisely, the estimates of the gain at about 4 Hz are of the same order of magnitude. On the other hand at about 12 Hz the peak of gain $|H_2(f)|$ is predominant compared with that of $|H_1(f)|$. This is caused by the fact that the input excitation $y_2(t)$ excites this piping model at this frequency stronger than $y_1(t)$, because $y_2(t)$ contains higher frequency components, which is nearly equal to the frequency associated with the second mode of this piping model. In the phase shift diagram as shown in Fig. 7, the rapid change of the phase shift can be obviously seen at the frequency where peaks of the gain are predominant.

Thus the frequency response characteristics as for the respective input motion can be estimated and examined by using the technique of multi-dimensional spectral analysis.

3. Examination of Results in Terms of Coherence Function

In an ordinary time invariant system with a single input and a single output a coherence function

$$\gamma_{yx}^2(f) = \frac{|S_{yx}(f)|^2}{S_{yy}(f)\,S_{xx}(f)} \tag{20}$$

can be defined. Here $S_{yy}(f)$ and $S_{xx}(f)$ are the power spectrum for input and output process respectively, and $S_{yx}(f)$ means the cross power spectrum between input and output process. This function may be considered as a measure of linear relationship between both signals and also a measure of noise existence caused in the input and output measuring devices for the system. Hence the coherence function can be used as a measure in determining the extent of the reliability of the estimation of a frequency response function for this system. It is judged by whether this function attains a theoretical maximum of unity in an appropriate frequency range.

Now this idea is developed to the multiple input system. The coherence function based on the relation between n input processes $\mathbf{y}(t) = \{y_1(t), y_2(t), \cdots; y_n(t)\}$ and output response process $x(t)$ can be determined using eq. (13).

$$\gamma_m^2(f) = \frac{1}{S_{xx}(f)}\,\mathbf{S}_{yx}(f)\,\mathbf{S}_{yy}^{-1}(f)\,\overline{\mathbf{S}}_{yx}'(f) = \frac{1}{S_{xx}(f)}\,\mathbf{H}(f)\,\overline{\mathbf{S}}_{yx}'(f) \tag{21}$$

where $\overline{\mathbf{S}}_{yx}'(f)$ denotes the complex conjugate transposed matrix $\mathbf{S}_{yx}(f)$. $\gamma_m(f)$ in eq. (21) is called multiple coherence function.

In multi-input problems it is frequently necessary to observe the relation between a specified input signal $y_i(t)$ and a output response $x(t)$. In this case, however, the values of coherence function due to the relation between $y_i(t)$ and $x(t)$ usually decreases as the correlation of this input with other input signals, $y_1(t), y_2(t), \cdots, y_{i-1}(t), y_{i+1}(t), \cdots; y_n(t)$. In order to suppress the effect of this spurious correlation, the coherence function for this relation is given by using "conditioned process"[7] or "residual process"[2] $\tilde{y}_i(t)$ and $\tilde{x}(t)$, which remove the existence of other input processes

$$\tilde{\gamma}_{ix}^2(f) = \frac{|\tilde{S}_{ix}(f)|^2}{\tilde{S}_{xx}(f)\,\tilde{S}_{ii}(f)} \tag{22}$$

where $\tilde{S}_{ii}(f)$ and $\tilde{S}_{xx}(f)$ mean "conditioned power spectrum" of $\tilde{y}_i(t)$ and $\tilde{x}(t)$ respectively, and $\tilde{S}_{ix}(f)$ means "conditioned cross power spectrum". These are called partial coherence function. From eq. (21) multiple coherence function is given by

$$\gamma_m^2(f) = \{S_{1x}(f)H_1(f) + S_{2x}(f)H_2(f)\}/S_{xx}(f) \tag{23}$$

and two types of partial coherence function can be written as

$$\left.\begin{aligned} \tilde{\gamma}_{1x}^2(f) &= \frac{|\tilde{S}_{1x}(f)|^2}{\tilde{S}_{11}(f)\tilde{S}_{xx}(f)} = \frac{|S_{1x}(f)|^2/\left|1-\{S_{2x}(f)S_{12}(f)\}/\{S_{1x}(f)S_{22}(f)\}\right|^2}{S_{11}(f)S_{xx}(f)\{1-\gamma_{2x}^2(f)\}\{1-\gamma_{12}^2(f)\}} \\ \tilde{\gamma}_{2x}^2(f) &= \frac{|\tilde{S}_{2x}(f)|^2}{\tilde{S}_{22}(f)\tilde{S}_{xx}(f)} = \frac{|S_{2x}(f)|^2/\left|1-\{S_{1x}(f)S_{21}(f)\}/\{S_{2x}(f)S_{11}(f)\}\right|^2}{S_{22}(f)S_{xx}(f)\{1-\gamma_{1x}^2(f)\}\{1-\gamma_{21}^2(f)\}} \end{aligned}\right\} \tag{24}$$

where $\gamma_{12}(f)$, $\gamma_{1x}(f)$ and $\gamma_{2x}(f)$ are ordinary simple coherence functions obtained for the system with pairs of single input and output $[y_1(t), y_2(t)]$, $[x(t), y_1(t)]$ and $[x(t), y_2(t)]$, respectively. Then the multiple coherence function and the partial coherence functions are computed by using these equations in terms of evaluated spectra for the present piping model system with two inputs discussed in the previous section.

Evaluated coherence functions are plotted in Fig. 8. Every curve maintains fairly low level at other frequency regions than about 2~5 Hz and 12 Hz where the magnitude of the gain of frequency response function $|H_1(f)|$ and $|H_2(f)|$ have predominant peak. This characteristic is due to fact that random wave forms of the two inputs $y_1(t)$ and $y_2(t)$ behave as that through narrow band pass filter, the predominant frequency is mainly equal to the natural frequency of the concrete primary structure. The obtained frequency response functions are provided extent of the reliability at the frequency range that the coherence functions are close enough to 1.0. While the estimate of partial coherence function $\tilde{\gamma}_{1x}(f)$ is larger than $\tilde{\gamma}_{2x}(f)$ in the range of 3~5 Hz, that of $\tilde{\gamma}_{2x}(f)$ becomes larger at the neighbourhood of 12 Hz. This means that at the coincidence of the natural frequencies of the both systems at 3~5 Hz the effect of the input $y_1(t)$ appears strongly to the output $x(t)$, but for second natural frequency, about 12 Hz, the effect of the input $y_2(t)$ is more dominant than that of $y_1(t)$.

In Fig. 9 the ratio of the respective partial coherence function to the multiple one is plotted for the two input piping model system. This is compared with the ratio of the ordinary simple coherence function which is obtained for the system that the respective input is assumed only the input with the multiple coherence function. From the figure the contribution of the specific input to the output response can be examined. It is also found in Fig. 9 the partial coherence function of the two inputs system is larger than the simple coherence function in the frequency of 1~4 Hz. Since two inputs $y_1(t)$ and $y_2(t)$ keep high correlation in the frequency range, the effects of the two inputs to the output response are almost equal. In this case, if the linear relation is assumed between both inputs, the system can be reduced to that with a single input by introducing an appropriate frequency response function $H_{12}(f)$ which describes the relation between $y_1(t)$ and $y_2(t)$.

On the other hand in the frequency range larger than about 8 Hz, the magnitude of the partial coherence function $\tilde{\gamma}_{1x}(f)$ tends to be smaller than the partial coherence function $\tilde{\gamma}_{2x}(f)$. This suggests that the contribution of the input $y_2(t)$ in this range is so strong that the effect of the input $y_1(t)$ may be negligible in practice. In the range of inbetween, 5~7 Hz, the partial coherence function for both inputs keeps predominant magnitude comparing with that of the simple coherence function. In this case, the multiple coherence function diminishes as shown in Fig. 8. This means that the estimate of the frequency response function is not reliable due to the disturbance of the additive noise caused in the measuring devices and the nonlinearity in the system.

As a result of this investigation using the experimental data of a model test of vibration, it is shown that the coherence function can be criteria to give the reliability of the estimated frequency response function for machine structure or equipment system which is subjected to the multiple seismic excitation during earthquake.

4. Conclusions

The vibrational characteristics of a piping model which is subjected to two seismic excitations from a building structure model at the floor of which the piping model is suspended are estimated by applying the multi-dimensional spectral analysis technique.

The results may be summarized as follows;

(1) The frequency response function for the two inputs model such as piping can be obtained by the multi-dimensional spectral analysis technique. Thus estimated function makes the component of the matrix which denotes the total frequency response characteristic for the multi-input system.

(2) By computing the value of the multiple coherence function and the partial coherence function which are specific to multi-input system, reliability of the estimated frequency response function at the appropriate frequency range can be examined.

Measure of the reliability for the computed results can be given in terms of the ratio of both coherence functions. Small value of the ratio is due to the effect of noise and nonlinearity in the multi-input system.

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Table 1. The Natural Frequency of the Two Inputs Piping Model

	without oil damper	with oil damper
1st	3.83	3.79
2nd	11.70	11.54
3rd	22.06	22.50

(Hz)

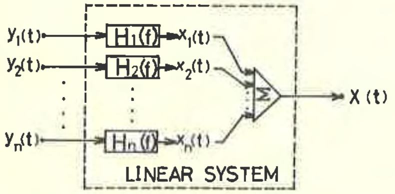


Fig. 1 Flowchart of Linear Multi-Input and Single-Output System

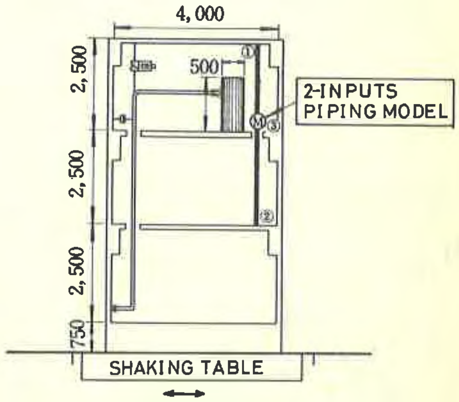


Fig. 2 Schematic Model of Concrete Structure, Equipment and Piping Model for Vibrational Test

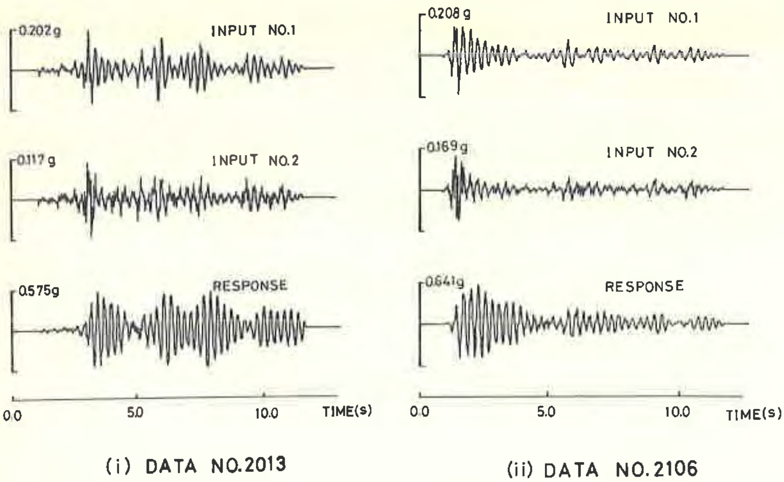


Fig. 3 Recorded Input and Output Response Wave Form of Two Inputs Piping Model Obtained through Vibration Test

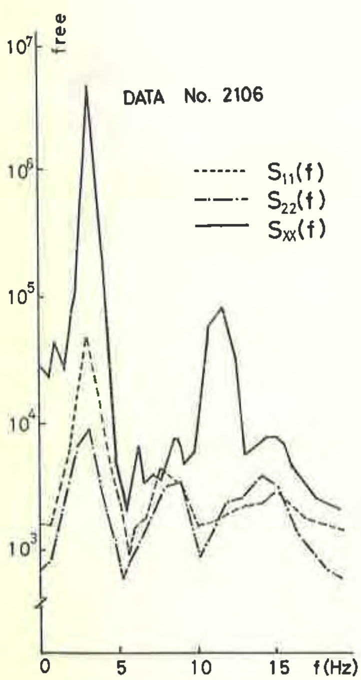


Fig. 4 Power Spectrum of the Input and Output Wave of Two Inputs Model ($S_{11}(f)$, $S_{22}(f)$ for Inputs, $S_{xx}(f)$ for Output)

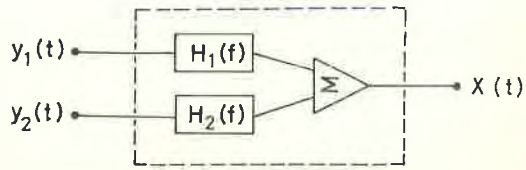
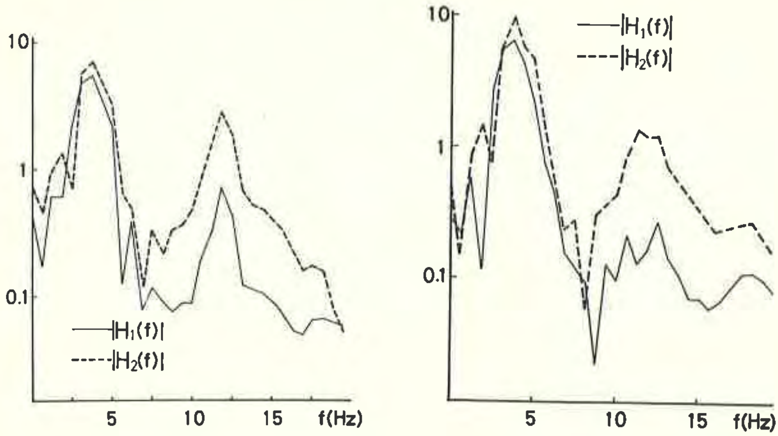


Fig. 5 Flowchart of Two-Inputs System



(i) DATA No. 2013 (ii) DATA No. 2106

Fig. 6 Gain Characteristics of Frequency Response Function as for Respective Input End

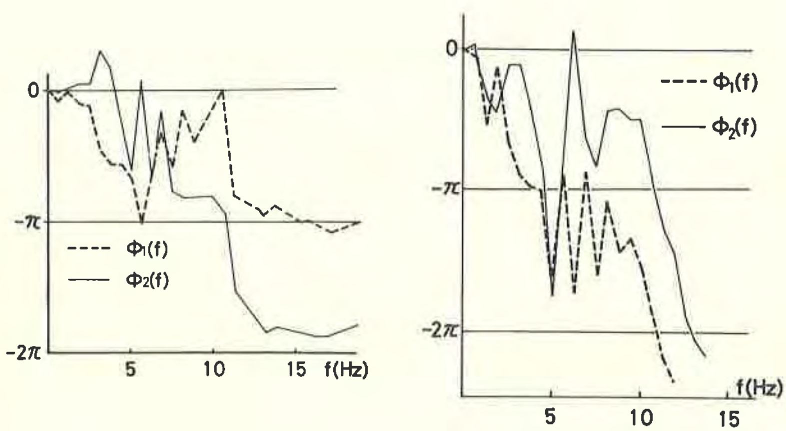


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Fig. 7 Phase Shift of Frequency Response Function as for Respective Input End

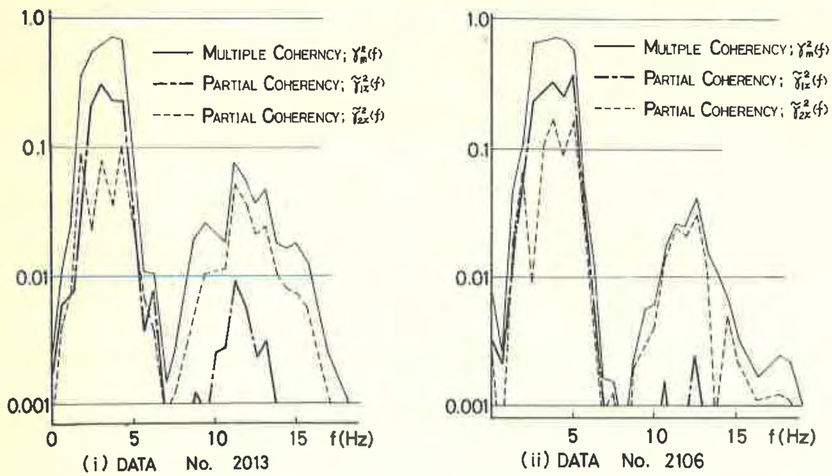

Fig. 8 Estimated Values of Multiple Coherence Function and Partial Coherence Functions for Two Inputs Piping Model

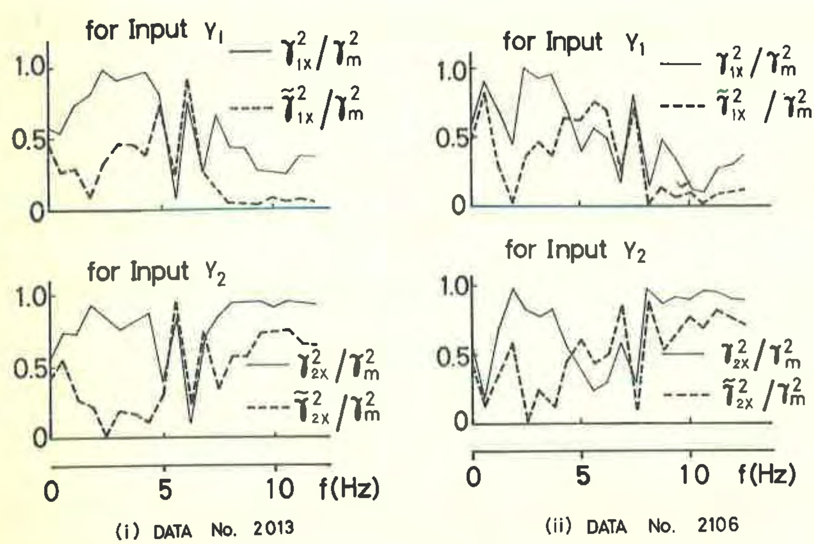

Fig. 9 The Ratio of the $\frac{\gamma_{ix}^2}{\gamma_m^2}$ and $\frac{\tilde{\gamma}_{ix}^2}{\gamma_m^2}$ as for Respective Input End