Stresses in the Bend Transitions of Steam Generator U-Tubes

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Abstract

By using formulas in existing literature, hoop stresses in the smallest radius U-bend transition region of the 22.23 mm [7/8 inch] OD x 1.27 mm [50 mil] uniform thickness U-tube were calculated. Prototypical temperature and pressure conditions were considered for tube ovalities in the -6 percent to + 6 percent range. The locations of the maximum tensile hoop stresses were found. With adjustments for local thickness variation, it was possible to pinpoint eventual axial crack initiation. The effects of residual stresses were not evaluated.

1. Introduction

The stress levels at the U-bend transitions of steam generator tubes remain the promising factor that could possibly clarify puzzling occurrences of tube cracking in those regions. This paper shows by the classical approach how the maximum hoop stresses may be computed in relation to locations in the U-bend and the ovality of the tube cross-section. Prior treatment of this problem was done by Cloud [1].

Prototypical pressures and temperatures were imposed on a standard U-bend geometry for the 22.23 mm [7/8 inch] OD tube with a 1.27 mm [50 mil] uniform thickness. The results were adjusted to account for local variation in tube thickness and the locations of the maximum tensile hoop stresses were finally indicated. Residual stresses that arise from tube manufacturing processes were not included.

2. Hoop Stresses Due to Thermal Conditions

The thermal conditions in the U-tube that produce hoop stresses in the U-bend cause (1) a relative movement between the hot leg (HL) and the cold (CL) and (2) a temperature gradient across the tube wall. Figure 1 shows the U-bend portion from the apex to the theoretical transition point (TTP), with the proper nomenclature indicated. In figure 2, the distance from the tube support plate (TSP) centerline to the TTP is designated "L" and the moment at any point (R, θ) due to the imposed loads is termed "M_s". For simplicity, it is assumed that the tube rotation at the TSP is negligible. Then, from figure 2:

$$M_s = FR\,(1 - \cos\theta) + PL\,(1 + R/L \sin\theta) - M_0 \qquad \text{eq. (1)}$$

The total strain energy in the U-bend may be given by:

$$U = 2\left(\frac{F^2L}{2AE}\right) + \int_0^L \frac{(Px - Mo)^2}{2EI}dx + \int_0^S \frac{M_o{}^2}{2EIK} ds + \int_0^L \frac{(Px - Mo + 2FR)^2}{2EI}dx \qquad \text{eq. (2)}$$

where K = Karman's rigidity factor $= \dfrac{1 + 12\lambda^2}{10 + 12\lambda^2}$ and $\lambda = \dfrac{Rt}{r^2}$.

The boundary conditions are $\partial U/\partial F = \Delta$, $\partial U/\partial P = C$ and $\partial U/\partial M_o = 0$, with Δ = net differential thermal movement of the tube legs and C = radial clearance of the tube in the TSP hole. The resulting simultaneous equations are:

$$\begin{bmatrix} 2(C1) & (C2) & -2(C3) \\ (C4) & (C1) & -(C1) \\ (C1) & (C3) & -(C3) \end{bmatrix} \begin{bmatrix} PL \\ FR \\ M_o \end{bmatrix} = \begin{bmatrix} \dfrac{2EIK\Delta}{\pi R^2} \\ \dfrac{EIKC}{\pi RL} \\ 0 \end{bmatrix} \qquad \text{eq. (3)}$$

The coefficients (C1) . . . (C4) are listed in table 1. The location are the maximum "M_s" was found by differentiating eq. (1):

$$\tan \Theta = -\frac{PL}{FR} \qquad \text{eq. (4)}$$

and the distance d (= R sin Θ) from the cold leg TTR was computed.

The hoop stresses around the tube section were calculated by Wahl's formula [2]:

$$\sigma_{h1} = \frac{M_s r}{I_o}\left(\frac{18\lambda \cos 2\phi}{1 + 12\lambda^2}\right) \qquad \text{eq. (5)}$$

where ø = angular location of the point on the tube circumference based on the conventions in figure 3. If the apparent moment is set as:

$$M' = M_s\left(\frac{18\lambda \cos 2\phi}{1 + 12\lambda^2}\right) \qquad \text{eq. (5a)}$$

then eq. (5) becomes transformed into the standard form:

$$\sigma_{h1} = \frac{M' r}{I_b} \qquad \text{eq. (5b)}$$

and it remains to find "r" and "I_b" for each case of ovality. Noting from eq. (5) that σ_{h1} varies as cos 2ø, its value is positive when ø = 0° or 180° (at the intrados or extrados), zero when ø = 45° or 135° and negative when ø = 90° or 270° (at the flanks).

When the tube is ovalized, "r_o" and "r_1" will vary around the perimeter. To determine these radii, it is necessary to find first the major and minor axes of the equivalent ellipse. Tube ovality "e" is usually expressed as "percent" and may be defined as the difference of the axis "b" at the flank and the axis "a" the entrados/extrados divided by the corresponding radius for the circular tube. Thus,

$$e = \frac{b-a}{r} \qquad \text{eq. (7)}$$

When b > a, the ovality is said to be positive or normal and the tube section is elongated along the flanks. When $b < a$, a negative or reversed ovality results and the tube section is elongated along the intrados/extrados line. See figure 3. By assuming that the tube perimeter stays constant before and after ovalization, a new relationship is derived:

$$a^2 + b^2 = 2r^2 \qquad \text{eq. (8)}$$

Eqs. (7) and (8) will determine "a" and "b", given "e" and "r". In the calculation, "r" is taken to be the mean radius of the original tube section so that "a" and "b" reflect the mean axes of the ovalized tube.

The ellipse for the tube is written in polar form, for convenience:

$$r = \frac{ab}{a^2 \sin^2 \phi + b^2 \cos^2 \phi} \qquad \text{eq. (9)}$$

where ϕ = the circumferential angle reckoned from the intrados. The values of "r" from eq. (9) are corrected by half the tube wall thickness in order to obtain "r_o" and "r_1" for use in eq. (5b).

The moment of inertia may be found by the formula:

$$I_b = a^3 C_b \qquad \text{eq. (10)}$$

where $C_b = [1/3k_o^2]\,[(1-k_o^2)\,F - (1 - 2k_o^2)\,E]$. "F" and "E" are complete elliptic integrals of the first and second kinds. The axis ratio, k_o, is always less than 1.

The through-wall thermal gradient also gives rise to hoop stresses in each tube cross-section of the U-bend. These may be calculated using Roark's formula [3]:

$$\sigma_{h2} = \pm \frac{E\alpha\Delta t}{2(1-\nu)\ln \frac{r_o}{r_1}} \left[1 - 2r_s^2 \frac{\ln\left(\frac{r_o}{r_1}\right)}{(r_o^2 - r_1^2)}\right] \qquad \text{(eq. 11)}$$

In the usual U-tube, t_1 (inside) is greater than t_o (outside). The hoop stress in eq. (11) for the outside fiber is positive with $r_s = r_1$. For the inside fiber, it is negative with $r_s = r_o$. The values of "r_o" and "r_1" have been previously found by eqs. (7), (8) and (9) for the cases of ovality considered in this work. Using the parameter values in table II, σ_{h2} was then calculated.

3. Hoop Stresses Due to Pressure Loading

Harvey [4] gives the hoop stress due to pressure loading on both sides of a thick cylinder as follows:

$$\sigma_{h3} = \frac{r_1^2 P_1 - r_o^2 P_o + (P_1 - P_o)\, r_o^2 r_1^2 / r^2}{(r_o^2 - r_1^2)} \qquad \text{eq. (12a)}$$

On the outside surface, $r = r_o$ and eq. (12a) becomes

$$\sigma_{h3} = \frac{r_1^2 P_1 - r_o^2 P_o + \Delta P\, r_1^2}{r_o^2 - r_1^2} \qquad \text{eq. (12b)}$$

When $r_1 = r_o - t$ is substituted and the higher powers of "t" are discarded, eq. (12b) is rewritten as:

$$\sigma_{h3} = (r_o/t - 2)\,\Delta P - P_o \qquad \text{eq. (12c)}$$

Similarly, on the inside surface ($r = r_i$), eq. (12a) becomes

$$\sigma_{h3} = \frac{r_i^2 p_i - r_o^2 p_o + \Delta p\, r_o^2}{r_o^2 - r_i^2} \qquad \text{eq. (13a)}$$

Again, substituting $r_o = r_i + t$ in eq. (13a),

$$\sigma_{h3} = (\frac{r_i}{t} + 1)\ \Delta P \qquad \text{eq. (13b)}$$

Eqs. (12c) and (13b) are used to find the basic hoop stresses due to pressure loading on the tube.

Two corrections for σ_{h3} are computed. The first relates to the toroidal effect due to the U-bend and the second accounts for the effect of local bending in the tube wall due to ovalization. The toroidal correction is given by:

$$K_1 = \frac{1 - (\frac{r}{2R}) \cos \phi}{1 - (\frac{r}{R}) \cos \phi} \qquad \text{eq. (14)}$$

where r = "r_o" for the outside fiber or "r_i" for the inside fiber
R = bend radius
ϕ = circumferential location around the tube as reckoned from the intrados

Based on figure 3, the local bending moment at any section of the internally pressurized tube in accordance with Timoshenko [5] may be expressed as:

$$M_b = \frac{p}{2} [\frac{a^2 b^2}{a^2 \sin^2 \phi + b^2 \cos^2 \phi} - \frac{b^2 C_a + a^2 C_b}{E}] \qquad \text{eq. (15)}$$

All variables in eq. (15) have been previously defined except

$$C_a = [1/3k_o^2]\ [(1 + k_o^2)\ E - (1 - k_o^2)\ F]$$

The bending stress is found by $\sigma_b = \pm \frac{6M_b}{t^2}$.

If M_b at 0° is in the direction shown in figure 3, then σ_b is negative on the outside fiber (OF) and positive on the inside fiber (IF).

4. Results of Calculations

Using the evaluation parameters on table II, eqs. (3) were solved: PL = 12.5643 Nm [111.1848 in-lbs], FR = 28.7474 Nm [254.3929 in-lbs] and M_o = 42.8516 Nm [379.2042 in-lbs]. The maximum "M_s" was located at Θ = 165.225° and d = 14.1681 mm [0.5578 in]. Table III shows the moments at each of the sections selected. Since their relative ratios are shown, it is only necessary to compute the stresses for one section and the one with the maximum moment was chosen. Table IV is a sample of the hoop stress values computed for every ϕ = 22.5° in the case of e = +2 percent. It is to be noted that the effects of tube thickness variation is included: 1.545 mm [60 mil] wall at the intrados and 1.168 mm [46 mil] wall at the extrados. The maximum tensile hoop stresses and their locations for the seven cases of ovality are set forth in table V.

5. Conclusion

The maximum tensile hoop stress in the 22.23 mm [7/8 inch] OD x 1.27 mm [50 mil] tube U-bend due to normal operating pressures and temperatures is located on the outside fiber for ovalities greater than or equal to zero and on the inside fiber for ovalities less than zero. This stress is at the intrados when the wall thickness is uniform but shifts to the extrados when the actual thickness variation is considered. Solely on this basis, the propensity to axial tube cracking will be strong at the extrados OF or extrados IF depending on the tube ovality.

6. References

[1] CLOUD, R. L., "Stresses in Bent Inconel Tubes", EPRI U-Bend Workshop, Denver, Colorado, August 20-21, 1980.

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[3] ROARK, R. J. and YOUNG W. C., Formulas for Stress and Strain, Fifth Edition, McGraw-Hill Book Company, 1975.

[4] HARVEY J.F., Theory and Design of Modern Pressure Vessels, Second Edition, Van Nostrand Reinhold Company, New York, 1974.

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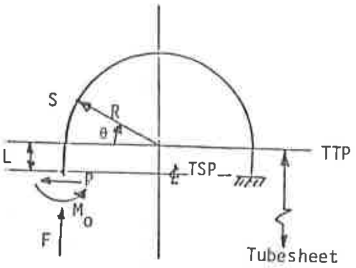

Figure 2. Diagram of the U-bend

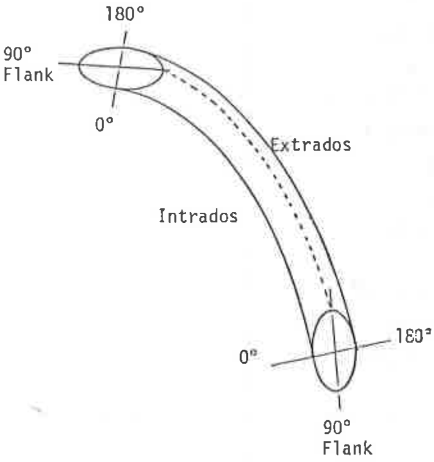

Figure 1. Nomenclature of the U-bend

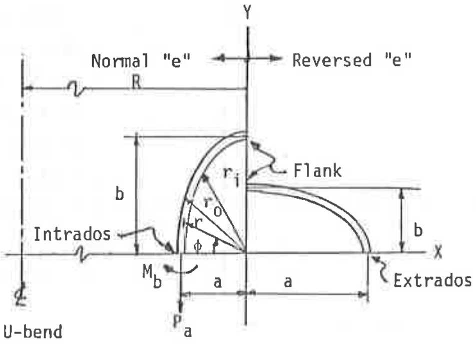

Figure 3. Cross-Section of Tube

TABLE I. COEFFICIENTS FOR EQUATIONS (3)

$$(C1) = 1 + \frac{KL}{\pi R} + \frac{2R}{\pi L}$$

$$(C2) = 3 + \frac{8KL}{\pi R} + \frac{4KIL_T}{\pi R^3 A}$$

$$(C3) = 1 + \frac{2KL}{\pi R}$$

$$(C4) = 1 + \frac{2KL}{3\pi R} + \frac{4R}{\pi L} + \frac{R^2}{2L^2}$$

TABLE II - PARAMETER VALUES FOR COMPUTATION

R	= 55.56 mm [2.1875 in]
L	= 92.08 mm [3.6250 in]
L_T	= 9072 mm [357.16 in]
Δ	= 1.25 mm [0.04906 in]
C	= 0.20 mm [0.008 in]
r	= 10.48 mm [0.4125 in]
t	= 1.27 mm [0.050 in]
A	= 83.6 mm^2 [0.1295907 in^2]
I	= 4589 mm^4 [0.0110253 in^4]
t_{HL}	= 292°C [557.5°F]
t_{CL}	= 282°C [540.0°F]
P_1	= 10006 Pa [2250 psi]
P_0	= 4047 Pa [910 psi]
E	= 130.74 MPa [29.4×10^6 psi]
α	= 14.13×10^{-6} mm/mm/°C]
	[7.85×10^{-6} in/in/°F]
λ	= 0.64279
k	= 0.398322
ν	= 0.3

TABLE III. BENDING MOMENTS DUE TO THERMAL MOVEMENT OF U-TUBE LEGS

SECTION	d INCH	θ DEGREES	M_s IN-LBS	RATIO M_s/M_s (MAX)
1	0	180.000	240.7664	0.965129
2	0.25	173.438	246.7675	0.989185
3	0.50	166.787	249.3677	0.999608
4*	0.5578	165.225	249.4655	1.000000
5	0.75	159.949	249.3508	0.995532
6	1.00	152.797	243.3005	0.975287

*Section 4 was used in the maximum hoop stress computations.

TABLE IV TOTAL HOOP STRESS FOR e = +2%

LOCATION		HOOP STRESS (PSI)				σ_{hT}	
SURFACE	ANGLE	σ_{h1}	σ_{h2}	σ_{h3}	σ_b	CONSTANT "t"	CORRECTED "t"
OF	0°	19609	3162	9007	5757	37535	-
	22.5°	13866	3163	8957	4117	30103	-
	45°	0	3164	8803	76	12043	-
	67.5°	-13866	3165	8559	-4088	-6230	-
	90°	-19609	3165	8252	-5850	-14042	-
	112.5°	-13866	3165	7922	-4088	-6867	-
	135°	0	3164	7629	76	10869	-
	157.5°	13866	3163	7428	4117	28574	-
	180°	19609	3162	7355	5757	35883	39548
IF	0°	17346	-3432	12839	-5757	20996	18296
	22.5°	12265	-3431	12770	-4117	17487	-
	45°	0	-3430	12562	-76	9056	-
	67.5°	-12265	-3429	12239	4088	633	-
	90°	-17346	-3429	11842	5850	-3083	-
	112.5°	-12265	-3429	11428	4088	-178	-
	135°	0	-3430	11071	-76	-7565	-
	157.5°	12265	-3431	10829	-4117	15546	-
	180°	17346	-3432	10742	-5757	18899	19998

TABLE V. LOCATION OF MAXIMUM TENSILE HOOP STRESSES

OVALITY (%)	MAXIMUM TENSILE HOOP STRESS (PSI)/LOCATION (DEGREES)			
	UNIFORM THICKNESS		ACTUAL THICKNESS	
	IF	OF	IF	OF
6	8737/0°	49152/0°	9735/0°	53391/180°
4	14904/0°	43268/0°	14039/0°	46383/180°
2	20996/0°	37535/0°	19998/180°	39548/180°
0	26584/0°	31539/0°	26566/180°	32439/180°
-2	32194/0°	25572/0°	33155/180°	28363/180°
-4	38289/0°	19847/0°	40286/180°	19507/0°
-6	44467/0°	13981/0°	47518/180°	14105/0°