Transactions, SMiRT-23
Manchester, United Kingdom - August 10-14, 2015
Division IX, Paper ID 769 (include assigned division number from I to X)

ADVANCED MONTE CARLO SIMULATION FOR THE ROBUST DESIGN AND MAINTENANCE OF FATIGUE-PRONE METALLIC COMPONENTS

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ABSTRACT

The design of inspection schedules is a complex optimization problem that requires the reliability to be assessed. The solution to this problem can be found balancing the costs associated to inspection/repair activities against the benefits related to the faultless operation of the infrastructure. The optimization aims at minimizing the total cost, obtained as the combination of maintenance and failure costs, by tuning some design parameters, such as the number, time and quality of inspections. The optimization problem is formulated as a time-dependent reliability-based optimization problem. The solution to this problem represents a real technological challenge, as the reliability assessment is a computationally intensive task, which may take up to few days on last generation processors. In this paper, an efficient and generally applicable numerical technique, which is capable of producing a solution in a very short amount of time (less than 1 hour), is proposed. The efficiency and accuracy of the proposed technique is shown by means of a literature example involving a fatigue-prone weld in a bridge girder.

INTRODUCTION

Preventive maintenance practice can be extremely cost-effective, as inspection and repair activities may prevent loss of serviceability or even partial collapse. However, making decisions as to whether and when performing inspections is a very complex task, especially on real-scale engineering systems.

The realistic quantification of costs associated to inspections, repair and failure, requires explicit consideration of the unavoidable uncertainties arising from the damage-propagation process, and from the inspection and repair activities. Uncertainties may come from the inherent variability of the damage-propagation process or from the lack of available knowledge about the process itself. Probability distributions are used as a comprehensive mean of representing the uncertainty. Within the generalized uncertainty model, the design of maintenance activities is an optimisation problem that requires the assessment of reliability box. Reliability-based optimisation methods, as described e.g. in Jensen (2002), are invoked to solve this problem. The number, times, and quality of inspections are the design variables of the optimisation, whereas the total cost and the failure probability are the objective and constraint functions, respectively. For the formulation and solution to time-dependent reliability-based optimisation problems see e.g. Valdebenito and Schuëller (2010).

In this paper, we propose a general methodology for the efficient solution of the time-variant reliability-based maintenance optimisation problem, where the reliability is assessed by means of p-boxes. No restrictions in terms of number of inspections and number of uncertain parameters can be found. The methodology is derived from the concept of forced Monte Carlo simulation, used to evaluate the availability of plants, see e.g. Zio and Marseguerra (2002), and it is exploited to efficiently assess the time-variant reliability conditional to the inspection outcomes, requiring only the execution of computationally inexpensive functions. Genetic Algorithms are used to drive the global optimiSation, as the cost and constraint functions are stochastic and therefore, no information about the derivatives can be efficiently used to converge to the minimum. This comes with quite some more numerical burden, which, however, can be significantly alleviated resorting to code parallelisation.

FORMULATION OF THE OPTIMISATION PROBLEM

The optimisation problem is formulated by defining objective, constraint functions and design variables. In order to define the objective function we look at the costs. Two main different classes of costs are considered:

- costs due to inspection and repair, $C_I + C_R$;
- costs of failure, C_F.
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It is assumed that manufacturing costs are deterministic as they are linked to construction and usage of materials. Note that the costs of repair and failure are obtained as expected values, $E[\cdot]$ as they come from the estimation of repair and failure probability respectively.

Inspection And Repair Costs

The cost due to inspections depends on inspection quality and time vector, $\boldsymbol{t}^{\mathrm{insp}}$, and can be expressed as

$$C_I(q, \boldsymbol{t}^{\mathrm{insp}}) = c_I \; q \; \eta(\boldsymbol{t}^{\mathrm{insp}}); \tag{1}$$

where, c_I, is a fixed unit cost, and q quantifies the quality of inspections. In the Eq.(1) the function

$$\eta(t) = \frac{1}{(1+s)^t}; \tag{2}$$

is a discount function of discount rate, s. Costs due to repair are present only if repair takes place. Hence they depend on the probability of repair, $p_R(q,t)$. The probability of repair, is linked to the probability of detecting the damage within an inspection, POD, which in turns depends on the inspection quality, q, the level of damage, $D(t)$, and the technique used to spot the flaw. For example, in fatigue-prone metallic components, non-destructive inspection (NDI) techniques can be used. NDI techniques have an associated probability of detection, which can be modelled as

$$POD(t) = (1 - p_0)\;(1 - \mathrm{e}^{\,q\;(f_1 - f_2\;D(t))}); \tag{3}$$

where, p_0, is the probability of not detecting a large crack, while f_1 and f_2 are constants that depend on the specific NDI technique.
The cost of repair can thus be expressed as

$$E[C_R(q, \boldsymbol{t}^{\mathrm{insp}})] = c_R \; E[p_R(q, \boldsymbol{t}^{\mathrm{insp}})] \; \eta(\boldsymbol{t}^{\mathrm{insp}}); \tag{4}$$

where, c_R is a fixed unit cost, and p_R is, a function of the quality and inspection times. Note that in some cases, the unit cost of repair can be very small or sometimes negligible compared to the cost of inspection.

Failure Costs

The cost of failure depends on the quality, q, as well as on the state of damage, $\mathrm{D(t)}$. Here, failure cost can be expressed as

$$E[C_F(q, \boldsymbol{t}^{\mathrm{insp}}, t)] = c_F \; E[p_F(q, \boldsymbol{t}^{\mathrm{insp}}, t)]; \tag{5}$$

where, c_F is a fixed unit cost associated with failure, partial collapse, or unavailability, and $\mathrm{p}_F(q, \boldsymbol{t}^{\mathrm{insp}}, t)$ is the failure probability. Note that the failure probability is a function of both the inspection time vector, and of the time when the reliability is assessed, t.

Total Costs

The total cost of maintenance is

$$E[C_M] = C_I + E[C_R] + E[C_F]. \tag{6}$$

Constraint And Cost Function

The maintenance problem can be generally formulated as a constrained optimization problem, where the constraint is the limit state level of safety that the system has to comply with. Here, the following formulation of the optimization problem is considered

$$\begin{aligned} \underset{q\in\mathbb{R}^+,\ \boldsymbol{t}^{\text{insp}}\in[0,T_M]^N}{\text{minimize}} \qquad & E[C_M(q,\boldsymbol{t}^{\text{insp}},t)] \\ \text{subject to} \qquad & p_F(q,\boldsymbol{t}^{\text{insp}},t) \leq p_F^{\text{critic}}; \end{aligned} \tag{7}$$

where, p_F^{critic} is determined by a prescribed limit state level. The problem of Eq.(7) is addressed using the penalty function

$$\psi(c) = 1 - e^{\alpha|\min(0,c)|}; \tag{8}$$

which is in turn a function of the following constraint

$$c = -\log_{10}\left(p_F(q,\boldsymbol{t}^{\text{insp}},t)\right) + \log_{10}\left(p_F^{\text{critic}}\right); \tag{9}$$

where, the constraint is satisfied if $c \geq 0$. The problem of Eq.(7) can, thus, be reformulated into an equivalent unconstrained problem, as

$$\underset{q\in\mathbb{R}^+,\ \boldsymbol{t}^{\text{insp}}\in[0,T_M]^N}{\text{minimize}} \quad E[C_M(q,\boldsymbol{t}^{\text{insp}},t)] + g\ \psi(c); \tag{10}$$

where, g is a penalty factor, which value can be chosen knowing approximately the order of magnitude of the objective function.

EFFICIENT ESTIMATION OF RELIABILITY BY MEANS OF ADANCED MONTE CARLO

The present numerical strategy is derived from the concept of forced MC simulation described in Zio and Marseguerra (2002), which is based on the computation of weights, w, to account for the probability of detection. After inspection the failure probability is conditional to the inspection outcomes thus, the task of computing this conditional probability at any point in time may be rather significant. The proposed strategy makes the computation of conditional probabilities very efficient, as only one full MC simulation is needed to compute the reliability for any inspection and observation time.

Time-Variant Reliability And Failure Probability Assessment

As the system, $S(t)$, evolves in time, so does the level of damage, $D(t)$, of specific components. The damage can be expressed as a function, $D = D(\boldsymbol{\theta}, t)$, of some input parameters $\boldsymbol{\theta}$, that can be used to quantify the level of damage.
In this paper, damage manifests as fatigue, and the computational model is governed by the Paris-Erdogan's law \cite{paris1963critical}. Thus, the input vector, $\boldsymbol{\theta}$, includes the initial crack length, the effective stress range, the shape ratio and any other coefficients of the law.

The time-variant reliability is obtained via definition of a critical threshold of damage, d^{thres}, as

$$r(t) = 1 - P[D(\boldsymbol{\theta}, t) \geq d^{\text{thres}}]; \tag{11}$$

where, both d^{thres} and $D(\boldsymbol{\theta}, t)$ are probabilistic quantities with associated distribution functions.
The time-variant reliability is obtained as

$$r(t) = 1 - \int_{D(\boldsymbol{\theta},t)\geq d^{\text{thres}}} h(\boldsymbol{\theta}) \, \mathrm{d}\boldsymbol{\theta}; \tag{12}$$

where, $h(\boldsymbol{\theta})$ is the joint density function of the random vector $\boldsymbol{\theta}$. By means of the MC method the time-variant failure probability, $p_F(t)$, can be calculated as

$$p_F(t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \mathcal{I}(\boldsymbol{\theta}, t) \; h(\boldsymbol{\theta}) \; \mathrm{d}\boldsymbol{\theta}; \tag{13}$$

where, $\mathcal{I}(\boldsymbol{\theta}, t) \in \{0, 1\}$, is the indicator function, which is 1 only if $D(\boldsymbol{\theta}, t) \geq d^{\text{thres}}$.

Formulation Of The Maintenance Problem

The maintenance problem requires the evaluation of the reliability, $r(t)$. With no inspections (N=0), the reliability can be assessed as in Eq.(12) and estimated as in Eq.(13). When inspections are considered, i.e. N>0, the reliability of the system is conditional to the inspection outcomes.
The reliability is a function of two different times: the observation time $t \in [0, T_M]$, when the reliability is assessed, and the inspection time, t^{insp}, when the inspections are performed.
In general, i.e. when N>1, the reliability is given by the conditional probability,

$$r(t) = 1 - P\left[D(t) \geq D^{\text{thres}} \mid t_1^{\text{insp}} < ... < t_N^{\text{insp}} < t\right]. \tag{14}$$

The optimal inspection time can be found between the two following limiting cases: If inspections are performed too early, $\boldsymbol{t}^{\text{insp}} << T_M$, nearly no damage will be found, and hence no repair will take place. As a consequence the reliability will only be improved marginally, or even not improved at all. On the other side, if the inspections are done too late, i.e. $\boldsymbol{t}^{\text{insp}} \simeq T_M$, the probability of detection would be large (because directly related to the level of damage), but it is likely that the system will have already failed, thus, the inspection will not be cost-effective.

Assumptions

In order to better illustrate the procedure and without restricting the generality of the approach two simple assumptions are made:

1. Any inspection is followed by only two outcomes: either the flaw is detected or not. If a flaw is detected repair takes place, which action is assumed to be perfect, i.e. after repair the level of damage is $D(t > t^{\text{insp}}) = 0$.

2. Only preventive maintenance is considered. If the critical threshold is exceeded at the time of inspection, the component is not repaired. That is, if failure has occurred, repair actions will not take place.

Classification of events and total failure probability

In order to calculate the reliability as defined in Eq.(14), mutually exclusive events are classified. Among all, four main events are identified:

1. the failure event, $F_i = [D(t_i^{\text{insp}}) > D^{\text{thres}}]$, at the time of the i-th inspection,
2. the failure event, $F_t = [D(t) > D^{\text{thres}}]$, at the observation time t with no inspections,
3. the repair/detection event, $R_i = [\delta(t_i^{\text{insp}}) = 1]$, at the time of the i-th inspection,
4. the event $\overline{R_i} = [\delta(t_i^{\text{insp}}) = 0]$, i.e. the event of non-repair/non-detection;

where, $\delta(t) \in \{0, 1\}$ is a binary random variable to characterize the outcome of inspections, which mass function is

$$\delta(t) = \begin{cases} 1 \text{ (success)} & \lambda_D(t) \\ 0 \text{ (failure)} & 1 - \lambda_D(t) \end{cases} ; \tag{15}$$

where, $\lambda_D(t) = POD(D_{\boldsymbol{\theta}}(t))$ is the likelihood of detecting the flaw during inspection. The probability of the failure event conditional to the inspection outcomes can be obtained as

$$P[F] = \sum_{j=1}^{N+1} P\left[F_{N-j+2} \cap (\overline{F_{N-j+1}})\right] \prod_{k=0}^{N-j+1} P\left[(\overline{R_k})\right] ; \tag{16}$$

where, for simplicity, the summation has been extended to N+1 to include the failure event at the observation time $F_t = F_{N+1}$.

Efficient Estimation Of Failure Probability Conditional To Inspection With MC Simulation

The expression for the estimation, by means of MC simulation, of the failure probability conditional to the inspection outcomes, is derived from Eq.(16) as

$$\hat{p}_F(t) = \frac{1}{N_S} \sum_{s=1}^{N_S} \left(\sum_{i=1}^{N} \left(1 - \left(1 - \delta_i^{\{s\}}\right) \mathcal{I}(\boldsymbol{\theta}^{\{s\}}, t_i)\, \mathcal{I}(\boldsymbol{\theta}^{\{s\}}, t_{i+1})\right) \prod_{k=0}^{i-1} \left(1 - \delta_k^{\{s\}}\right) + \mathcal{I}(\boldsymbol{\theta}^{\{s\}}, t) \prod_{k=1}^{N} \left(1 - \delta_k^{\{s\}}\right)\right); \tag{17}$$

where, $t_i = t_i^{\text{insp}}$, $\delta_i = \delta(t_i^{\text{insp}})$, and the indicator function is

$$\mathcal{I}(\boldsymbol{\theta}, t) = \begin{cases} 1, & \text{if} \quad D_{\boldsymbol{\theta}}(t) \geq D^{\text{thres}} \\ 0, & \text{otherwise} \end{cases} . \tag{18}$$

In Eq.(17) the failure probability is estimated with a finite number of samples, N_S, on the generated samples $\boldsymbol{\theta}^{\{s\}}$. Eq.(17) requires a full simulation for every inspection time vector, $\boldsymbol{t}^{\text{insp}}$, as new inspection outcomes have to be generated. This limitation can be removed noting that the expected value of $\delta(t)$ equals the probability of detection, as

$$w(t) = E[\delta(t)] = \lim_{N_S \to \infty} \frac{1}{N_S} \sum_{s=1}^{N_S} \delta(t)^{\{s\}} = E\left[POD\left(D(\boldsymbol{\theta}, t)\right)\right]. \tag{19}$$

The expected value, $E[\delta(t)]$ is indeed the weight to look for in order to improve the efficiency of the estimation. Using this weight, it is no longer necessary to generate new outcomes any time a different inspection time is picked. This will make it possible to use results coming from the same MC simulation for every selected inspection and observation time. The failure probability is estimated via the expression

$$\hat{p}_F(t) = \frac{1}{N_S} \sum_{s=1}^{N_S} \left(\sum_{i=1}^{N} \mathcal{I}(\boldsymbol{\theta}^{\{s\}}, t_i) \left(1 - (1 - \hat{w}_i)\, \mathcal{I}(\boldsymbol{\theta}^{\{s\}}, t_{i+1})\right) \prod_{k=0}^{i-1} (1 - \hat{w}_k) + \mathcal{I}(\boldsymbol{\theta}^{\{s\}}, t) \prod_{k=1}^{N} (1 - \hat{w}_k) \right); \tag{20}$$

where, this time the inspection outcomes are only linked to the original sample set $\boldsymbol{\theta}^{\{s\}}$. An equivalent, alternative and more efficient way to compute Eq.(20), which can be vectorised in a computer code, is by means of

$$\hat{p}_F(t) = \frac{1}{N_S} \sum_{s=1}^{N_S} \left(\mathcal{I}(\boldsymbol{\theta}^{\{s\}}, t) \ \prod_{j=1}^{N} 1 - \hat{w}_j \ \left(1 - \mathcal{I}(\boldsymbol{\theta}^{\{s\}}, t_j)\right) \right). \tag{21}$$

Total And Partial Probability Of Repair

By means of the direct approach, the probability of repair can be calculated for the $\mathrm{i\text{-}th}$ inspection as

$$p_{R_i} = \lim_{N_S \to \infty} \frac{1}{N_S} \sum_{s=1}^{N_S} \prod_{k=1}^{i} \delta_k^{\{s\}}. \tag{22}$$

By inverting the order of summation and product sequence, the probability of repair for the $\mathrm{i\text{-}th}$ inspection can be calculated as

$$p_{R_i} = \lim_{N_S \to \infty} \prod_{k=1}^{i} \frac{1}{N_S} \sum_{s=1}^{N_S} \delta_k^{\{s\}} = \prod_{k=1}^{i} w_k. \tag{23}$$

The estimator for the total probability of repair, after all inspections have been performed, is obtained using a finite sample set of N_S samples as

$$\hat{p}_R = \prod_{k=1}^{N} \hat{w}_k. \tag{24}$$

EXAMPLE OF A FATIGUE PRONE METALLIC COMPONENT

In this section a welded connection prone to fatigue, taken from Lukić and Cremona (2001), is under consideration. Welds are particularly weak to damage accumulation as imperfections may form and grow under cyclic loading. The analysed weld with associated defect is shown in Fig.(1). From the picture it

can be appreciated that the crack propagates from the weld toe towards the bottom mining the integrity of the structural connection.
The crack propagation phenomenon is modelled using the Paris-Erdogan law, by means of

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\,(\Delta K)^m\,; \tag{25}$$

where, a is the crack length, N is the number of loading cycles, ΔK is the stress intensity factor range, and C and m are two material parameters. In this study no threshold is put on the ΔK values, thus, it is assumed that all loading cycles count on damage. Eq.(25) is solved implicitly by means of numerical integration. The stress intensity factor range is a function of the crack length

$$\Delta K = Y(a)\ M(a)\ \Delta S\ \sqrt{\pi a}; \tag{26}$$

where, $Y(a)$ is the stress intensity correction factor, and $M(a)$ is the stress concentration factor, which are also function of the crack shape, a/c, the flange thickness, b, the flange width w, the weld height, h, and the weld angle, ϕ, see e.g. Figure (1).

Table 1: Mission time and annual frequency of cyclic load

Name	Value	Unit	Description
ν	4.5	$[10^6\ \mathrm{cycles/year}]$	Annual number of cycles
T_M	50	[years]	Mission time

In order to solve Eq.(25) a mission time, T_M, of the structural component has to be selected, as shown in Table(1). From the rain-flaw histogram (loading analysis) it is possible to derive the annual number of cycles, ν, which multiplied by the mission time, in years, provides the total number of cycles. Thus, the number of cycles at the target life time is $N_{\max} = \nu * T_M$.

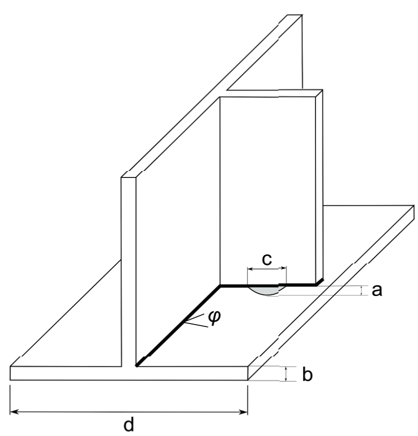

Figure 1: Illustration of the welded joint detail and crack geometry

Uncertainties

Input quantities are modelled with continuous probability distribution. The parental distributions are provided in Lukić and Cremona (2001). The uncertainty model is constructed from the nominal values of mean and standard deviation of Table(2). Note that the material quantities C and m show a strong negative Gaussian correlation, which is usually close to -0.99.

Table 2: Probability distributions for the input quantities of the example

Random Variable		Mean, S. Deviation and C. of Variation		
θ_i	Distribution	$\mathrm{E}[\theta_i]$	$\mathrm{Std}[\theta_i]$	$\mathrm{CoV}[\theta_i](\%)$
a_0 $[\mathrm{mm}]$	*Lognormal*	0.125	0.045	36.0
ΔS $[\mathrm{MPa}]$	*Gamma*	7.800	0.100	1.3
C^* $[10^{-13}\ \mathrm{mm/cycles}]$	*Lognormal*	2.500	0.923	36.9
m^*	*Normal*	3.000	0.040	1.3
a_f $[\mathrm{mm}]$	*Gumbel*	0.500	0.050	10.0
$\mathrm{a/c}$	*Lognormal*	0.400	0.160	40.0
d $[\mathrm{mm}]$	*Gamma*	812.0	8.100	10.0
b $[\mathrm{mm}]$	*Gamma*	31.60	3.200	10.1
h $[\mathrm{mm}]$	*Gamma*	8.400	0.700	8.3
ϕ $[\mathrm{deg}]$	*Gamma*	35.00	2.000	5.7
*Correlation: $\rho(\ln C,\ m) = -0.99$				

Definition Of Constraint And Cost Function

It is recalled that the inspection time vector, $\boldsymbol{t}^{\mathrm{insp}}$, and the inspection quality, q, are the design variables of the problem. The input unit costs of maintenance and failure are reported in Tab. (3).

Table 3: Unit costs used for the numerical example

Cost	Value	Description
c_I	10^4	Unit inspection cost
c_R	10^2	Unit repair cost
c_F	10^5	Unit failure cost
s	0.01	Discount rate

Fig. (2) shows how the total cost displays on graph as a function of the inspection time, as the inspection quality is fixed as $q = 1$. Although the cost of failure may appear quite smooth, every point of the curve is obtained by estimating the failure probability and therefore it is associated with an estimation error. In this example, the achieved estimation error on the failure probability is quite small, i.e. $\leq 10^{-6}$.

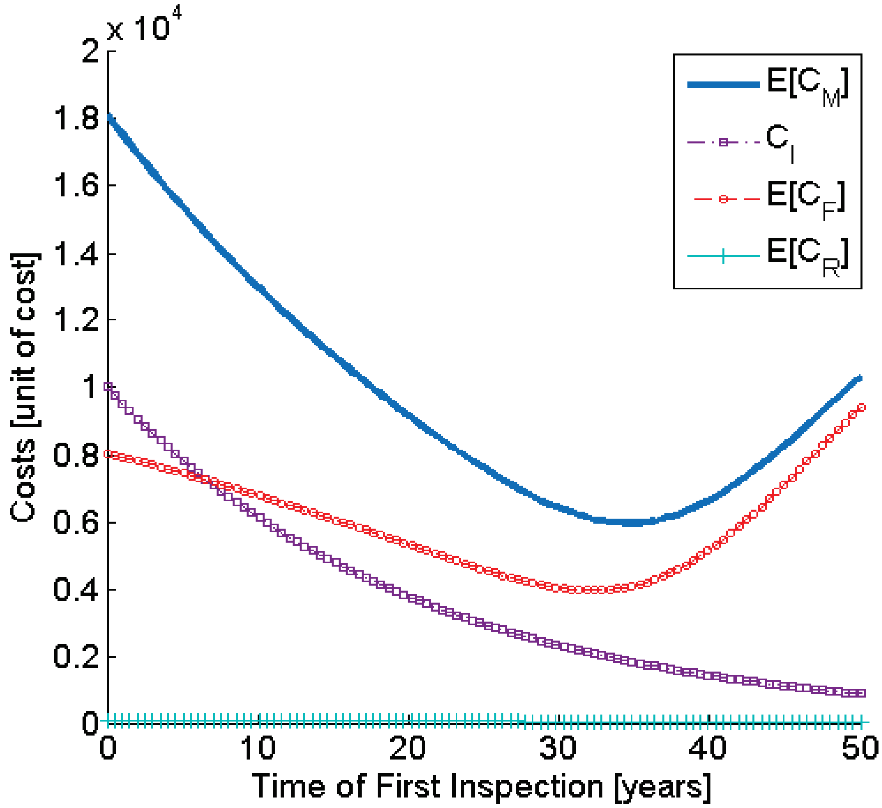

Figure 2: Curves of costs obtained with q=1

The minimum is located at $\mathrm{t}^{\mathrm{insp}*} = 36$ years, as shown in Fig. (2), where it is also possible to appreciate that the expected cost of repair, $E[C_R]$, is smaller compared to the others and contributes a little to the solution of optimum inspection time.

Solution To The Constrained Optimisation With N Inspections

The solution to constrained optimisation problem is found using Genetic Algorithm (GA). The choice for GA is justified by the stochastic nature of the objective function and constraint, which do not display as a smooth function.
As a consequence, information about the derivatives on both objective and constraint cannot be exploited. Results from the constrained optimization as the number of inspections increases are obtained fixing a critical threshold of failure probability to $\mathrm{p}_F^{\mathrm{critic}} = 10^{-3}$. The variance of the failure probability estimator obtained for this problem is $\mathrm{Var}[p_F] \simeq 5 \cdot 10^{-7}$, thus, it is very accurate.

Table 4: Minimum cost, failure probability and corresponding optimum inspection times

N	p_F	$\min C_M$	q*	$\boldsymbol{t}^{\mathrm{insp}}$ [years]
5	$5\ 10^{-1}$	$4.3\ 10^5$	0.6	$\{25.6, 25.7, 25.8, 26.3, 27.0\}$
6	10^{-3}	$4.4\ 10^6$	87.6	$\{0.7, 7.4, 13.9, 21.1, 31.6, 45.4\}$
7	10^{-3}	$4.2\ 10^6$	73.0	$\{0.7, 4.6, 11.4, 17.0, 24.8, 33.9, 49.0\}$

9	10^{-3}	3.9 10^6	51.2	$\{0.5, 4.0, 8.3, 11.4, 16.0, 20.8, 27.0, 32.7, 41.8\}$
11	10^{-3}	4.0 10^6	43.2	$\{0.7, 1.7, 5.5, 10.3, 13.9, 16.8, 20.9, 25.7, 31.6, 38.7, 45.4\}$
15	10^{-3}	4.1 10^6	33.5	$\{0.02, 1.5, 4.5, 7.3, 10.5, 12.3, 15.4, 18.8, 22.4, 24.8, 29.2, 33.9, 41.8, 48.3, 50\}$
19	10^{-3}	4.0 10^6	25.4	$\{0.02, 1.0, 1.9, 4.6, 7.7, 9.3, 11.3, 13.9, 16.3, 18.0, 20.4, 22.9, 25.6, 29.1, 32.2, 37.0, 40.2, 47.8, 50\}$

From Table 4 it is possible to see how at least 6 inspections are needed to meet the required reliability constraint, as with 5 inspections the failure probability is $p_F = 0.5 > 10^3$. It is, thus, necessary to perform a number of inspections N>5 in order to find a suitable solution. Note that as the number of inspection increases the minimum total cost does not necessarily have to increase, as the quality of the single inspection may decrease. About the inspection times, despite they come from the solutions of a stochastic optimization, they results quite evenly distributed, although, there seems to be preferable to run more inspections within the first 10 years.

CONCLUSION

In summary, A general and efficient methodology for the optimisation of maintenance schedules has been presented. The maintenance of a system is a challenging engineering task, where the estimation of costs requires the consideration of heterogeneous uncertainties arising from the damage propagation process and from the inspection/repair activities. The methodology exploits the concept of forced MC simulation to maximize the efficiency, without reducing the accuracy. The strategy makes use of the computation of weights that emulate the outcomes of the preventive maintenance. The strategy is capable of assessing the reliability of the system at any given time and for any number of inspections, performing only one full reliability analysis, which results can subsequently be used, with nearly no additional computational cost.
In the numerical example, the proposed methodology permitted the calculation of the minimum number of inspections necessary to comply with the given reliability constraint. Moreover, by solving the stochastic optimization problem, it was possible to identify the inspection times that minimise the total cost while satisfying the imposed constraint. For the specific example under consideration, it was found that at least 5 inspections are needed, and that there is no significant difference in terms of total maintenance costs as the number of inspections increases.

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