THEORY OF PUMP INDUCED PULSATING COOLANT PRESSURE IN PRESSURIZED WATER REACTORS

L.E. PENZES

Combustion Division, Combustion Engineering, Inc., Windsor, Connecticut 06095, U.S.A.

SUMMARY

The theory of pump induced pulsating pressure distributions in a PWR coolant annulus is developed. The calculated pressure distribution can then be applied to predict the dynamic responses of the reactor internals. The mathematical analysis is formulated in accordance with the linearized Navier-Stokes equations by assuming a compressible, inviscid liquid. These equations are combined to form a single equation in terms of the unknown pressure distribution. The boundary conditions are two concentric rigid walls in the radial direction and any combination of closed, open, and piston-spring supported end conditions in the axial direction. The pulsating pump pressure which induces the pressure fluctuation in the annulus is prescribed at a small opening of the outer cylindrical wall (pump inlet of the reactor). This formulation of the problem determines a time dependent, mixed boundary value problem. For this case, the separation of variables technique cannot be applied and a solution of this problem would require a lengthy procedure utilizing numerical analysis techniques.

Instead of using these methods, an approximate solution is obtained by introducing the concept of time dependent body force in the governing differential equations. With this conceptual substitution for the actual loading, the time dependent, mixed boundary value problem can be represented as a forced vibration problem with homogeneous boundary conditions. This problem can then be solved by the method of normal modes.

First, the free vibration modes are obtained by applying the previously defined homogeneous boundary conditions. Using the modes of free vibration, the forced vibration solution with the fictitious body force is then developed. The body force, which replaces the pulsating surface pressure conceptually, is placed close to the pulsating pressure surface at the pump inlet. It acts over a volume having the same surface geometry and small radial thickness.

The numerical value of the body force, P, is determined by using an initial estimated volumetric body force, P_{est}, which is then adjusted after the numerical calculation is completed to satisfy the ratio of the prescribed boundary condition, q_0, and calculated surface pressure, q_{cal}, obtained from P_{est}.

Numerical examples are provided which give the pressure distribution in the axial and circumferential directions of the annulus for various configurations of one and/or several pumps. Using different radial thicknesses for the volume of the body force, a convergence study was also conducted with excellent numerical results.

1. Introduction

Flow induced vibrations have caused many problems in reactor operation. These problems have necessitated analysis of reactor intervals to investigate vibratory stresses and stabilities. Some possible causes of the vibratory stresses in reactor internals are turbulent flow, von Karman Vortices, flow induced self excited vibration, etc. [References 1, 2, 3, and 4]. Because of the general complexity of this problem, we restrict our analysis to the pressure fluctuations of the reactor coolant caused by pump pulsation. This pulsation phenomenon is common in most turbomachinary, and may be associated with the pump speed or its higher harmonics or the blade passing frequency and the corresponding higher harmonics. Although all of these exciting frequencies are deterministic, the magnitude of the pressure fluctuations in the pumps becomes a very complicated problem and usually can only be resolved by an experimental determination. Pump induced vibration has had an interesting history, References [5, 6], and no satisfactory theoretical solution for the magnitude of the pressure fluctuation has yet appeared. Considering these facts and the complexity of the flow induced vibration problem in general, our analysis will be restricted in nature by some simplifying physical assumptions.

It will be assumed that the pump induced pressure fluctuation is known at the inlet of the reactor with the corresponding exciting frequency. In reality, the fluctuating pressure from the pump travels from the pump through the pipe line into the reactor and in some cases through the steam generator. With these conditions, the problem could be a traveling wave phenomenon and a general analytical solution of this case would be a formidable task. Because of the conceptual similarities of this problem with longitudinal oscillations and the Pogo effect of missiles [6, 5], we have assumed that the gross effect of the many traveling waves can be represented (approximately) as a standing wave phenomenon. In the design analysis, our primary interest is the structural response which can be computed easier by using the solution of pressure standing waves [6, 5].

2. Mathematical Formulation of the Problem

For the purpose of clarity, the mathematical derivation is first presented in summary form.

The mathematical analysis is formulated according to linearized Navier-Stokes equations, assuming a compressible, inviscid fluid and the concept of time dependent body forces as forcing functions. In this case, the time dependent mixed valued boundary value problem is replaced approximately by a forced vibration problem with homogeneous boundary conditions. The solution for the unknown pressure distribution is developed by using the method of normal modes in two steps: (1) Determine the free vibrations in terms of the liquid frequencies and pressure mode shapes; and (2) The pressure distribution is obtained utilizing the free vibration solution at the solution of forced vibration equation.

2.1 Basic Derivations

The differential equations for forced vibration were formulated by use of Berry and Reissner paper [7]. Modifying those equations by the time dependent body force technique according to Schlichting and Lamb [8 and 9] it follows that, the equations of equilibrium become:

$$\rho_o \frac{\partial \overline{v}}{\partial t} = \overline{P} - \text{grad } p \qquad (1)$$

where: ρ_o = Reference density (constant); $\overline{v}$ = Velocity vector; $\overline{P}$ = Volumetric forcing function vector; p = Total pressure; and t = time.

The equation of compressibility is expressed in the terms of fluctuating pressure as:

$$q = p - P_o = c_o^2 (\rho - \rho_o) \qquad (2)$$

where: q = Fluctuating (dynamic) pressure; P_o = Reference pressure; c_o = Reference sound velocity.

The equation of continuity follows from Berry and Reissner's eq. (3.5) as: [7]

$$\frac{\partial \rho}{\partial t} + \rho_o \text{ div } \overline{V} = 0 \qquad (3)$$

Differentiating eq. (3) with respect to time, t, substituting density, ρ, from eq. (2) and the expressions for velocity components from eq. (1) into the resulting equation, the governing differential equation for our problem becomes: (in terms of the unknown fluctuating pressure):

$$\nabla^2 q = \frac{1}{c_o^2} \frac{\partial^2 q}{\partial t^2} + \text{div} \cdot \overline{P} \qquad (4)$$

An induced forced vibration in the liquid is represented by eq. (4). Eliminating the forcing function, $\overline{P}$, eq. (4) is reduced to the well-known wave equation in terms of fluctuating pressure as:

$$\nabla^2 q = \frac{1}{c_o^2} \frac{\partial^2 q}{\partial t^2} \qquad (5)$$

Figure 1 illustrates our mathematical model with the necessary notation. This idealized hydrodynamic model may give us a basic understanding of the relationships between the pulsating body force and the periodic pressure fluctuations at any location of the liquid including the inner and outer surfaces with the pump inlet of the reactor.

The solution of this problem can now be obtained by using the method of normal modes.

2.2 Solution of Free Vibration

If we assume that the solution of wave eq. (5) is represented by the phenomenon of standing waves, then the separation of variables technique can be applied. In this case, the pressure is expressed as:

$$q = q_1(t) q_2(z) q_3(\theta) q_r(r) \qquad (6)$$

The substitution of eq. (6) into wave eq. (5) yields a set of ordinary differential equations. The solution of these equations can be easily obtained as:

$$q_1 = Ae^{i\omega t},\ q_2 = B_1 \sin\alpha z + B_2 \cos\alpha z,\ q_3 = D_2 \cos m\theta,\ q_4 = C_3 J_m(\lambda r) + C_4 Y_m(\lambda r) \qquad (7)$$

where: A, B_1, B_2, C_3, C_4, D_2 = Arbitrary constants; z, θ, r = Axial, circumferential, and radial coordinates; ω, α, m = Separation constants; λ = Expression in the terms of separation constants; and J_m, Y_m = Bessel functions of the First and Second Kind.

Because of physical reasons, the circumferential wave numbers must be chosen to be integers; otherwise, the separation constant, m, becomes 0, 1, 2, etc. Specifying the boundary conditions, the separation and integration constants can be determined up to a single set of arbitrary constants.

In the axial direction, the following types of boundary conditions are considered: open, closed, and piston-spring supported. Any combination of these configurations can be applied. Since our problem is reduced to a single variable, the dynamic pressure, q, the boundary conditions must be formulated accordingly. The following equations can be easily derived from eqs. (1) and (2) by ignoring the forcing function or References [9, 10, and 11] should be consulted.

Boundary Condition of Open End

This end condition can be easily visualized in terms of pressure as:

$$q_2\Big|_{z=z_1} = 0 \qquad \text{where } z_1 = \text{Axial coordinate.} \tag{8}$$

Boundary Condition of Closed End

This boundary condition may be obtained from axial components, of the equilibrium eq. (1) after ignoring the forcing function $\bar{P}$, free vibration, and considering that the value of the acceleration is zero at the rigid wall, then:

$$\frac{\partial q_2}{\partial z}\Big|_{z=z_1} = 0 \qquad \text{where } z_1 = \text{Axial coordinate.} \tag{9}$$

Boundary Condition of Piston-Spring Supported

Figure 2 illustrates the general idea. The equilibrium of forces at the ideal piston leads to the following relation:

$$q_2\Big|_{z=z_1} = \frac{k}{\rho_o \omega_{nms}^2 A} \frac{\partial q_2}{\partial z}\Big|_{z=z_1} \tag{10}$$

where: ω_{nms} = Natural frequency of the liquid; A = Area of piston; k = Fictitious spring constant.

The fictitious spring constant may be determined from the appropriate test results.

Boundary Condition in the Radial Direction

If we consider rigid walls in the radial direction, then the boundary condition can be formulated in a manner similar to eq. (9) as:

$$\frac{\partial q_4}{\partial r}\Big|_{r=r_1} = 0 \qquad \text{where } r_1 = \text{Radial coordinate.} \tag{11}$$

For specific application to our model, we consider the following boundary conditions in the axial direction: open-open, closed-open, and closed-closed ends. Substitution of the expression for q_2 of Eq. (7) into expressions (8) and (9) leads to the appropriate mode shapes in the axial direction. The results of the derivations are summarized in Table 1.

The radial mode shape, q_4, can be obtained assuming that rigid walls are enclosed in the radial direction. In this case, boundary condition 11 may be expressed in more detail as:

$$\frac{\partial q_4}{\partial r}\Big|_{r=a_1} = 0 \qquad \frac{\partial q_4}{\partial r}\Big|_{r=a_2} = 0 \tag{12}$$

where: a_1 = Inner radius of the annulus; and a_2 = Outer radius of the annulus.

Substituting function q_4 of expression (7) into boundary condition 12, the unknown frequencies, ω_{nms}, and the ratio, η_{nms}, of the arbitrary constants can be obtained as:

$$J'_m(\lambda_{nms}a_2) = \eta_{nms} Y'_m(\lambda_{nms}a_2) \tag{13}$$

where: $$\eta_{nms} = -\frac{J'_m(\lambda_{nms}a_1)}{Y'_m(\lambda_{nms}a_1)}, \quad ' = \frac{\partial}{\partial r}, \quad \lambda_{nms} = \left[\frac{\omega^2_{nms}}{c^2_o} - \alpha^2\right]^{1/2} \tag{14}$$

Eq. (13) is the frequency equation and once the end conditions in the axial direction are specified and the constants α_n are determined from Table 1, the unknown frequencies, ω_{nms}, can be computed.

The computation of the piston-spring supported end condition is similar to the other discussed cases and it will not be given here in detail.

As in the solution for free vibration, the pressure function q, as eigenfunctions, can now be obtained by combining eqs. (7) and Table 1 as:

$$q = \sum_{n=j}^{\infty} \sum_{m=0}^{\infty} \sum_{s=0}^{\infty} A_{nms} e^{i\omega_{nms}t} Q_{nms} \tag{15}$$

where: $$Q_{nms} = (B_1 \sin\alpha_n z + B_2 \cos\alpha_n z)[J_m(\lambda_{nms}r) + \eta_{nms} Y_m(\lambda_{nms}r)]\cos m\theta \; ; \tag{16}$$

j, B_1, B_2 = 0 or 1 from Table 1; A_{nms} = Arbitrary constants.

2.3 Solution of Forced Vibration

The time dependent mixed boundary value problem will be solved approximately by using the concept of forced vibration. The pulsating surface pressure will be replaced by a pulsating body force which will be described mathematically by a thin volume element. The body force is placed near to the location of the pulsating surface pressure.

Assuming a periodic pump pulsation, the driving force of the liquid is described according to the radial component P_r as:

$$P_r = P\cos\omega_p t \tag{17}$$

where: ω_p = Driving frequency; $P = P_o$ = Constant at Volume V_1, which is bounded as:

$$a^* \le r \le r; \quad z_1 \le z \le z_2; \quad -\theta_1 \le \theta \le \theta_1$$

$P = 0$, everywhere else in the liquid, this volume is designated as V_2 and the other components $P_\theta = P_z = 0$ everywhere in the liquid.

Substituting force component 17 into differential eq. (4), the equation of forced vibration is defined for our problem as:

$$\nabla^2 q = \frac{1}{c^2_o}\frac{\partial^2 q}{\partial t^2} + \frac{P}{r}\cos\omega_p t \tag{18}$$

where the condition of P is determined by eq. (17).

Let us assume, that the pulsating pump pressure is primarily steady state and, therefore, the transient part (homogeneous solution) will be damped out after some time. In this case, our interest is only the particular solution of eq. (18). The particular solution can be expressed by the method of normal modes as:

$$q_p = \sum_{n=j}^{\infty} \sum_{m=0}^{\infty} \sum_{s=0}^{\infty} C_{nms} Q_{nms} \cos\omega_p t \tag{19}$$

The substitution of relation (19) into differential eq. (4) and using the properties of normal modes, yields the driving function P in terms of normal modes as:

$$\frac{1}{c_o^2} \sum_{n=j}^{\infty} \sum_{m=0}^{\infty} \sum_{s=0}^{\infty} C_{nms}(\omega_p^2 - \omega_{nms}^2)\, rQ_{nms} = \begin{cases} P_o \text{ at } V_1 \\ 0 \text{ at } V_2 \end{cases} \tag{20}$$

Multiplying both sides of expression (20) by Q_{nms}, integrating both sides over the total volume domain and utilizing the orthogonal properties of functions, Q_{nms}, the unknown coefficients, C_{nms}, are:

$$C_{nms} = \frac{c_o^2 \int_{a^*}^{a_2} \int_{-\theta_1}^{\theta_1} \int_{z_1}^{z_2} P_o Q_{nms}\, dr d\theta dz}{(\omega_p^2 - \omega_{nms}^2) \int_{a_1}^{a_2} \int_{o}^{2\pi} \int_{o}^{\ell} r\, Q_{nms}^2\, dr d\theta dz} \tag{21}$$

where a^* = radius determining the thickness of the body force.
After computing the values of C_{nms}, the pressure distribution of the liquid can be computed at any point of the annulus or at the boundaries according to eqs. (15) and (21) as:

$$q_p = (\cos\omega_p t) \sum_{n=j}^{\infty} \sum_{m=0}^{\infty} \sum_{s=0}^{\infty} C_{nms} Q_{nms} \tag{22}$$

where: j, B_1 and B_2 are defined by Table 1; and η_{nms} is determined by relation (14).

The volumetric body force, P_o, must also satisfy the prescribed pump fluctuating pressure, q_o, at the pump inlet of the reactor as:

$$q_o = \sum_{n=j}^{\infty} \sum_{m=0}^{\infty} \sum_{s=0}^{\infty} C_{nms} Q_{nms} \Big|_{\substack{r=a_2 \\ z=\frac{z_1+z_2}{2} \\ \theta=\theta^*}} \tag{23}$$

where: θ^* = Circumferential coordinate of the pump inlet; q_o = Fluctuating surface pressure at the pump inlet.

Because of the nature of eq. (1), the body force, P, cannot be expressed explicitly in terms of the pressure, q. For the computation of coefficients C_{nms}, the calculation starts with an estimated value of P_o. The calculated pressure, q_p, will usually not satisfy condition 23; however, the calculated values of C_{nms} have a linear relationship with the estimated value of body force, P_o. The correction of P_o to the desired value can easily be adjusted according to eq. (23).

Because the time dependent mixed valued boundary value problem was solved approximately with the method of forced vibration, the accuracy of the approximation must also be investigated. In formula (21), the radial thickness of the body force is defined as a known small quantity, assuming that the body force occupies a small volume close to the fluctuating surface pressure. Since eq. (23) must also be satisfied, this expression gives us the additional required condition. For practical purposes, these results may also demonstrate that the arbitrary selection of thickness, within a reasonable range, does not change the pressure distribution in the liquid. It will be shown in the numerical computation that the maximum difference in the pressure distribution at the inner surface (core support barrel) is less than one percent.

2.4 Configuration of Several Pumps

So far, we have restricted our analysis to a pressure distribution due to one pump according to eq. (22). Using the concept of superposition, eq. (22) can be generalized for configurations of several pumps as:

$$q_{p,k} = \sum_{k=1}^{N} \sum_{n=j}^{\infty} \sum_{m=0}^{\infty} \sum_{s=0}^{\infty} C^k_{nms} \cos\omega_p t \; Q^k_{nms} \cos(m\theta + \phi_k) \tag{23}$$

where: $$Q^k_{nms} = Q_{nms}(\cos m\theta)^{-1}; \tag{24}$$

N = Number of pumps; ϕ_k = The phase angle of the pump relative to the global coordinate system; C^k_{nms} = Generalized coefficient, designating the different fluctuating inlet pressure for each pump.

The angle, ϕ_k, is determined as a combination of physical location of the pump relative to a global coordinate system and the relative phase angle of the fluctuating pressure between the different pumps.

3. Numerical Examples

Some important numerical examples are presented here. The computation covers the configurations of one, two, and three pumps, with the corresponding boundary conditions of open ends. Due to the similar nature of computation, the boundary conditions of closed-open ends are restricted to a configuration of one pump. A convergence study, considering different radial thicknesses of the body force is also studied.

3.1 Boundary Conditions for Open Ends

The dimensions of the liquid annulus are determined by the following quantities: a_1 = 76 in; a_2 = 86 in; ℓ = 328 in; z_1 = 54 in; z_2 = 94 in; θ_1 = 13.30

The sonic velocity of the liquid is C_o = 3850 ft/sec. With these quantities, the natural frequencies of liquid annulus can be computed from eq. (13), (14), and from Table 1, using the proper values of α_n. Considering boundary conditions for open ends, the frequencies in cps of the annulus are presented in Table 2.

The frequencies of Table 2 are computed corresponding to the lowest set of flexural modes. The frequencies of the so called "shear or thickness modes" can also be determined; however, they are so high, that they have no effect on our present computations. For example, the computed lowest "flexural frequency" in f_{111} = 115 cps from Table 2. The corresponding lowest "shear frequency" is f_{112} = 1172 cps, which corresponds to an order of magnitude increase.

The pressure distribution in the annulus will be computed with the knowledge of the driving frequency (first blade passing frequency of the pump) $\omega_p/2\pi$ = 100 cps and with the known pressure of the pump inlet duct as q_o = 10 psi. Let us start our computation with an assumed volumetric body force $P_o = P_{o,est}$ = 1.45 lb/in^3 and radial thickness of the body force, $a_2 - a^* = a_2 - a_1$ = 10 in.

Using expressions (21) and the computed frequencies from Table 2, the coefficients C_{nms} of the pressure distribution are computed and the results of the computation are shown in Table 3. Once the number of terms in C_{nms} is satisfactory, using the linear relation of the body force P in the expression of C_{nms}, the estimated body force value P_o can be adjusted to the desired value. The actual value of the body force must also satisfy condition (23); however, the volumetric body force, $P_{o,est}$ = 1.45 lb/in^3 would lead to a surface pressure of $q_{o,est}$ = 9.3515 psi. The adjustment can be obtained easily as:

$$P_o = P_{o,est} \frac{q_o}{q_{o,est}} = (1.45) \frac{(10)}{(9.3515)} = 1.55 \text{ lb/in}^3 \quad (25)$$

Substituting the computed values of C_{nms} from Table 3 into expression (20) will give the pressure distribution in the liquid annulus. For the purpose of an easy visualization the pressure distribution in the axial direction is shown in Figure 3.

Figure 4 demonstrates the circumferential pressure distribution. Both figures are drawn in normalized scales.

3.2 Study of Convergence According to the Radial Thickness of the Body Force and the Mode Shape in the Radial Direction

So far, it has been assumed that the radial thickness of the body force is relatively small compared with the average radius of the annulus. In the previous computation, the radial thickness of the body force was taken equal to the thickness of the annulus as: $(a_2 - a^*) = (a_2 - a_1) = 10$ in. A question can arise as to how much the body force thickness influences our solution [Reference 2]. Truncating the series after 8 terms, the pressure distributions are computed at the inner surface of the annulus with the corresponding radial thicknesses 0.1, 1, 5, and 10 inches for the body force. Table 4 presents the results of computations, where the last column shows the ratios of the inner and outer surface pressures. This table clearly indicates that the solution of our problem is uneffected by radial variations in the thickness of the body force, probably as far as the approximation remains between reasonable limits.

3.3 Configurations of Several Pumps

Figure 6 illustrates a schematic view of a configuration of three pumps. Figure 5 shows the radial pressure distribution through the thickness of the liquid. As the computation shows, the pressure distribution is nearly constant through the thickness, which looks reasonable considering that the thickness of the annulus is 10 inches and the average radius of the annulus is 81 inches.

If it is assumed that the fluctuating pressures of the pumps are in phase, then only the circumferential locations of the pump inlets need be considered. For this case, eq. (37) was applied to compute the configurations of two and three pumps in phase. With these considerations, the circumferential components of pressure distributions in normalized scale, are given with the corresponding equations shown in Figure 7 for two pumps and in Figure 8 for three pumps. It may be worthwhile to mention that the axial component of the pressure distributions in unaltered compared with the configuration of one pump (Figure 3).

3.4 Boundary Conditions of Closed-Open Ends

Since the computation is almost identical with the case of open ends, no details will be given here. The difference can be obtained from Table 1, using the proper values of α_n and the corresponding axial mode shapes. Applying the data of the previous case and carrying out the computation, the axial distribution of the pressure is shown in Figure 9. Since for this case, the circumferential distribution of the pressure looks very similar to Figure 4, it is not given here.

Acknowledgement

The author wishes to express his thanks to Ms. Irene Greaves for her excellent work with the typing of the manuscript. Also, a note of appreciation to Dr. E. J. Brown for his many helpful comments and suggestions.

REFERENCES

[1] LESUEUR, C., MILAN, D., PAYAN, G., "Etude des vibrations aleatoires d'origine hydrodynamique de certaines structures du bloc reactor Phenix", Nuclear Engineering and Design 18 (1972) 1972, pp 279-303.

[2] CHEN, Y. N., "Flow-Induced Vibration and Noise in Tube-Bank Heat Exchangers due to von Karman Streets", ASME 67- Vibr-48.

[3] CHEN, S. S., WAMBSGANSS, M. W., "Parallel-Flow-Induced Vibration of Fuel Rods", Nuclear Engineering and Design 18; 1972, pp 253-278.

[4] PAIDOUSSIS, M. P., "An Experimental Study of Vibration of Flexible Cylinders Induced by Nominally Axial Flow", Nuclear Science Engineering 35(1) 1969, pp 127-138.

[5] GOLDMAN, R. L., "Elimination of the Pogo Instability from the Gemini Launch Vehicle", Dynamic Sbability of Structures, Proceedings of an International Conf. held at Northwestern University, October 18-20, 1965, pp 157-166, Pergamon Press.

[6] PENGELLEY, C. D., "Natural Frequency of Longitudinal Modes of Liquid Propellent Space Vehicles", Journal of Spacecraft, Vol. 5, No. 12, pp 1425-31, December 1968.

[7] BERRY, J. G., REISSNER, E., "The Effect of an Internal Compressible Fluid Column on the Breathing Vibrations of a Thin Pressurized Cylindrical Shell", Journal of the Aeronautical Sciences, May 1958, pp 288-294.

[8] SCHLICHTING, H., "Boundary Layer Theory", Pergamon Press, 1955, page 49.

[9] LAMB, S. H., "Hydrodynamics", Sixth Ed. Dover Publication, pp 264, 265, and 268.

[10] RAYLEIGH, J. W. S., "Theory of Sound", Vol. II, Dover Publication.

[11] WOOD, A., "Acoustics", Dover Publication, 1960.

[12] MEIROVITCH, L., "Analytical Methods in Vibrations", The MacMillan Co. Collier-MacMillan Limited, London, 1969.

TABLE 1

Boundary Conditions	Axial Mode Shapes	α_n	n	Remarks
Open-Open	$B_1 \sin\alpha_n z$	$n\pi/\ell$	1, 2, 3...	$B_1 = 1$ $B_2 = 0$
Closed-Open	$B_2 \cos\alpha_n z$	$n\pi/2\ell$	1, 2, 3...	$B_1 = 0$ $B_2 = 1$
Closed-Closed	$B_2 \cos\alpha_n z$	$n\pi/\ell$	0, 1, 2, 3...	$B_1 = 0$ $B_2 = 1$

where: ℓ = Length of annulus; and B_1, B_2 = Arbitrary constants.

TABLE 2

m	f1m1	f2m1	f3m1	f6m1
0	1880			
1	115	168	229	432
2	195	230	278	460
3	280	305	344	
4	370	390	420	

TABLE 3

Values of C_{nm1}

m \ n	1	2	3	6
0	0.0092			
1	9.85	2.67	0.902	-0.199
2	1.405	1.363	0.692	-0.2105
3	0.557			

TABLE 4

$(a_2 - a^*)$	$q(a_2) = q_o$	$q(a_1)$	$q(a_1)/q_o$
10	10	10.01000	1.001000
5	10	10.00993	1.000993
1	10	10.00891	1.000891
0.1	10	10.00873	1.000873

COORDINATE SYSTEM

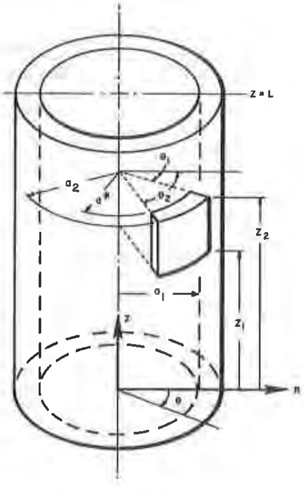


FIG. 1

BOUNDARY CONDITION OF PISTON-SPRING SUPPORTED CASE

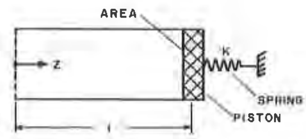


FIG. 2

AXIAL PRESSURE MODE SHAPE

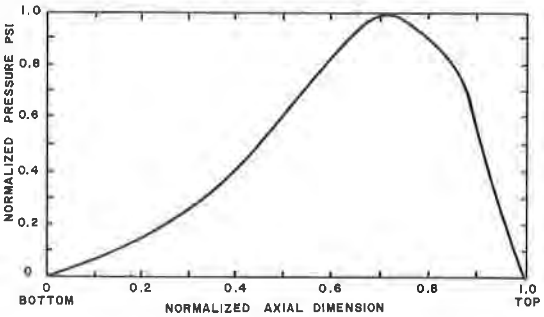
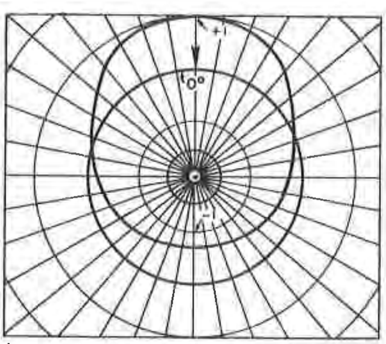


FIG. 3

CIRCUMFERENTIAL VARIATION IN PRESSURE FOR ONE PUMP



$\phi_1(\theta) = 0.74 \cos\theta + 0.18 \cos 2\theta + 0.082 \cos 3\theta$

FIG. 4

RADIAL PRESSURE DISTRIBUTION THROUGH THE LIQUID ANNULAS

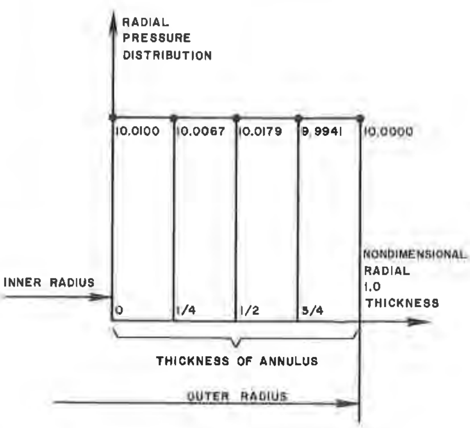


FIG. 5

SCHEMATIC CONFIGURATION OF THREE PUMPS

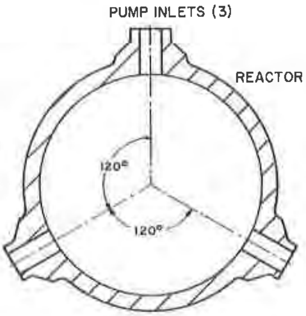
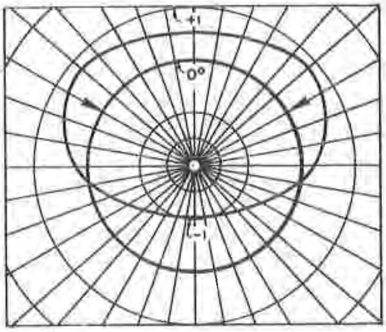


FIG. 6

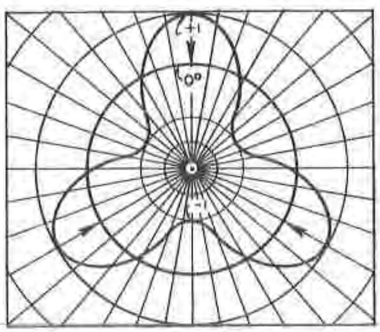
CIRCUMFERENTIAL VARIATION IN PRESSURE
FOR TWO PUMPS - IN PHASE



$\phi_2 = 0.98 \cos\theta - 0.23 \cos 2\theta - 0.22 \cos 3\theta$

FIG. 7

CIRCUMFERENTIAL VARIATION IN PRESSURE
FOR THREE PUMPS



$\phi_3 = \cos 3\theta$

FIG. 8

AXIAL PRESSURE MODE SHAPE

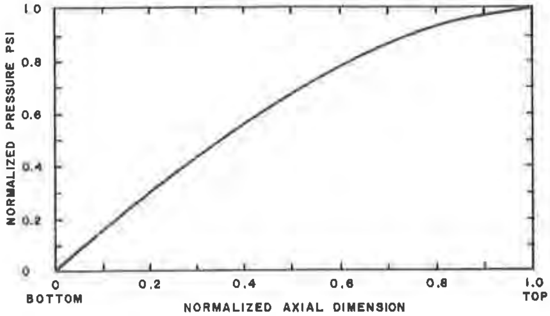


FIG. 9