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# Seismic reliability of slope stability considering spatial variability of soil properties

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ABSTRACT: A comprehensive method for the reliability analysis of layered systems under earthquake loading is proposed in which the inherent variability of earth materials are considered as well as their spatial variations and the uncertainty of the specified earthquake loadings, and the stresses for a given slope are determined through static finite element analysis. Using Monte Carlo simulation, the method evaluates the total failure probability of a slope considering the correlations among potential sliding surfaces and its effect on reliability. The results of case studies of homogeneous and layered earth slopes indicated that spatial variability of soil tends to reduce the estimated probability of sliding failure, that it is significant to consider the correlation between potential failure surfaces in assessing the failure probability and that the calculated probability appears to be not sensitive to the spatial correlation distances within the values reported for actual soils.

## 1 INTRODUCTION

The stability of natural and man-made earth slopes continues to be of great engineering interest, as such slopes are often a part of a constructed system. Under earthquake shaking, the instability of earth slopes may pose special hazard to other constructed facilities; consequently, the seismic safety or reliability of slopes deserves special consideration.

The properties of earth material are highly variable, including the density, modulus, and strength. Moreover, these properties also vary spatially. For these reasons, the assessment of the safety or reliability of a given slope under a specified earthquake load requires the consistent consideration of the inherent variabilities of the earth material properties as well as their spatial variations, and the uncertainty in the specified earthquake loading. Such a stochastic problem has been examined previously(e.g., Suzuki 1990).

A comprehensive analysis method of the reliability of a given slope is proposed. The unique features of the method consist of:

1. the variability of each soil property is characterized as a random field;
2. the stresses corresponding to a given sample field are determined through 2-dimensional finite element analysis;
3. failure of a slope is possible through multiple sliding surfaces defined by the Fellenius method; and
4. the probability of failure of a slope is evaluated through Monte Carlo simulation.

The method is illustrated with a homogeneous system as well as with a 3-layered system. Several conclusions pertinent to the reliability of earth slopes are identified; also, the results of the analyses can be used to appraise the current design of earth slopes.

## 2 APPROACH TO PROBLEMS

For evaluating the influence of the variability of earth material on the safety or reliability of a given slope under a specified earthquake load, soil properties of any given layer such as density, modulus and strength are defined by the mean value and coefficient of variation(c.o.v.) and the corresponding spatial variation is defined in terms of the correlation length. The earthquake loading which is also generally considered highly variable is similarly defined; however, the loading term is assumed to act uniformly on the entire deposit.

### 2.1 Homogeneous earth slope model

First, a homogeneous earth slope with an underlying base layer subjected to an earthquake load is evaluated for its safety against sliding through its most critical circular sliding surface as determined by the Fellenius method; nominal soil properties, and horizontal and vertical seismic intensities are assumed.

Next, the soil properties of the slope and seismic intensities are redefined as randomly varying with given c.o.v. around the nominal values. The slope is then modeled by 2-dimensional finite elements and the spatial variability of soil properties is introduced through a 2-dimensional stochastic field defined by the auto correlation function:

$$(1) \qquad \rho[x,y] = \exp\left[-\frac{x}{l_x} - \frac{y}{l_y}\right]$$

where $x$ and $y$ denote horizontal and vertical separation distances, and $l_x$ and $l_y$ are auto-correlation parameters in horizontal and vertical directions in $x$, $y$ coordinate, respectively.

An earth slope model thus provided with spatially varying random soil properties subjected to a set of randomly sampled seismic intensities are solved by 2-dimensional static finite element analysis to determine the stresses. The resultant stresses along each potential sliding surface as defined by the Fellenius method are used to calculate the resisting force and driving force; from which the performance function for each surface is evaluated; i.e.,

$$(2) \qquad S_{ij} = resisting \quad force - sliding \quad force$$

where $i$ and $j$ stand for the $i$-th surface in the $j$-th sample in the Monte Carlo simulation.

The above procedure is used repeatedly to determine the number of times in which $S_{ij}$ is negative among the total number of trials, from which the probability of failure, $P_{fi}$ for the slope $S_i$ is evaluated.

After obtaining the probability of sliding for each potential surface, the correlation coefficients between arbitrarily selected potential surfaces are calculated to identify the "representative" failure surfaces(Ang & Tang 1984). The reliability of the slope is then determined by the sum of the failure probabilities of the representative failure surfaces.

### 2.2 Layered earth slope model

The important factors influencing the stochastic slope stability were examined in the above homogeneous slope model in highly idealized manner, therefore, the reliability analysis of a layered system which generally appears in practical engineering problems is examined here. For this case, a similar procedure as described above is also applicable.

## 3 APPLICATION TO SPECIFIC SLOPES

### 3.1 Homogeneous Deposit

The model slope is shown in Figure 1 and the soil properties selected for the homogeneous earth slope with an underlying base layer are shown in Table 1.

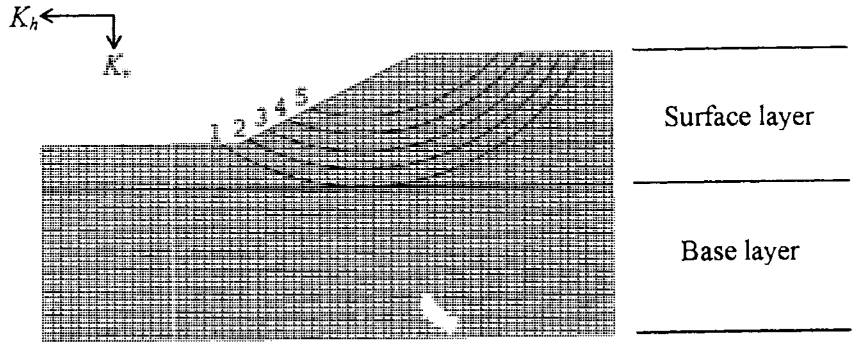

Figure 1 Homogeneous model: numerals denote potential sliding surfaces

Table 1 Soil properties used for homogeneous slope

| | | Unit weight $\gamma_i$ (t/m$^3$) | Young's modulus $E_i$ (t/m$^2$) | Cohesion $C_i$ (t/m$^2$) |
|---|---|---|---|---|
| Surface layer | avrg. | 1.59 | 8,740 | 4.70 |
| | c.o.v. | 0.017 | 0.141 | 0.202 |
| Base layer | avrg. | 1.80 | 47,900 | 10.0 |
| | c.o.v. | — | — | — |

In practice, the auto-correlation parameters, $l_x$ and $l_y$, are not well-defined for a particular soil layer except in rare cases; therefore, these quantities are assumed for clay deposits within the ranges of correlation lengths reported for actual soils(e.g., Matsuo 1984, Vanmarcke 1988).

The earthquake loading was defined with $K_h$=0.15 and $K_v$=0.5 $K_h$ and c.o.v. of 50% for all the case studies. Thirteen case studies were performed which can be classified into three types as follows:

The soil properties of a layer are assumed as:

A. uniform with average values given and spatially perfectly correlated; e.g., deterministic analysis method;

B. stochastically independent random variables defined by average values and c.o.v.s, but completely correlated in space(i.e., both $l_x$ and $l_y$ are infinite);

C. stochastically independent random variables defined by average values and c.o.v.s, and partially correlated in space(i.e., both $l_x$ and $l_y$ are finite).

In evaluating $P_{fi}$s of five potential sliding surfaces determined by Fellenius method, for each trial of Monte Carlo simulation, the stresses of a model defined by 2-dimensional stochastic field were calculated by 2-dimensional FEM. The results obtained from the above three types of calculations are shown in Table 2 for the critical surface 1 only in Figure 1(each case is identified as one of these three types A, B, and C).

Table 2 Results of case studies for the critical surface 1 of homogeneous slope

| Case | $E_1$ ($t/m^2$) | $\gamma_1$ ($t/m^3$) | $C_1$ ($t/m^2$) | $l_x$ (m) | $l_y$ (m) | N (times) | $F_{s1}$ (avrg.) | c.o.v. | $P_{f1}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1-1 | A | A | A | — | — | 1 | 1.164 | — | — |
| 1-2 | B | B | B | ∞ | ∞ | 500 | 1.201 | 0.286 | 0.300 |
| 1-3 | A | C | C | 30.0 | 1.0 | 500 | 1.191 | 0.218 | 0.226 |
| 1-4 | C | A | C | 30.0 | 1.0 | 500 | 1.192 | 0.219 | 0.234 |
| 1-5 | C | C | A | 30.0 | 1.0 | 500 | 1.195 | 0.208 | 0.194 |
| 1-6 | C | C | C | 1.0 | 1.0 | 500 | 1.198 | 0.215 | 0.218 |
| 1-7 | C | C | C | 5.0 | 1.0 | 500 | 1.197 | 0.221 | 0.222 |
| 1-8 | C | C | C | 10.0 | 1.0 | 500 | 1.196 | 0.222 | 0.230 |
| 1-9 | C | C | C | 30.0 | 1.0 | 500 | 1.192 | 0.220 | 0.226 |
| 1-10 | C | C | C | 50.0 | 1.0 | 500 | 1.191 | 0.219 | 0.226 |
| 1-11 | C | C | C | ∞ | 1.0 | 500 | 1.198 | 0.222 | 0.226 |
| 1-12 | C | C | C | 30.0 | 3.0 | 500 | 1.200 | 0.232 | 0.238 |
| 1-13 | C | C | C | 50.0 | 3.0 | 500 | 1.202 | 0.230 | 0.226 |

From Table 2, the slope is conventionally evaluated as having the factor of safety $F_{s1}=1.164$ and the probability of failure itself is unknown; however, if only the variability of soil properties is introduced, $F_{s1}$ is increased by 3% with a probability of sliding of $P_{f1}=0.300$. If the other stochastic parameters are considered, corresponding $F_{s1}$s remain approximately constant among the three types A, B, and C; however, $P_{f1}$s differ slightly with the soil property to which variability is considered.

The probabilities of failure of the other four potential sliding surfaces in Figure 1 were calculated and the correlation coefficients between pairs of surfaces were evaluated, with the auto-correlation parameters shown in Table 3.

Table 3 Correlation coefficients between potential sliding surfaces

| $l_y$ | 1m | | | | | | 3m | | ∞ |
|---|---|---|---|---|---|---|---|---|---|
| $l_x$ | 1m | 5m | 10m | 30m | 50m | ∞ | 30m | 50m | ∞ |
| case | 1-6 | 1-7 | 1-8 | 1-9 | 1-10 | 1-11 | 1-12 | 1-13 | 1-2 |
| 1 vs. 2 | .9230 | .8820 | .8702 | .8579 | .8559 | .8667 | .9219 | .9083 | .9989 |
| 1 vs. 3 | .8959 | .8024 | .7648 | .7282 | .7203 | .7175 | .7956 | .7462 | .9929 |
| 1 vs. 4 | .8462 | .6998 | .6431 | .5895 | .5738 | .5650 | .6493 | .6346 | .9791 |
| 1 vs. 5 | .7328 | .5483 | .4828 | .4230 | .4057 | .3622 | .4879 | .5130 | .9512 |
| 2 vs. 3 | .8859 | .8187 | .8042 | .8011 | .8038 | .8093 | .8982 | .8741 | .9973 |
| 2 vs. 4 | .8144 | .6592 | .6115 | .5862 | .5864 | .5858 | .7362 | .7144 | .9875 |
| 2 vs. 5 | .7301 | .5451 | .4821 | .4300 | .4136 | .3697 | .5492 | .5656 | .9644 |
| 3 vs. 4 | .8317 | .7324 | .7034 | .6859 | .6811 | .7054 | .8680 | .8477 | .9961 |
| 3 vs. 5 | .7089 | .5321 | .4751 | .4208 | .4053 | .4030 | .6700 | .6574 | .9803 |
| 4 vs. 5 | .7084 | .5929 | .5648 | .5290 | .5148 | .5455 | .8199 | .8110 | .9938 |

Table 3 shows that the neighboring pairs of potential failure surfaces have strong correlation, exceeding 0.7 for the auto-correlation parameters of natural deposits.

In this case, the representative sliding surfaces will consist of the surfaces 1, 4, and 5, and the probability of failure of the sliding surface 1 ranges between 0.226 and 0.238 for $l_y$=1~3m and $l_x$=30~50m. Further, the total probability of sliding of this slope which considers all the potential sliding surfaces was found to range between 0.232 and 0.244.

These values may be compared with $P_{f1}$=0.300 for the Case 1-2 in Table 2 which considers the randomness of soil properties but assumes perfect spatial correlation.

When buildings or other facilities are located near the foot or shoulder of a slope, the local failure of the slope through one of the non-critical surfaces may be important.

Therefore, the contribution of the other potential sliding surfaces and their probabilities should be considered.

## 3.2 LAYERED SYSTEM

For a more practical slope stability problem, a three layered deposit is analyzed. The model and soil parameters are shown in Figure 2 and Table 4, respectively. The potential sliding surfaces are also shown in the figure.

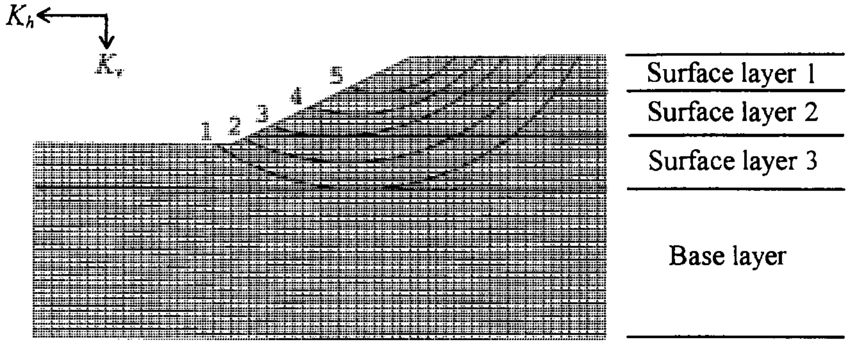

Figure 2 The model of three layered deposit; numerals denote potential sliding surfaces

Table 4 Soil properties of 3-layered slope

| | | Unit weight $\gamma_i$ (t/m$^3$) | Young's modulus $E_i$ (t/m$^2$) | Cohesion $C_i$ (t/m$^2$) |
|---|---|---|---|---|
| Surface layer 1 | avrg. | 1.62 | 3,990 | 3.0 |
| | c.o.v. | 0.029 | 0.235 | 0.289 |
| Surface layer 2 | avrg. | 1.63 | 7,140 | 4.1 |
| | c.o.v. | 0.026 | 0.235 | 0.244 |
| Surface layer 3 | avrg. | 1.63 | 16,060 | 5.5 |
| | c.o.v. | 0.025 | 0.168 | 0.227 |
| Base layer | avrg. | 1.80 | 47,900 | 10.0 |
| | c.o.v. | — | — | — |

Based on the results obtained in the previous case, spatial correlation parameters, $l_y$ and $l_x$, are selected as 1m and 30m, respectively; in addition, the perfectly correlated case is also examined. The number of Monte Carlo simulation is 500. The seismic coefficients are $K_h$=0.15 and $K_v$=0.5 $K_h$ with c.o.v. of 50%, respectively.

The results are shown in Table 5. Using the results obtained through Monte Calro simulation, the correlation coefficients between potential failure surfaces are also evaluated.

Table 5 Results of reliability analyses of three-layered slope

| Sliding Surface | Case 2-1 (A) Deterministic | Case 2-2 (B) Stochastic but $l_y$ & $l_x$=∞ | | | Case 2-3 (C) Stochastic and $l_y=1_m$ & $l_x=30_m$ | | |
|---|---|---|---|---|---|---|---|
| | $F_{si}$ | $F_{si}$ | c.o.v. | $P_{fi}$ | $F_{si}$ | c.o.v. | $P_{fi}$ |
| 1 | 1.260 | 1.301 | 0.288 | 0.208 | 1.286 | 0.224 | 0.142 |
| 2 | 1.379 | 1.420 | 0.274 | 0.104 | 1.438 | 0.220 | 0.058 |
| 3 | 1.419 | 1.456 | 0.322 | 0.126 | 1.451 | 0.218 | 0.034 |
| 4 | 1.903 | 1.954 | 0.319 | 0.022 | 1.915 | 0.228 | — |
| 5 | 2.428 | 2.493 | 0.373 | 0.034 | 2.506 | 0.263 | 0.002 |
| All | — | $P_f$ = 0.288 | | | $P_f$ = 0.174 | | |

In this three-layered system, the sliding surfaces 1, 3, and 5 of Case 2-2, and surfaces 1, 3, 4 and 5 of Case 2-3 were found as the representative surfaces, respectively. On the basis of these representative sliding surfaces(see Ang and Tang 1984 on PNET), total probability of failure of the slope in each case becomes as follows;

Case 2-2; $P_f = P_{f1} + P_{f3} + P_{f5} = 0.368$
Case 2-3; $P_f = P_{f1} + P_{f3} + P_{f4} + P_{f5} = 0.178$

If the unrealistic Case 2-2 is excluded, the results obtained from Case 2-3 compares favorably with the result given in the last line of Table 5, which was obtained directly by Monte Carlo for the union of all potential sliding surfaces.

## 4 DISCUSSION OF RESULTS AND CONCLUSIONS

Soil properties of natural or man-made earth slopes are highly variable; moreover, these variabilities are spatially correlated. Properly, in assessing the safety or reliability of a given slope under earthquake loadings, a slope and its surrounding deposit may be modeled as a random field.

On this basis, the results of the study indicated the following relative to the safety of earth slopes:

1. spatial variability tends to reduce the estimated probability of sliding failure; neglect of the spatial variability will, therefore, over-estimate the real failure probabilities;
2. in assessing the failure probability, the correlation between potential failure surfaces is significant and need to be considered; and
3. the calculated failure probability appears to be not sensitive to the correlation lengths that are within the ranges reported for actual soils.

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