

A BASIS FOR DESIGN ON ALLOWABLE PLASTIC STRAIN

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ABSTRACT

Two basic fracture modes are examined, namely plastic instability in pure shear and pore formation and joining. The necessary conditions for the former are (1) a free surface, (2) a stress gradient, and (3) the material must simulate the ideally plastic body. The latter is likened to the "separation" phenomenon in boundary layer flow. Possible interactions of those modes in the creep range are discussed. Ramifications of the instability mode in plane stress situations are explained.

1. INTRODUCTION

The most basic limitation to the load-bearing capacity of a component in a machine or a structure is the capacity of the material to accommodate geometric discontinuities (notches) while under load. Such discontinuities must be anticipated in real engineering structures. The loading may be static, dynamic, or repeated (fatigue loading).

For the purpose of this paper, chemical environments are ignored, as are degradation processes such as wear. The emphasis is largely on static loading at ambient temperature and at elevated temperatures at which creep becomes a factor. The role of the notch will be examined in terms of events related to plastic straining, rather than elastic stress concentration effects.

2. BASIS OF FRACTURE IN METALS

It appears generally recognized at present that fracture in metals occurs as the result of plastic flow. The theoretical fracture strength of a metal can be achieved only under special conditions, as in whiskers of the order of one micron in diameter. Real metals in the bulk on loading follow the sequence (1) elastic action, (2) plastic action, and (3) fracture. There are too many possibilities of relaxation processes to achieve the theoretical strength in bulk materials.

The second requirement for fracture is the localization of plastic flow, referred to here as plastic instability. The intense localization of flow is a required event to produce conditions necessary for the fracture event. Some viewpoints on approaches to instability phenomena are summarized in Table I.

The most familiar mode to the metallurgist of localization of flow is the phenomenon of "necking" in the tensile test. It becomes evident on analysis of this event that it does not constitute a pure material instability, but depends to a large degree on the geometry of the test bar. There are variations of the cross-sectional areas and/or variations in metallurgical structure along the gage length which give rise to attempts to neck all through the plastic zone. [1] Ultimately one of these attempts is successful and necking proceeds to failure. A gage section with a perfectly uniform cross-section and perfectly uniform structure would be devoid of any basis for necking.

What is needed is an instability criterion which is purely a material instability, rather than one that depends, at least, in part, on geometrical factors.

3. SUGGESTION OF VAN ITERSON

An interesting suggestion was made by a Dutch plastician, F. K. Th. Van Iterson, [2] based on observations on the flow of clay, namely the idea of the plasticized state, or a fourth state of matter between the solid and liquid state. This plasticized state can occur in directions of pure shear, in which direction the planes or surfaces are devoid of extensional strains ($\epsilon = 0$). When such a state is developed, all the strain is concentrated along these directions. Van Iterson gave the expression for the angle between the direction of pure shear and the reference axis as

$$\cos 2 = \frac{1}{3} \left[\frac{(\sigma_1 - \sigma_2) - (\sigma_2 - \sigma_3)}{(\sigma_1 - \sigma_3)} \right] \quad (1)$$

It is now recognized that the directions of the plasticized state are directions of the "characteristics," or slip lines in slip line theory for plane strain and plane stress states. The slip lines (surfaces) have two distinguishing properties: (1) the stress state meets the flow condition for the material, and (2) they are directions of pure shear ($\epsilon = 0$). Thus, plastic flow concentrates in directions of pure shear, and we may term this phenomenon "plastic instability in pure shear."

Thomas [3] treats the growth and decay of discontinuities of velocity over a slip surface in an ideally plastic solid. The surfaces, again, are "characteristic" surfaces, which are treated as wave surfaces. A fixed surface (ξ) is called a surface of stability if every slip discontinuity, (V), over it is damped out. A surface of instability is one in which the discontinuity will not be damped out. A surface of instability is subject to fracture and is called a fracture surface.

4. DISCONTINUITY SURFACES

The phenomenon of plastic instability in pure shear can be interpreted in terms of an interesting concept in plasticity theory, namely a surface of "discontinuity in tangential velocity," [4] illustrated in Figure 1.

The material flows in the plastic zone with a velocity V_0 behind the discontinuity surface, and with velocity V_1 beyond the discontinuity surface, which is a slip line (surface in three dimensions). Continuity demands that there be no discontinuity in the normal velocities ($V_{n0} = V_{n1}$). A discontinuity in tangential velocity, however, is allowable. The tangential velocity discontinuity is $\Delta V_t = V_{t1} - V_{t0}$. When the tangential velocity discontinuity is activated, the deformation is concentrated in pure shear and the material on either side of the

surface becomes rigid. A "β" discontinuity will be reflected at a rigid boundary as an "α" discontinuity, as illustrated in Figure 2.

The tangential velocity discontinuity surface is a strictly theoretical concept in plasticity theory, sometimes used in an apologetic spirit. It is the basis of method of upper bound solution, in which rigid sections are conceived as sliding over discontinuity surfaces. All the deformation is considered to take place along the discontinuity surfaces.

Research in this laboratory, [5] however, has demonstrated conclusively that tangential velocity discontinuities, indeed constitute a basic fracture initiation and propagation mode. The conditions found necessary to activate such surfaces are the following:

- a) There must be a free surface.
- b) There must be a stress gradient.
- c) The material must simulate a perfectly plastic body.

The elastic-ideal plastic and the rigid-ideal plastic response is illustrated in Figure 3. In the ideal plastic state, the material is at the flow condition. The stress state cannot be raised because it quickly relieves the stress increment by rapid flow back to the flow condition. The Lüders-Piobert phenomenon in low carbon steels is an example of this state. The rigid-ideal plastic state is used in slip line solutions for mathematical simplicity.

The concept of activation of surfaces of discontinuities in tangential velocity has been confirmed in metals and alloys in several instances, as listed below:

- a) In heat treated high strength steels. [5b]
- b) As "heat lines" in forging. [6]
- c) Fracture in the side-pressing of cylinders. [7]
- d) Fracture of flat tensile specimens of low carbon steel through the temperature embrittlement range. [8]
- e) Instability as the basis of the Portevin-LeChatelier effect in copper-zinc alloys (serrated stress-strain curves). [9]

5. SECOND BASIC PROCESS OF FRACTURE INITIATION

The activation of surfaces of discontinuity in tangential velocity is seen from the above evidence to be a basic mode of fracture initiation and propagation. The second basic mode is the formation of pores and the joining of pores. Materials which fail by the former mode are termed homogeneous materials, whereas those that fail by the latter mode are termed heterogeneous materials.

It is observed that pores are formed almost universally at substrates, or the interfaces between the matrix and second phase particles such as inclusions. There are several suggestions as to how these "decohesion" events occur. The author prefers to regard the process as analogous to the phenomenon of "separation" in boundary layer flow, illustrated in Figure 4. [10] The matrix flows past the particle in a stream line fashion. At a critical velocity, a turbulent flow condition arises and the flow at positions behind the particle actually reverses direction over the substrate, thus generating pores. The sphere is the geometry most likely to generate pores. The probability of occurrence of decohesions in flowing over a flat surface is negligible.

The significance of a pore is illustrated in Figure 5. The pore immediately furnishes two of the three conditions to activate instability in pure shear (discontinuity surface), namely a stress gradient and a free surface. If the material can achieve the ideal plastic

state, two pores can be joined by an instability, which constitutes the basic step in fracture propagation. Such processes lead to "internal tearing."

There is very likely a temperature above which the material is viscous to such a degree that the material cannot achieve the ideal plastic state. The material now cannot activate discontinuity surfaces and instabilities cannot lead to fracture and contribute to joining of pores. Pores must be joined by receding away of the ligament by plastic flow, equivalent to 100% reduction of area in the tensile test. Materials which cannot activate instabilities, thus, must join pores by this mechanism, which requires considerably more flow.

6. CHARACTERIZATION OF NOTCH DUCTILITY

The role of discontinuity surfaces in the fracture event was disclosed by research in this laboratory on material parameters which give rise to the "strength transition" in toughness, as depicted in Figure 6. The familiar temperature transition in steels is also depicted in the figure. Parallel programs were conducted: (a) study of the instability phenomenon employing the torsion test, and (b) study of the development of a crack in slow bending incrementally of Mesnager U-notch impact bars (notch radius-1mm, notch depth-2mm).

A. Initiation and Propagation of Fracture

The stages of initiation and propagation of fracture in the U-notch bars of heat-treated high strength steels are illustrated in Figures 7 and 8. The early stage of initiation in the plastic zone under the notch is pictured in Figure 7. Of particular importance is the definite displacement of the two segments on either side of the discontinuity, bearing out the notion that the initiation is the result of activation of a surface of discontinuity in tangential velocity. The propagation is illustrated in Figure 8. The discontinuity surface must make an angle of 45° to a free surface in plane strain. The fracture penetrated along a slip line in a logarithmic spiral path. A very interesting feature is the shift from, say, an α -slip line, to a β -slip line, and back to an α -slip. Such a course can be traced out on the theoretical slip line field for the U-notch, presented in Figure 9.

The crack was observed to penetrate to a certain distance in a stable manner (weak instability). At a critical distance, the crack propagates in an unstable manner (strong instability). These observations suggest two plasticity parameters that govern notch ductility, namely the root strain at which the plastic strain concentrates along characteristics, termed ϵ^* , and the depth to which the crack can penetrate in a stable manner, termed l^* (tolerance for weak instabilities). The smaller the value of ϵ^* , the lower is the size of the plastic zone and the notch ductility. The smaller the value of l^* , the less is the tolerance for weak instabilities and the lower is the notch ductility. It is judged that of the two parameters, ϵ^* is the more important, in general.

B. Instability in Torsion

The torsion test has some inherent advantages in studying instability in pure shear: (a) the planes of pure shear lie perpendicular to the specimen axis, (b) necking does not occur in directions of zero extensional strains, (c) the specimen inherently has a stress gradient, which can be controlled.

The occurrence of such instabilities in the strength transition in toughness was confirmed by the torsion tests. The general behavior observed was (a) uniform deformation,

(b) localization of deformation in bands perpendicular to the axis of the specimen, and (c) fracture along the interface of one of the bands of concentrated plastic flow. The basic plasticity parameters so obtained are γ_{ins} , the shear strain at which the flow begins to localize, and the "strength of the instability" expressed as $\frac{1}{\gamma_f - \gamma_{ins}}$, where γ_f is the strain at fracture. The parameter γ_{ins} corresponds to ϵ^* under the notch, and $\frac{1}{\gamma_f - \gamma_{ins}}$ corresponds to l^* . It is interesting that in a high strength aluminum alloy, and in alpha, alpha-beta, and beta titanium alloys, there was no evidence of tolerance for weak instabilities. A single strong instability is observed to form, which quickly goes to rapid fracture ($\gamma_{ins} = \gamma_f$ in such cases).

The appearance of instability bands in torsion specimens of AISI 4340 steel (217 ksi T.S.) twisted at several strain rates is demonstrated in Figure 10. There is a greater tolerance for weak instabilities at slower strain rates, and the total strain to fracture decreases with increasing strain rate. Thus, strain rate is an important factor determining ϵ^* or γ_{ins} .

The stress gradient was found to have a direct bearing on ϵ^* , decreasing with increasing stress gradient. This is thought to be the basis of the notch acuity effect on the load-bearing capacity of a component containing a notch. The correspondence of experimentally determined values of γ_{ins} and γ_{ins} converted from the state of stress in the Mesnager U-notch is presented in Figure 11, for AISI 4340 steel through the strength transition. The effect of the stress gradient is evident in the varying diameters of the torsion bars and comparing the U-notch and V-notch. When the stress gradient in the torsion bar matches fairly well that in the U-notch, the correspondence in the two plasticity parameters is encouragingly good. Such agreement in this basic parameter determined by two experimental methods strengthens the proposition that this indeed is a basic plasticity parameter which determines notch ductility when the fracture is initiated by an instability in pure shear.

The dependence of the strain to onset of instability in torsion (γ_{ins}) on strain rate for AISI 4340 steel at two strength levels is illustrated in Figure 12. The general tendency is a reduction of this parameter with increasing strain rate. Non-ferrous materials (Ti alloys, 7075 Al) show less dependency on strain rate. Preliminary data on the effect of temperature on γ_{ins} are presented in Figure 13 for the same steels and strength levels. The temperature dependency tends to be complicated, showing maxima and minima.

7. FOUNDATIONS FOR A DESIGN SYSTEM BASED ON ALLOWABLE PLASTIC STRAIN

The two basic modes of fracture initiation have been described, namely the activation of a surface of tangential velocity discontinuity (plastic instability in pure shear), and the formation of pores (cavitation) and joining of pores. The joining process may occur by an instability in pure shear, or by receding array of the ligament by flow (100% reduction of area). Fracture initiation, thus, may be (1) instability controlled, or (2) cavitation controlled.

A. Instability Control

The three requirements for activation of the instability in pure shear are (1) a free surface, (2) a stress gradient, and (3) the simulation of the body of the ideally plastic state. If the material fractures while the stress-strain curve is rising, it can be assumed that the fracture initiation is cavitation controlled.

A basic requirement, accordingly, is to characterize the material as to its susceptibility to mounting an instability in pure shear. This can be envisioned as an envelope of the form

$$\epsilon^* = f(\dot{\epsilon}, \sigma_{ij}, \frac{d\sigma}{dx}, T, \alpha) \quad (2)$$

where ϵ^* is the critical root strain at a discontinuity, $\dot{\epsilon}$ is the strain rate, σ_{ij} is the stress state, $d\sigma/dx$ is the stress gradient, T is the temperature, and α is the metallurgical structure, which includes prior strain and strength level.

This envelope is most conveniently determined by use of the torsion test. The stress state in torsion can be converted to other stress states by the significant stress-significant strain relationship. Quantitative application of such an envelope to predict the maximum allowable static load must await the development of satisfactory elastic-plastic solutions under a notch.

B. Cavitation Control

Much less is known about the conditions necessary for cavitation, or pore formation. More attention is needed in characterizing the possible changes in rheological states as the straining of a metal continues. The change to ideal plastic state is an example of a rheological transformation.

If one accepts the model of pore formation in metals and alloys as analogous to the phenomenon of separation in boundary layer flow of fluids, then a characterization of conditions for pore formation may have the following form

$$V^* = f(\mu, \rho, \beta) \quad (3)$$

where V^* is the critical velocity across a substrate, μ is the viscosity of the metal, ρ is the density, and β is the geometry of the substrate.

Such a formulation requires the determination of the viscosity of the metal as a function of the amount of plastic strain. The sphere is the most effective geometry for producing pores.

C. Creep

Two important phenomena are activated in the creep range, namely recovery by dislocation climb and grain boundary sliding. The latter leads to cavitation preferentially along grain boundaries, which eventually joins up in a typical grain boundary fracture. There is increasing evidence that wedge fracture initiation and grain boundary fracture initiation should not be regarded as two separate modes, but are the result of cavitation at preferred sites.

The rate of cavity formation at grain boundaries is controlled principally by the concentration of substrates (inclusions), and possibly ledges in the grain boundary. Creep ductility is not related primarily to the rate of pore formation, but rather by the rate of joining of pores. The pores may be joined by a instability in pure shear, or by the much slower process of receding of the ligament. There is very likely a temperature above which the material becomes viscous to a degree that it cannot simulate an ideally plastic body and instabilities cannot be activated. The matrix properties, thus, are important in relation to the possibility of mounting an instability, and possibly in controlling the ratio of grain boundary sliding to the total strain.

D. Interaction of Plastic Instability and Creep Failure

A schematic diagram depicting the interaction of instability control and cavitation control at elevated temperatures is presented in Figure 14.

Two paths to fracture initiation and propagation at temperatures below the creep range are represented, namely by the plastic instability, or by pore formation and joining (cavitation). In the latter case (termed ductile fracture), the possibility is indicated of a critical strain rate at which pores can join by instabilities. The rate of propagation at this strain rate is expected to show a significant increase. Propagation rates in instability fractures are expected to be inherently rapid.

The onset of the creep range is signalled by the onset of grain boundary sliding. The cavitation here changes from general pore formation (Case II) to preferential pore formation in the grain boundaries. The situation in Case I (instability control) is that at temperatures just inside the creep range, fracture may occur by the instability mode. At somewhat higher temperatures, the initiation may occur by cavitation but, the joining may be by the instability mode. At still higher temperatures, no instabilities can occur and the joining of pores occurs by flow.

This general scheme emphasizes the role of the instability in determining the possibility of dangerous unstable or catastrophic fracture, in the possibility of accelerating ductile fracture propagation, and in generating low fracture ductilities in the creep range.

E. Repeated Loading

No work has been done so far on the role of the instability in fatigue crack initiation and propagation. Fatigue fractures in the creep range are quite similar to those in creep. It is likely, thus, that the ability to join pores by the instability should have an important influence in reducing fatigue resistance. Fatigue propagation at temperatures below the creep range probably occurs by the instability at a critical plastic strain. The strain is enhanced by the Bauschinger effect in the case of cycling from tension to compression to tension.

F. Instability in Plane Stress

There are some interesting aspects of instability fractures in the biaxial stretching of sheet, or relatively thin plate. This is a biaxial stress state (plane stress) which allows plastic strain in three directions; the stress perpendicular to the plane of the sheet is assumed to be zero.

The slip lines in plane stress do not intersect orthogonally, as in plane strain. [11] The position of the slip lines depends on the ratio of the stresses $\left(\frac{\sigma_2}{\sigma_1}\right)$, as illustrated in Figure 15. For ratios from 0 to 0.5, the slip lines lie within the plane of the sheet and useful deformation will be limited by the onset of the instability fracture. For ratios from greater than 0.5 to 1.0, the slip lines are out of the plane of the sheet and the instability fracture cannot occur. Useful deformation will be limited by the well known tensile instability (load instability).

The case of the ratio equal to 0.5 is particularly interesting. The α and β slip lines now coincide, or merge into one. Their directions will coincide with that of σ_1 if the factor f' has a value of +1.0, and with the direction of σ_2 if f' has a value of -1.0. The latter case coincides with the situation in a closed and cylindrical pressure vessel or piping. The direction of σ_2 is the longitudinal direction of the vessel. This situation explains why longitudinal welds are so dangerous in such vessels. It is along such welds that there is the possibility of joining internal defects by the shearing instability. In unwelded piping, longitudinal splits can occur as the result of instability.

8. SUMMARY

A basis for design on allowable plastic strain is presented for static loading at ambient temperatures and for elevated temperatures into the creep range. It is based on two basic modes of fracture initiation: (a) plastic instability in pure shear, and (b) pore formation (cavitation). Pores may be joined by this instability or by plastic flow. The applications to fatigue loading and to plane stress situations are discussed.

9. ACKNOWLEDGEMENTS

The model herein presented is the result of several years of consideration and observations on flow and fracture in metals by the author and the many contributions of my collaborators: Professor G. W. Powell, John H. Bucher, Y. H. Liu, J. T. Cammett, Robert Purdy, Jr., C. A. Griffis, J. C. Jasper, R. H. Ernest, A. K. Chakrabarti, Capt. W. J. Meyerer, J. M. Vanecko, D. Firrao, D. McGarry, Dr. K. Tanaka, and E. N. C. Dalder.

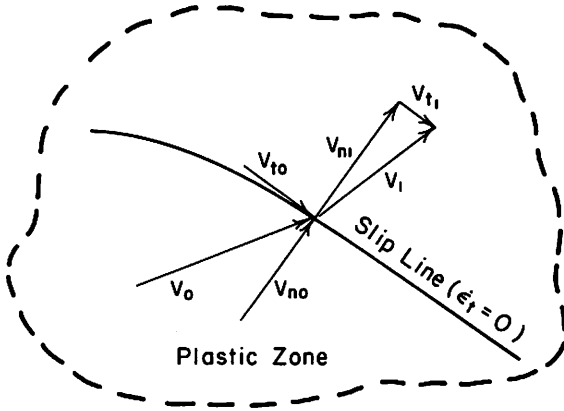
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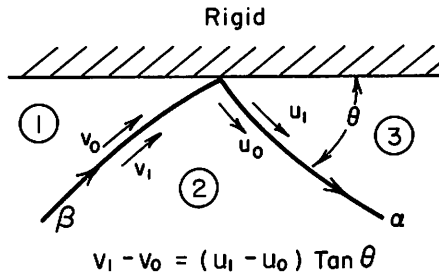
TABLE I.
Classification of Approaches to Instability

| <u>Dynamical Systems</u> | <u>Metallurgical Viewpoints</u> | <u>"Static" Plasticity</u> | <u>"Dynamic" Plasticity</u> |
|--------------------------|--|--|---|
| 1. Liapunov | 1. Necking in Tension ($dL = 0$) | 1. Maximum Load | 1. Tracy V. Thomas-- propagation of plastic waves on slip surfaces |
| 2. Poincare' | 2. Taylor-Elam, Rotation of single crystal in slip | 2. Swift's criterion | |
| 3. La Grange | 3. Adiabatic flow | 3. Uniqueness-- R. Hill | 2. N. Cristescu-- dynamic deformation of plastic bodies |
| | | 4. Van Iterson--flow in directions of pure shear | |



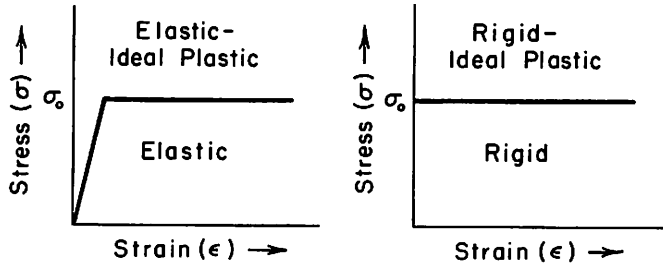
Continuity Condition: $v_{no} = v_{ni}$
Tangential Velocity Discontinuity
 $\Delta v_t = v_{t1} - v_{to}$

Figure 1.



Tangential Velocity Discontinuities are Reflected at Rigid Boundaries

Figure 2.



Condition for Ideal Plastic State

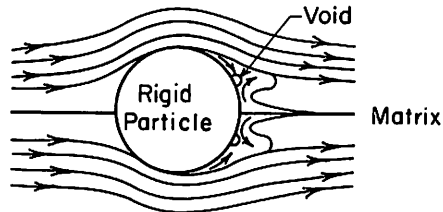
I. Flow Condition $f(\sigma_{ij}) = 0$

II. No Useful Work in Stress

Cycling: $d\sigma_{ij} d\epsilon_{ij}^{(p)} = 0$

Figure 3.

Phenomenon of Separation (Decohesion)



Critical Condition

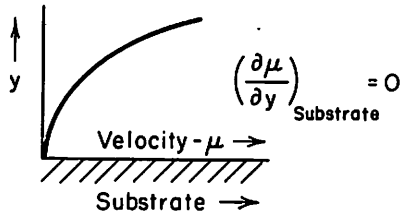


Figure 4.

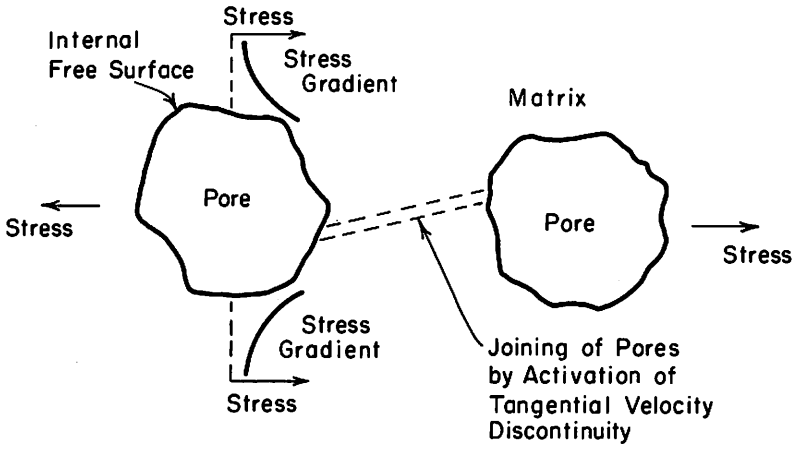


Figure 5.

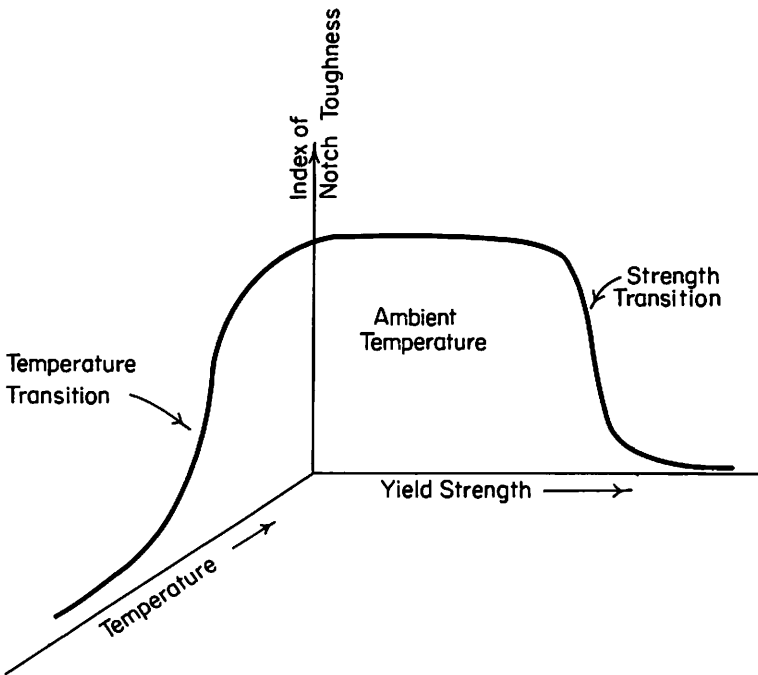


Figure 6. Schematic depiction of the strength transition and temperature transitions in notch toughness in steels.

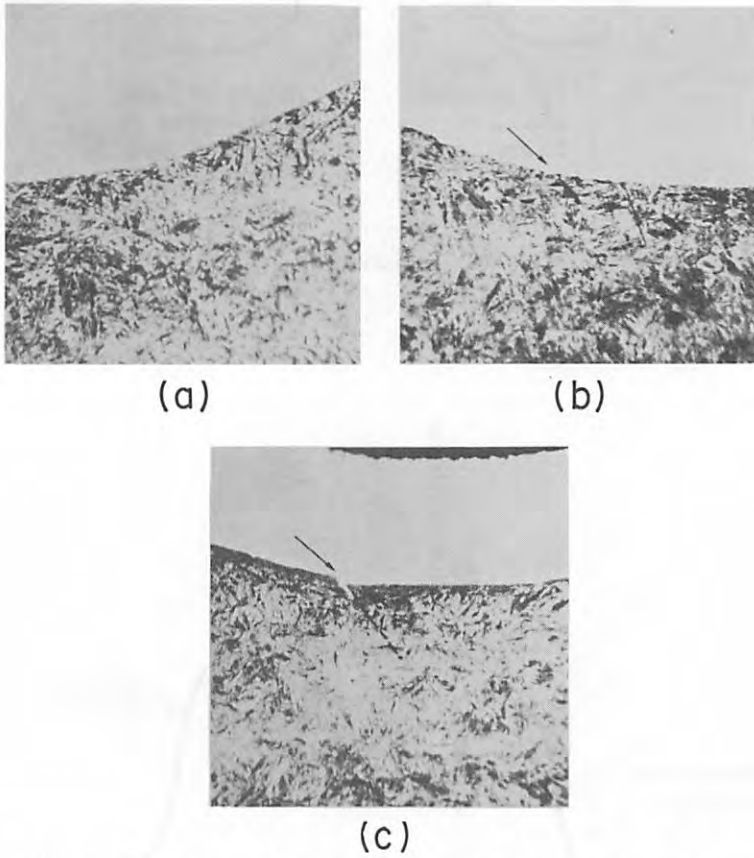
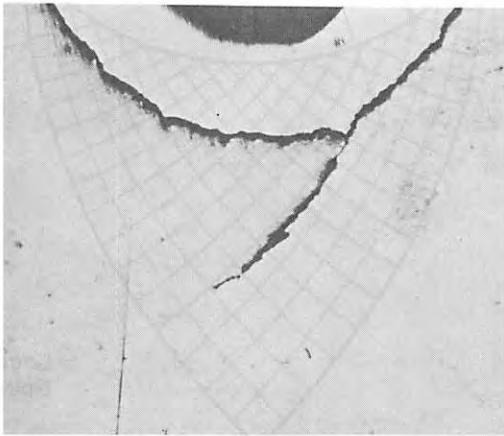


Figure 7. Initiation of fracture at the notch surface in a Mesnager U-notch impact bar of a high-strength, heat-treated alloy steel (Reference 5d).



(a)



(b)

Figure 8. Propagation of fracture along slip lines (discontinuity surfaces) in the Mesnager U-notch impact bar (Reference 5d).

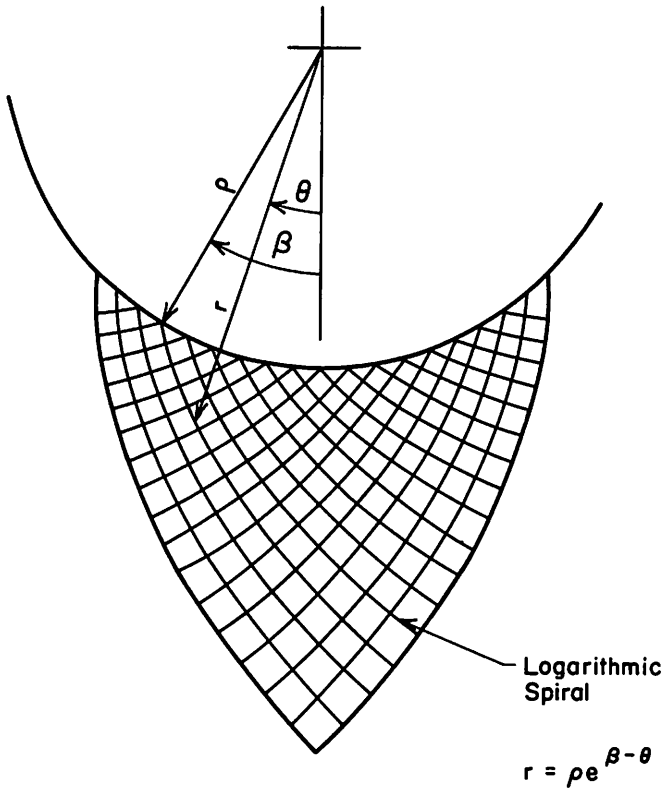


Figure 9. Theoretical slip line field for the U-notch.

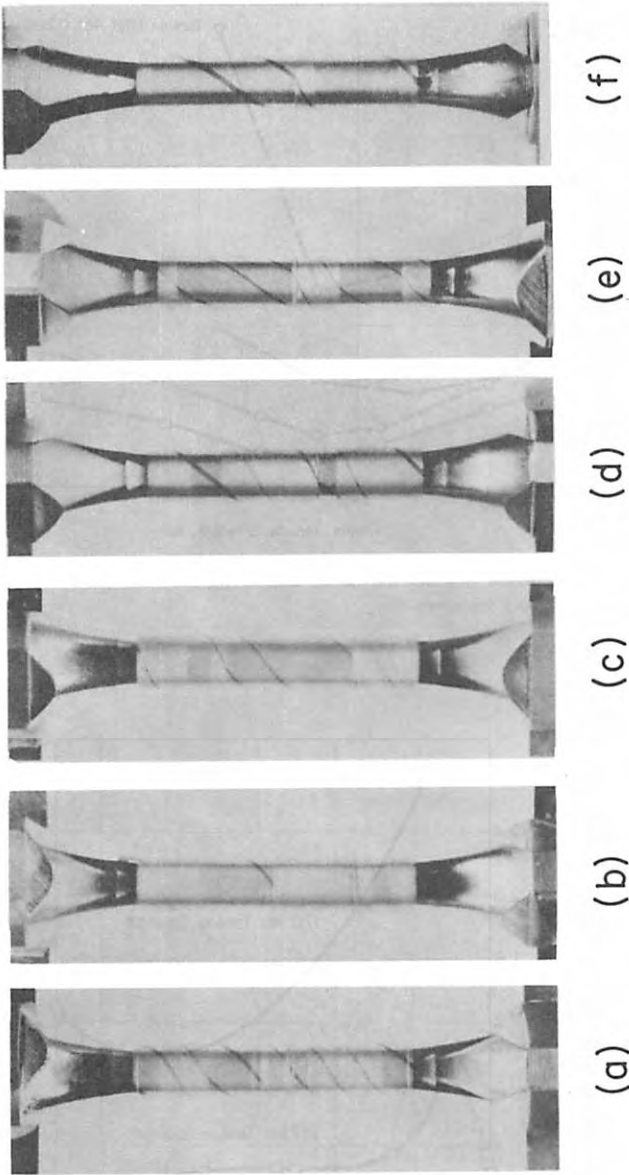


Figure 10. Photographs showing the appearances of torsion specimens of AISI 4340 steel (217 ksi T.S.) twisted at the various shear strain rates indicated (1.5X) (Reference 5c).

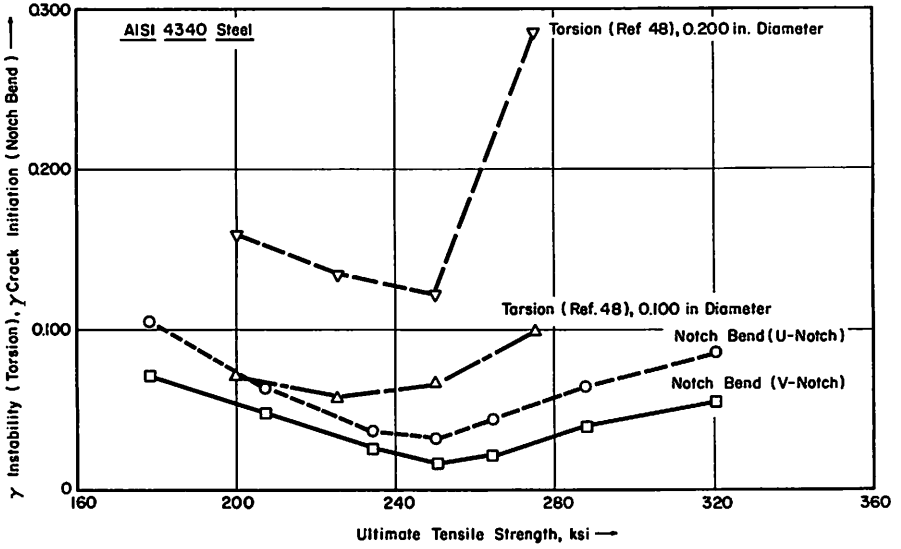


Figure 11. Comparison of γ_{ins} from the torsion test to γ_{ins} converted from c^* measured in the U-notch bar (Reference 5d).

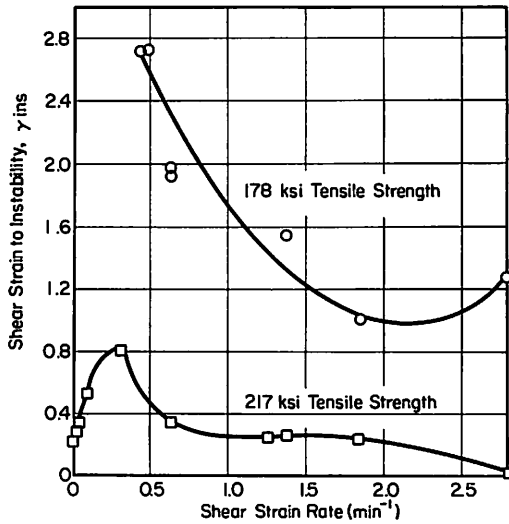


Figure 12. Effect of shear strain rate on the shear strain to instability for AISI 4340 steel at two strength levels.

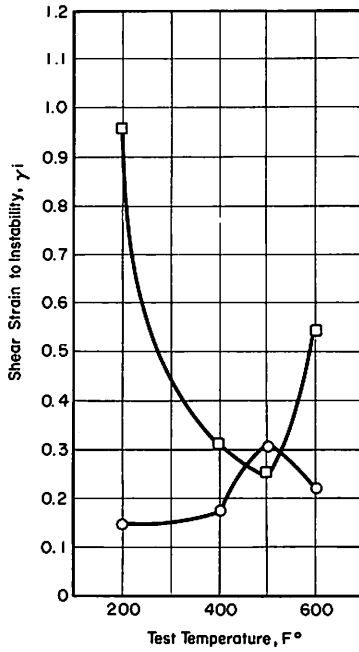


Figure 13. Shear strain to instability versus test temperature at the shear strain rate of 0.2 min^{-1} for AISI 4340 steel at two strength levels.

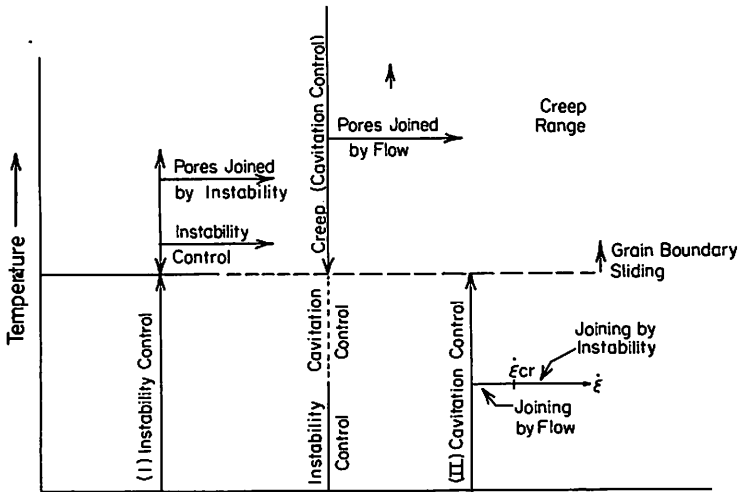


Figure 14. Schematic diagram illustrating the interaction between fracture by instability control and fracture by cavitation control.

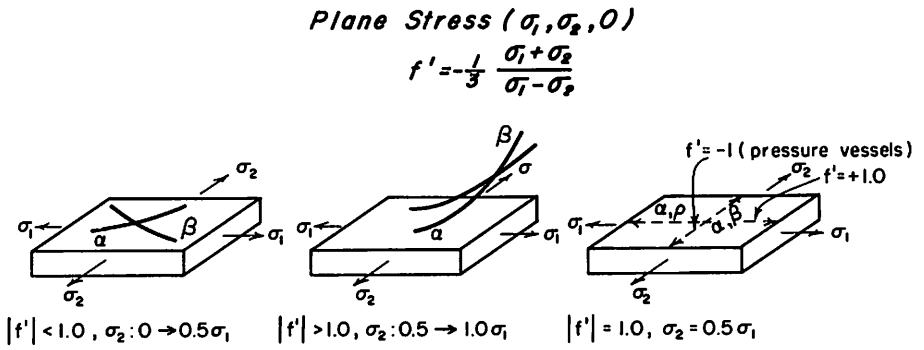


Figure 15. Instabilities in plane stress (biaxial stretching).

DISCUSSION

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Q

If slip field boundaries start and finish at free surfaces, can you suggest the slip velocity field which applies in plane strain at a notch ?

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A

The matter of slip velocity fields was not considered, nor is the author, as a metallurgist, skilled in calculating this field. It was only pointed out that a discontinuity surface must be a slip line surface.