Transactions of the **13th International Conference on Structural Mechanics in Reactor Technology** (SMiRT 13), Escola de Engenharia - Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil, August 13-18, 1995

Reliability of a protective channel with lognormal downtimes by the device of stages

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ABSTRACT-Lognormally-distributed repair times of a single protective channel mean that its reliability analysis cannot be performed by a Markovian model anymore. The device of stages is used to transform the channel state transition diagram so that all transition rates become constant and a Markovian model is then obtained. The aim is to calculate the plant accident rate where the protective channel is installed.

1. INTRODUCTION

The reliability analysis of protective channels of process plants has long been developed and many features have been considered in its modeling. Particularly, Oliveira and Amaral Netto (1987) calculated the accident rate of a plant equipped with a single protective channel and analyzed the influence of the demand rate and of the repair rate on the plant accident rate. However, in their modeling they considered that all the times involved in the analysis followed exponential distributions, so as to allow for using a Markovian model.

On the other hand, it has long been recognized that repair times do not follow an exponential distribution in general, Singh and Billinton (1972, 1977), for the very reason that this distribution is memoriless and so it does not allow to taking into account the experience gained with successive repairs.

For this reason, it is advisable to try alternative methodologies for considering the time-dependent reliability analysis of systems, as is the case with the single channel protective channel. In other words, methods for transforming a Nonmarkovian model into a Markovian one are desirable. Among the available methods, Cox and Miller (1965), Singh and Billinton (1977), we will apply the device of stages to the single protective channel by considering that its repair times follow a lognormal distribution. This distribution has been recognized as one of the adequate distributions to model repair times, Singh and Billinton (1972), Singh et alii (1973), Emoto and Schafer (1980) and Malayia and Su (1982).

We will present the reliability analysis of the single protective channel considering that its repair times follow a given lognormal distribution and so the device of stages will be applied in order to transform the Nonmarkovian model into a Markovian one. The adquate combination of stages will be discussed and then the plant accident rate will be evaluated. The behavior of the repair rate will be displayed and commented for the intent is to obtain a curve that presents an initial transient behavior characterized by a monotonic increasing trend followed by a rather steady-state behavior. Physically, this means that experience gained with repairs is reflected in the repair rate until a limit is reached: that of the administrative times involved in the repair that cannot be lowered indefinitely.

2. THE DEVICE OF STAGES WITH LOGNORMAL REPAIR TIMES

If the logarithm of a random variable follows a normal distribution, then the random variable follows a lognormal distribution whose density function is given by:

$$f_T(t) = \frac{1}{\sqrt{2\pi}\, t\alpha} \exp\left\{ -\frac{1}{2}\left[\frac{\ln(\rho t)}{\alpha} \right]^2 \right\}, \tag{1}$$

where ρ and α are the mean and standard deviation of the natural logarithm of the random variable. Considering the mean μ and the standard deviation σ of the associated normal distribution, one has:

$$\rho = \ln\mu = \frac{1}{2}\ln\left[\left(\frac{\sigma}{\mu} \right)^2 + 1 \right] \tag{2}$$

and

$$\alpha^2 = \ln\left[\left(\frac{\sigma}{\mu} \right)^2 + 1 \right] \tag{3}$$

The repair rate for the lognormally distributed repair times is given by:

$$\mu(t) = \frac{\dfrac{1}{\sqrt{2\pi}\, t\rho} \exp\left\{ -\dfrac{1}{2}\left[\dfrac{\ln\rho t}{\alpha} \right]^2 \right\}}{1 - \Phi\left(\dfrac{1}{\alpha} \ln\rho t \right)} \tag{4}$$

where $\Phi(.)$ represents the distribution function of the standard normal distribution.

The problem of calculating the accident rate for a plant equipped with a single protective channel whose downtimes are lognormally distributed was approached earlier, Costa Nunes *et alii,* (1994), in the sense that detailed considerations were made concerning the approach of stages. However, it was not the intent to calculate the accident rate.

The device of stages allows for approximating a time-dependent transition from a state to another one by a combination of fictitious states (the so called stages) whose transition rates are all constant. In this sense, a Markovian model results.

The Markovian model developped in Oliveira and Amaral Netto (1987) considers that the single protective channel can be in one of three states: state 1 means that the channel is working; by a failure transition with constant rate λ (the channel failure rate), it comes to state 2, to a condition of unrevealed failure. Finally, under a demand (whose constant rate is given by ν) the channel reaches state 3, where the failure is revealed. At this point, repair begins with a rate μ(t) given by Eq. (4). The channel leaves state 3 after the repair is finished and then it comes to state 1 with a rate given by γμ(t), where γ is the probability of no human error under repair. Alternatively, there is the transition to state 2 with a rate of (1-γ)μ(t), meaning that an unperfect repair has been performed. Oliveira and Amaral Netto (1987) considered that the channel repair was perfect, and so γ=1.

Examining Eq. (4) , Singh and Billinton (1972, 1977), Singh et alii (1973), possible combinations of stages for approximating it are a group of stages in series in series with two stages in parallel or, alternatively, two groups of stages where each group comprises two stages in series.

The choice of the combination of stages that better represents Eq. (4) is perfomed in two steps. Initially, the Laplace transform of the density function is examined.However, this procedure is not helpful in the case of the lognormal distribution for its Laplace transform cannot be written as a rational function. If this were possible, the roots of the denominator could give an indication of a candidate combination of stages.

In the second step, the selected combination is compared with the repair rate given by μ(t) to check whether the properties of the repair rate are preserved under the approximation.

For the case of the lognormal distribution, the first step is bypassed and so possible combinations of stages are selected by trial and error. As already mentioned, two candidate combinatons are:

(a) a group of stages in series in series with two stages in parallel; and

(b) two groups of stages in series linked in parallel.

As the transition rates of the stage combination are all constant, then one has to write down the density function of the combination and then match its moments, Soong (1981), to those of Eq. (4). The number of moments to be matched depends on the number of parameters to be estimated, Singh and Billinton (1977).

3. DISCUSSION OF THE APPLICATION

The case analyzed is that of a single channel whose repair times are lognormally distributed with mean 24 hr and standard deviation 12 hr. The channel failure rate is equal to 10 per year and the channel proof test interval is equal to 1 week.

The better approximation was obtained with the first combination discussed in the last section, that is, a group of stages in series linked to two stages in parallel. The density function for this combination is given by, Singh and Billinton (1977):

$$f_0(t) = \sum_{k=1}^{2} \omega_k \rho_k \left(\frac{\rho}{\rho - \rho_k} \right)^{\alpha} \left\{ e^{-\rho_k t} - e^{-\rho t} \sum_{i=1}^{\alpha} \frac{\left[(\rho - \rho_k) t \right]^{i-1}}{(i-1)!} \right\} \tag{5}$$

where, the symbols will be better understood if one takes a look at Figure 1, which presents the state transition diagram after the approximation by the device of stages has been performed.

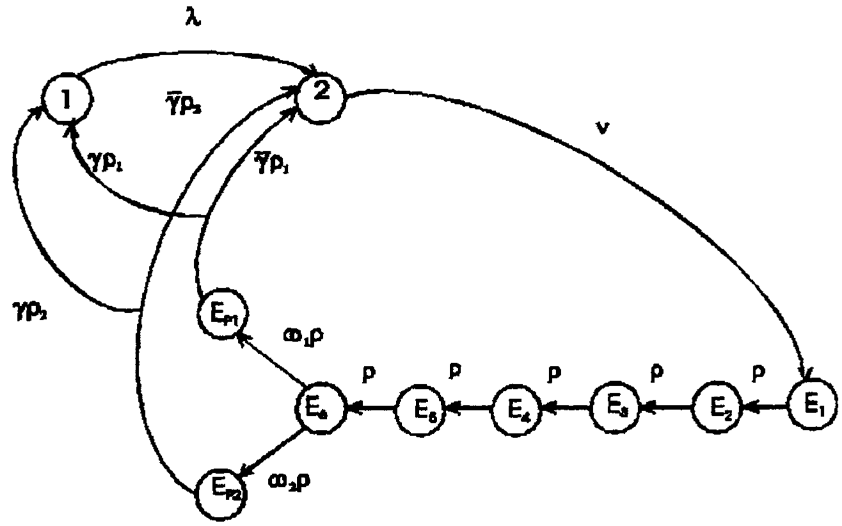

Figure 1. State transition diagram for the single protective channel after the approximation by the device of stages.

As can be seen from Figure 1, the former state 3 was transformed into a combination of 8 stages: stages E_1 to E_6 in series linked in series to stages E_{p1} and E_{p2} which in turn are linked in parallel. The failure rate (λ) and the demand rate (ν) are not affected by the transformation. On the other hand, the transition rate from stage E_1 to stage E_2 is equal to ρ (which is a constant parameter). This is the same transition rate for the remaining transitions between stages until stage E_6 is reached. Here, the transition rate is also equal to ρ but now there is a probability ω_1 that the chosen arrival stage is E_{p1} and a complementary probability ω_2=1-ω_1 that the chosen one is E_{p2}.

Once stage E_{p1} is eventually reached, then the next transition has a rate of ρ_1 and a probability equal to γ of choosing state 1, so that the transition rate to state 1 is equal to $\gamma\rho_1$. Alternatively, the transition rate to state 2 is equal to $\bar{\gamma}\rho_1$. The same reasoning applies to transitions from stage E_{p2}.

The moment matching technique had to be applied to estimate the five parameters needed to perform the transformation that resulted in the state transition diagram of Figure 1.The matching was performed to the first five moments of the distributions displayed in Eqs. (1) and (5). These parameters are: ρ, ω_1, ρ_1, ρ_2, and α, where this latter is the number of stages linked in series. It should be noted that ω_2 can be immediatly estimated for it is equal to 1-ω_1, as already discussed.

It is interesting to note that the former state 3 was replaced by the combination of the 8 stages shown in Figure 1 and that the transition rate from state 2 remains the same, that is, a demand puts the channel in stage E_1 where the failure is revealed and the repair begins. However, the residence time in the former state 3 is modeled by a sequence of stages in order to make the residence times follow exponential distributions. The combination of stages is left with constant transition rates, shown in the figure.

For the case at hand, the estimated parameters are: $\rho = 4.39x10^{-1}/hr$, $\omega_1 = 0.109901$, $\rho_1 = 6.53x10^{-1}/hr$, $\rho_2 = 1.03x10^{-1}/hr$, and $\alpha = 6$.

The next step is to determine the approximated repair rate and then compare it to the one given by Eq. (3). This comparison is presented in Figure 2, which considers a time period of approximately 4 days. It may be seen that the approximation is quite good and an interesting feature is that the transient behavior of the repair rate lasts for about 30 hours, then it reaches a maximum and next decreases monotonically as a straight line with negative slope almost equal to -1.

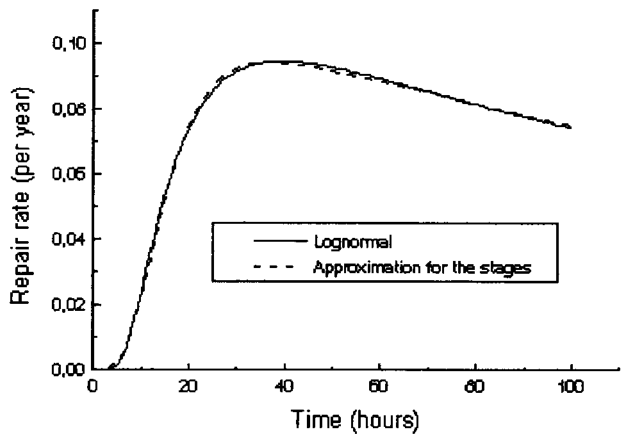

Figure 2. Approximated repair rate for the lognormal distributed repair times of the single protective channel.

With the approximated repair rate, the channel model becomes a Markovian one and, according to Figure 1, a set of 10 coupled differential equations must be solved to obtain the plant accident rate. This latter is given by:

$$\eta = \frac{\nu}{\tau_p} \int_0^{\tau_p} \left[p_2(t) + p_{E_1}(t) + \ldots + p_{E_6}(t) + p_{E_{p1}}(t) + p_{E_{p2}}(t) \right] dt \tag{6}$$

where τ_p is the proof test interval and the $p_{(.)}$ are the state probabilities, that is, the probabilities of being in the different states. Eq. (6) models the situation where the channel online repair is allowed. Otherwise, only the $p_2(t)$ term should be kept inside the brackets.

Figure 3 displays the results obtained from Eq. (6). It may be seen that the behavior of the plant accident rate is as expected. Specifically the accident rate is plotted against the channel demand rate. This demand rate is supposed to be constant and it may be interpreted as the rate of occurrence of disturbances at the plant which may be viewed as a homogeneous Poisson counting process with rate given exactly by ν.

In this sense, the protective channel should respond to the disturbances and as the number of these latter grows the plant accident rate is expected to become greater. The difference between the two repair policies is clear only for $\nu>30$ per year.

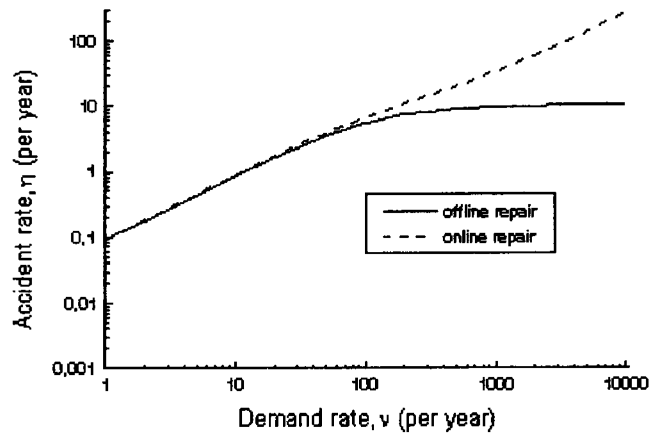

Figure 3. Plant accident rate considering the lognormally distributed repair times for the protective channel

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