Evaluation of Three Dimensional J Integral of the CT Specimen in Elasto-Plastic States

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SUMMARY

J integral is now accepted as the most useful parameter with which to describe the stress-strain field of the crack tip, and the criterion of the crack propagation. Many experiments and analytical studies on the J integral have been carried out. In the experimental studies to determine the J_c value, the shape and size of the specimens are recommended by ASME. These shapes and sizes are decided upon as to satisfy the condition of the plane strain. In analytical studies, J values are evaluated by several methods, assuming that the specimen is in the plane strain condition.

To compare the experiments with the analyses, it is necessary to study the thickness effect of these specimens on the J value. In this study, the three dimensional J integral is used to evaluate these thickness effect. The expressions of the three dimensional J integrals have been introduced and have been applied to several elastic problems by the authors in previous papers[1,2]. Now in this study, they are applied to the elasto-plastic problems.

1 Compact Type specimen is analyzed by using the finite element method. J values are evaluated at several points along the crack front. J values near the free surface are smaller than those obtained in the center of the plate.

J_M, the J value evaluated by using Merkle-Corten's conventional equation[3], and J values evaluated by two dimensional analyses are also obtained and compared to each other. It is shown that the J_M value and the J value of the center of the specimen thickness agree. The reasons for the decrease of the J value near the free surface are discussed by considering each term of the J integral expression. Finally, the average value of three dimensional J integral is estimated by using two dimensional results.

1. EXPRESSION OF THE THREE DIMENSIONAL J INTEGRAL --- THE EQUATION OF J_x

As reported before[1], the J integral has three components, J_x, J_y, and J_z, in the three dimensional space. To evaluate the J integral of CT specimen, it is enough to consider only one component, J_x, because of the symmentricity of the specimen. By referencing Fig.1, J_x is expressed as

$$J_x = \int_\Gamma (W\delta_{jx} - \sigma_{ij}u_{i,x})ds_j - \int_S (\sigma_{iz}u_{i,x})_{,z}dS. \tag{1}$$

Each term of eq.(1) is broken down as follows:

$$J_x = J_{S1-1} + J_{S1-2} + J_{S2+S3}, \tag{2}$$

$$J_{S1-1} = \int W\delta_{jx}ds_j, \tag{3}$$

$$J_{S1-2} = -\int_\Gamma \sigma_{ij}u_{i,x}ds_j, \tag{4}$$

$$J_{S2+S3} = -\int_S (\sigma_{iz}u_{i,x})_{,z}dS. \tag{5}$$

Where J_{S1-1} and J_{S1-2} are related to the circular integral along Γ and J_{S2+S3} is also related to both surfaces of the cross section in Fig. 1.

2. TEST SPECIMEN

Fig.2 shows the shape and the dimensions of 1 CT specimen. The material of the specimen is assumed to be A533B steel, and the constitutive equations are determined by the experiment , as shown in Fig.3, and in the following:

$$\sigma \leq \sigma_{YS}; \qquad \varepsilon = \sigma/E, \tag{6}$$

$$\sigma \geq \sigma_{YS}; \qquad \varepsilon = \sigma/E + \{(\sigma/E')^n - (\sigma/E')^n\}. \tag{7}$$

where σ_{YS}, the yield stress is 0.5492GPa(56.0kgf/mm^2), n=3.0, E=206GPa(21000kgf/mm^2), and E'=1.609GPa(164.1kgf/mm^2).

Fig.4 shows the mesh division of the specimen. The external force is applied to point A, and the displacement is measured at point B. Paths to evaluate the J values are also shown in Fig.4 by heavy dark lines. 6 paths are used and the average of them are shown in the following results.

Fig.5 shows the mesh division along the plate thickness direction. Three types of mesh division are used: one-layer, two-layer, and three-layer. As it is considered that stresses and strains change largely near free surface, the thickness of the center of the specimen is larger than others in two and three layer divisions. In the two-layer division, the width of the second layer is 80% of the plate thickness, and in the three-layer division, the width of the third layer is 57% of it.

Two dimensional analyses are also carried out by using the same mesh division as shown in Fig.4 and assuming both conditions of plane stress and plane strain.

3. RESULTS AND DISCUSSIONS.

Fig.6 shows the load versus the load line displacement curves. The results of the three dimensional analyses appear between those of the two dimensional analyses. The results of the two-layer and the three layer analyses coincides well, and they are nearer to the result of plane strain analysis.

Fig.7 shows the shapes of the plastic zones created near the crack tip when the deformation is 0.3mm. The shape of the center of the specimen is similar to that of plane strain analysis and near the free surface it is similar to that of the plane stress analysis.

Fig.8 shows the J_x values along the plate thickness. The results of the plane stress and the plane strain are also shown in this figure. J_x values in the center of the plate are larger than those of the nearby surface, and as the deformation proceeds, they increases rapidly. These results implies that the J_x value at the center of the specimen becomes larger than some critical value at first, so they agree with the experimental results that the fracture first begins at the center of the CT specimen.

By using the results of Fig.8, J'_x value, the average of the J_x is obtained by the following equation.

$$J'_x = \frac{1}{B}\Sigma J_{xi} b_i, \tag{8}$$

where J_{xi}, and b_i mean the J_x value of each layer and the width of it, respectively, and B is the thickness of the plate. J_M, the conventional J value is also obtained by using the results of Fig.6.

Fig.9 and Fig.10 show the J_x values versus the displacements curves of the two-layer and the three-layer division. In these figures, J'_x , and J_M values are also shown. In both figures, it is shown that though the curves of each layer are separate from other, the J'_x and J_M values are similar to the values of the layer in the center of the specimen. The J_M value is especially similar to them. These results mean that the experimental evaluation of J value by using Merkle-Corten's equation is nothing other than an evaluation of the J value in the center of the CT specimen. As the fracture first occurs in the center of the plate, so J_M can be useful as the parameter of fracture initiation.

Fig.11(a)-(d) shows the contributions of each term of eqs.(3)-(5) to the J_x value for each deformation stages. As shown in these figures, the diminution of the J_x values at the free surface is caused by the third term, J_{S2+S3}, alone. The other two terms contribute largely to the value of the J integral, but they don't contribute to the diminution of it. Then, it is obvious that the stress components concerning the z axis, the plate thickness direction, have a great effect on this diminution.

Fig.12 shows the change of the σ_z value along the z axis. It is shown that the σ_z value near the free surface becomes reasonably small. This is the main reason for the diminution of the J_x values at the free surface.

Last, the two equations are used to estimate the three dimensional J value using two dimensional analyses.

(i) Assuming that J_x value is the linear function of the z axis, then $\bar{J}_x$ is defined as follows from eq.(8).

$$\bar{J}_x = \frac{1}{2}(J_{x\ pl.stress} + J_{x\ pl.strain}) \tag{9}$$

(ii) Assuming that the plane strain region occupies nearly 70% of the plate thickness, and the remaining thickness is in the plane stress condition, $\bar{\bar{J}}_x$ is defined by

$$\bar{\bar{J}}_x = 0.7 \times J_{x\ \mathrm{pl.strain}} + 0.3 \times J_{x\ \mathrm{pl.stress}} \tag{10}$$

Fig.13 shows the $\bar{J}_x$ value and the J'_x values of three types of layer division. Though there are some discrepancies (7% when the displacement is 1.1mm), they are generally considered to agree. Fig.14 shows the $\bar{\bar{J}}_x$ value and J'_x values. They agree very well even when the large deformation occurs. So it is reasonable to conclude that nearly 70% of the plate thickness is in the plane strain condition in the case of 1 CT specimen. Also it is noticed that the three J'_x values for each type of division coincides well.

By changing the plate thickness, it is possible to carry out the same calculation ultimately finding the smallest sized specimes that can possibly be used to determine the J_c value.

4. CONCLUDING REMARKS

1 CT specimen is analysed and the three dimensional J integrals are obtained in elasto-plastic states.

i) J_x value, along the crack front, becomes smaller nearby the free surface and the largest at the center of the specimen.

ii) J_M, the experimental method to evaluate the J value, coincides well to the J_x value at the center of the specimen.

iii) The decrease of J_x value near free surface are mainly caused by the decrease of the stress components concerning to the z axis.

iv) By assumeing the plane strain region as 70% of the plate thickness, J_x values are well estimated by using the results of two dimensional analyses.

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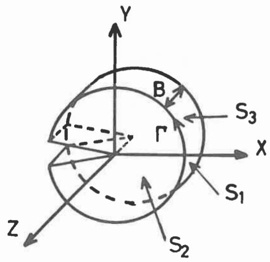

Fig.1 The cross section of three dimensional crack.

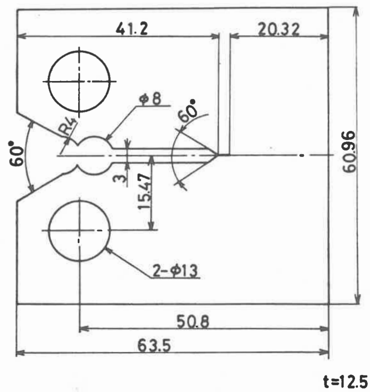

Fig.2 The shape and dimensions of 1 CT specimen.

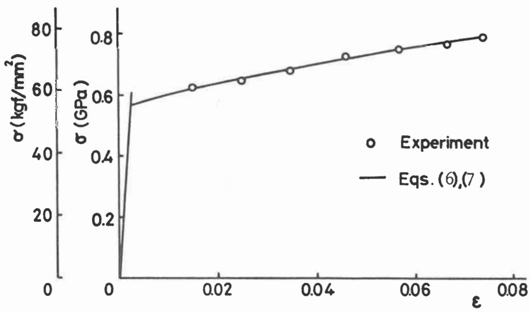

Fig.3 The stress-strain relation of A533B steel.

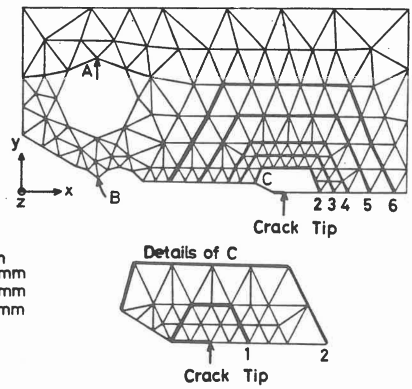

Fig.4 The mesh division of the specimen.

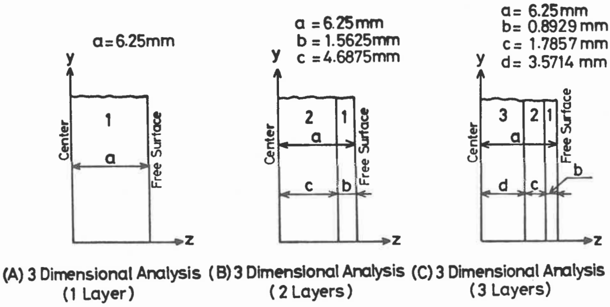

Fig.5 The mesh division along plate thickness.

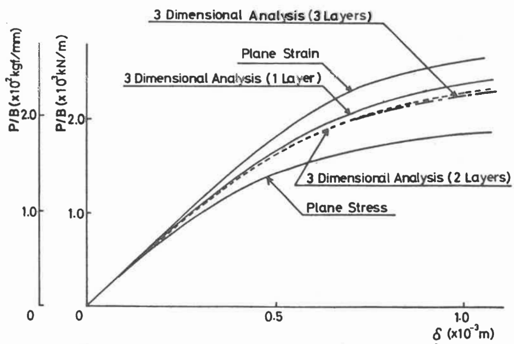

Fig.6 The load(P)-displacement(δ) curves.

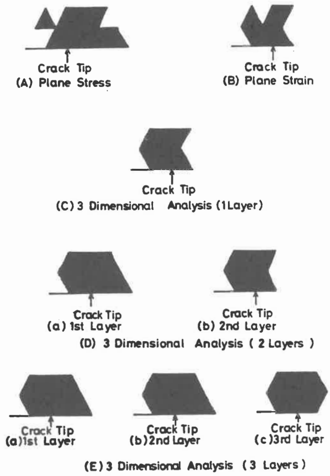

Fig.7 The shapes of the plastic zones. (δ=0.3mm)

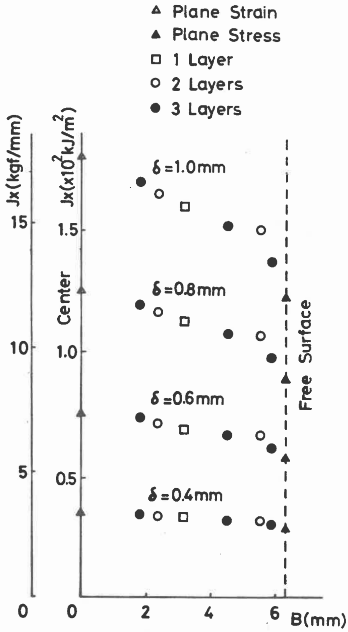

Fig.8 The J_x value along the plate thickness.

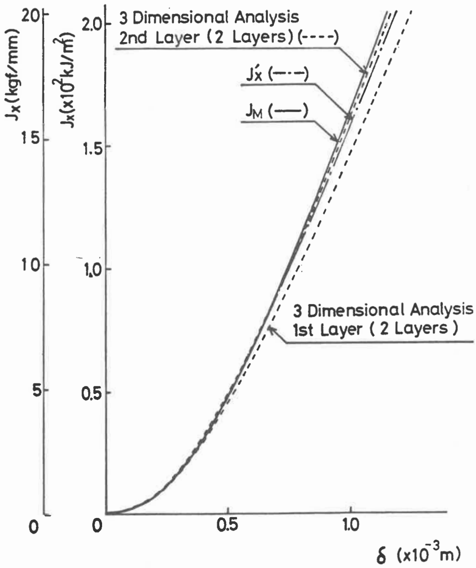

Fig.9 J_x-δ curves. (Two-layer division)

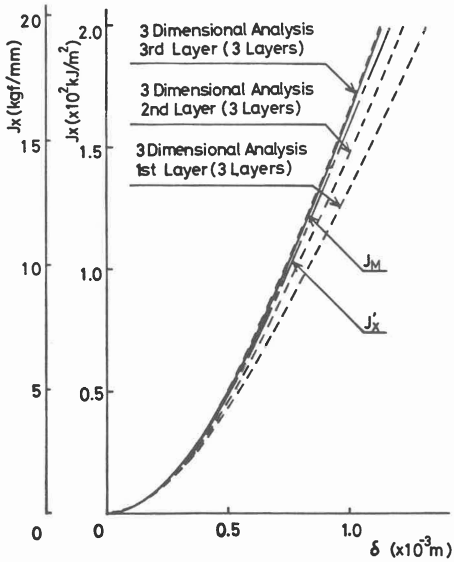

Fig.10 J_x-δ curves. (Three-layer division)

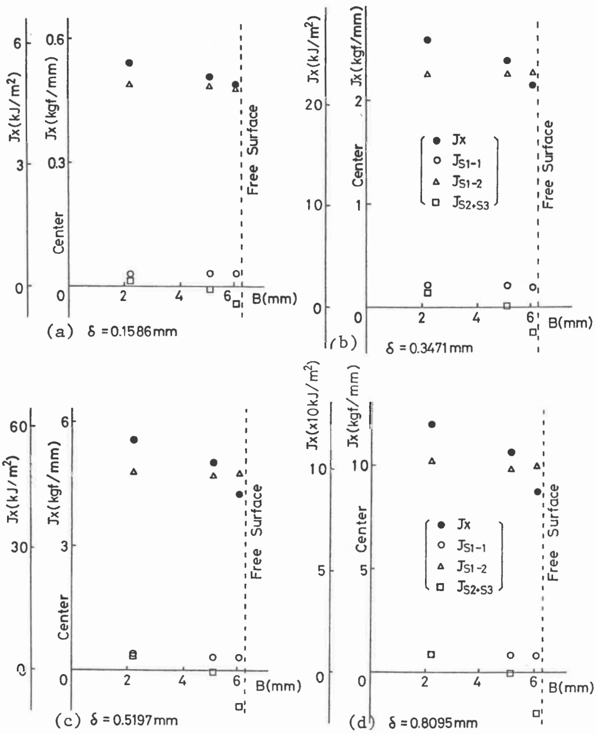

Fig.11 (a)-(d) The contrinutions of each term to the J_x value.

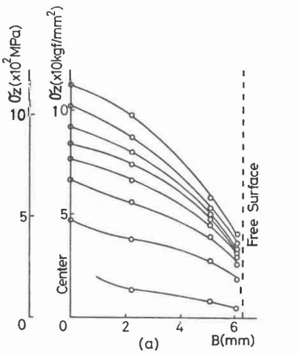

Fig.12 The change of σ_z value along the plate thickness.

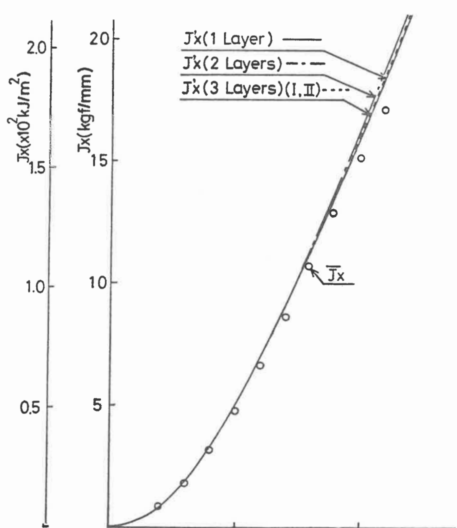

Fig.13 J_x' and $\overline{J}_x$-δ curves.

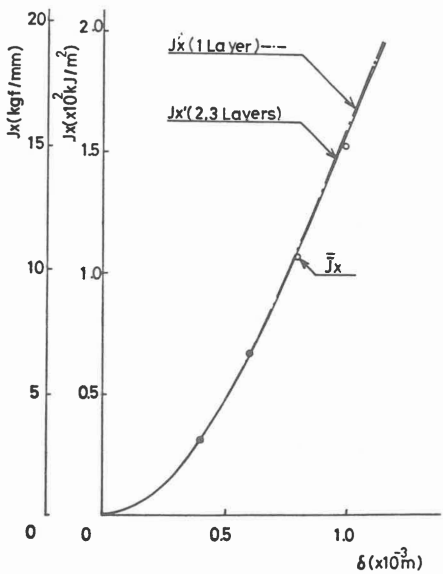

Fig.14 J_x' and $\overline{\overline{J}}_x$-$\delta$ curves.