

## ON CONSTITUTIVE MODELLING IN FINITE ELEMENT ANALYSIS

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### Abstract

This *compact* contains a brief introduction to the problems involved in constitutive modeling as well as an outline of the final paper to be submitted. Attention is focused on three important areas: (1) the need for using theoretically sound material models and the importance of recognizing the limitations of the models, (2) the problem of developing stable and effective numerical representations of the models, and (3) the necessity for selection of an appropriate finite element mesh that can capture the actual physical response of the complete structure. In the final paper, we will be presenting our recent research results pertaining to each of these problem areas.

## 1. Introduction

The issues of formulating realistic representations for the nonlinear constitutive behavior of materials and of establishing effective implementations through finite element procedures have been subjected to intense research activity during the last few years [1 - 6]. It is widely recognized that, to enhance and exploit our present capabilities for finite element analysis, we are in much need of improved constitutive descriptions that can be implemented in an effective manner. When considering the development of such material descriptions we are basically concerned both with adequate analytical formulations and with the numerical aspects of implementing and using the resulting models in appropriate finite element approximations.

Basically, if we seek to characterize materials for the analysis of nuclear reactor components and other structures, there are available at least the following classes of mathematical models: hypo-elasticity, time-independent elasto-plasticity, and various characterizations of creep [6]. For each of these models some degree of theory is available to describe important phenomena, such as temperature effects, strain-hardening, strain-softening and so on. In some cases it may also be convenient and adequate not to distinguish between time-independent plasticity and creep but rather to lump both physical phenomena together as visco-plasticity [7].

In seeking to describe an actual material, a library of conventional theories has therefore developed. Thus, if the problem is to represent a specific material behavior, the appropriate mathematical model must be selected and the limitations of the model, assessed by comparison with the actual response being simulated, must be recognized. An example which demonstrates the degree of choice that can arise here may be found in the many mathematical descriptions of concrete: this material has been modeled using both hypo-elastic and elastic-plastic material descriptions that are each based on quite different assumptions [1, 8 - 10].

For the actual usage of a theoretical description of a material, it is not sufficient for the model to be just physically sound and capture in theory the major response characteristics: the model must also be computationally tractable; namely, it must be possible to implement the model in a stable and effective manner. As an example, it may be noted here that although many of the basic creep models currently in use in finite element analysis were proposed long ago, it is only now that computationally stable and accurate response predictions using these simple models for actual structural components are becoming feasible [11, 12].

The major difficulties that are encountered in the effective implementation of nonlinear material models are those of stability, accuracy and cost. The model formulations and implementations should be as stable as possible, thus allowing relatively large load or time steps, and use of these large incremental steps should still allow the response to be predicted with sufficient accuracy. The cost of analysis will generally decrease when relatively large load or time steps can be employed in the solution, but it must also be recognized that the model must not be too costly (e.g. due to excess equilibrium iteration) in its usage per solution step.

In addition to the above difficulties of describing the material behavior in a theoretically sound and numerically effective manner, in some finite element analyses the

spatial discretization used can have a profound effect on the prediction of the response [2, 13, 14]. In essence, the finite element mesh must be able to represent the specific modes of deformation of interest, including those that can be predicted from analytical considerations, and special regard must be given to this aspect of finite element modeling. For example, in the analysis of elastic-perfectly-plastic conditions, the finite element mesh must be able to represent the incompressibility constraints of perfect plasticity [12], and in the analysis of peak load or ultimate failure conditions, the finite element mesh must be able to capture phenomena like strain localizations that can develop into intense shear zones or slip-surfaces [2, 4, 15]. Therefore, apart from using an appropriate material description, also a proper finite element mesh must be chosen to model the complete problem.

## 2. Brief Outline of the Paper

The objective in this presentation is to describe some recent results of our work on constitutive formulations for tractable numerical implementation, and to discuss these results in the context of the overall problem of material modeling for finite element analysis. In the final paper we will first briefly describe the basic features of the hypo-elastic, elastic-plastic and creep material models that we have developed and implemented. The basic material assumptions used in these models are commonly employed and we briefly discuss the major assumptions and shortcomings as related to the characterization of real material response, with some emphasis on more recent concerns about modifications to account for non-classical effects (such as localization and more general unstable material characteristics).

A major goal in the development and implementation of the above material models was to obtain improved numerical representations and solution strategies. In this respect, we will present next the approaches that we have developed, and discuss in detail the analysis of thermo-elastic-plastic and creep response, including temperature dependence of material moduli. Some consideration will be given also to common artifacts for modeling unstable (e.g. softening) behavior and their possible sources of error.

In the third section of the paper we will address the problem of selecting an appropriate finite element mesh for the analysis of material nonlinear response. Here we emphasize the analysis of limit load conditions and strain localizations, where special considerations are necessary.

Finally, we will present the finite element solutions of a number of demonstrative sample analyses. These results will help to substantiate the theoretical observations made earlier in the paper and at the same time they will illustrate to some degree the applicability and shortcomings of the constitutive modeling capabilities generally available at present.

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