COMPARISONS OF THE PERCENTAGE POINTS OF DISTRIBUTIONS WITH THE SAME FIRST FOUR MOMENTS, CHOSEN FROM EIGHT DIFFERENT SYSTEMS OF FREQUENCY CURVES

by

E.S. Pearson, University College, London

N.L. Johnson,* University of North Carolina at Chapel Hill

and

I.W. Burr, Purdue University

1. PURPOSE OF THIS INVESTIGATION

Our object is to study the extent to which the probability integrals of members of different systems of univariate, unimodal frequency curves, $y = f(x)$, having identical first four central moments, are in agreement. If the variables, x, are standardized so as to have a zero mean and unit standard deviation, we are proposing to investigate to what extent the two "shape" parameters $\sqrt{\beta_1} = \mu_3/\sigma^3$ and $\beta_2 = \mu_4/\sigma^4$, often described as measuring skewness and kurtosis, can provide estimates for several different systems of distributions, of the probability integrals

$$P = \int_{-\infty}^{X} f(x)dx \ .$$

2. HISTORICAL SUMMARY OF THE DEVELOPMENT AND USES OF FREQUENCY CURVES

2.1. Fitting to observational data

It was realized towards the end of the 19th century that the frequency distributions of many series of observed, continuously distributed data could not be adequately represented by the normal or Gaussian probability law, for which $\sqrt{\beta_1} = 0$, $\beta_2 = 3.0$. Two systems of non-normal distributions were then put forward:

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(a) The Gram-Charlier, or rather similar Edgeworth systems depending on series expansions involving the normal function and its derivatives.

(b) The Pearson system, based on the solution of the single differential equation

$$\frac{1}{y}\frac{dy}{dx} = \frac{-(c_1+x)}{c_0 + c_1 x + c_2 x^2} .$$

For both systems the parameters of the curves are expressible in terms of the mean μ_1' (which for a standardized variable is zero) and the higher moments about the mean, μ_i $(i = 2,3,4,\ldots)$. In graduating an observed frequency distribution by a theoretical curve it was for long the practice to equate the moments of the latter to those of the former distribution. While the Pearson curves of system (b) require no more than the first four moments, the series expansions of system (a) allow for the introduction of more moments and therefore might be expected to provide a closer fit. However, in practice, this possibility provides little advantage when attempting to graduate observed data subject to sampling errors, because the higher moments of the data, m_i, are subject to standard errors increasing rapidly with i.

If we compare like with like, e.g. use only four moments in either system, we find as shown by Barton and Dennis (1952)* that outside a rather restricted region in the β_1, β_2 field, the Gram-Charlier and Edgeworth curves may cease to be positive definite and unimodal. Although we shall not be concerned in this report with the use of frequency curves in graduating observational data, this weakness, as well as pressure on time and space, influenced us in deciding to exclude curves of the systems (a) from our investigation.

2.2 New uses for systems of frequency curves

Apart from using mathematical curves to graduate observed frequency distributions, it was realized that, in the development of statistical theory,

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* See Draper and Tierney (1972) for further comments on this region and some additional points.

these curves had two other very useful functions:

(a) They could be used to represent approximately the sampling distribution of a statistic in cases where the true distribution was difficult to derive explicitly, but its moments were known or at any rate calculable.

(b) They could be used to represent population distributions in studies of the robustness of tests and in procedures of estimation which had been based on the assumption of parental normality.

The pioneer work in the direction (a) seems to have been taken by Student (W.S. Gosset) who in his investigation (1906) on how to treat the mean and variance in very small experimental samples, where the variables could be assumed to be normally distributed, took a number of illuminating steps:

(i) first he derived the 3rd and 4th moments of the sample estimate of variance, s^2, the expectation of s^2 and its 2nd moment being already known;

(ii) then he realized that the values of $\beta_1(s^2)$ and $\beta_2(s^2)$ were those of a Pearson Type III or gamma distribution;

(iii) assuming this last to be the true distribution of s^2, and having shown that in samples from a normal population, $\bar{x}$, the mean and s^2 were independent, he deduced the sampling distribution of $z = t/\sqrt{\nu} = (\bar{x} - \mu_1')\sqrt{n}/s'$ (where $\nu = n-1$ and $s'^2 = \Sigma(x-\bar{x})^2/n$) and found this to be a Pearson Type VII curve. Student was unaware that Abbé and Helmert had previously proved mathematically that s^2 unquestionably had this Type III form, a result which Fisher also confirmed in 1915. However, Student's line of approach is one which has since been followed where no true distribution is known, only the moments.

A number of years later it was again Student (1927) who broke fresh ground by calculating a table of approximate upper 10, 4 and 2 percentage points of the range (w) in samples of n = 2(1)10 from an N(0,1) population, using Pearson curves having the moments recently published (Tippett, 1925; Pearson, 1926). A fuller and more accurate table of percentage points for range, using the best available estimates of the moments to be used in fitting a Pearson curve, was published a few years later (Pearson, 1932), and when accurate percentage points were computed *ab initio* (Pearson and Hartley, 1942) it was realized how closely the "Pearson curve fitting" procedure had given the true values.

At about the same time as these approximations to the distribution of the range were being developed, one of us (Pearson, 1930, 1931) had used the work of Fisher (1928, 1929) and Wishart (1930) to derive approximations to the lower and upper 5 and 1 percent points of

$$\sqrt{b_1} = m_3/s^3 \qquad \text{and} \qquad b_2 = m_4/s^4$$

in samples of n from a normal population, using Pearson Type VII and Type IV curves having the correct first four moments. The results were only given for relatively large samples, i.e. for $\sqrt{b_1}$, $n \geq 50$, and for b_2, $n \geq 100$.

When Johnson (1949) developed his new system of S_B and S_U frequency curves he used a rather different method of exploring the similarity between Pearson and Johnson curves having the same first four moments. His procedure was to calculate the expected group frequencies of:

(a) An S_B and a Pearson Type I curve, both fitted using the same four moments to an observed frequency distribution* of N = 631,682 observations for which $\sqrt{b_1} = 0.318$, $b_2 = 2.430$.

(b) An S_U and a Pearson Type IV curve fitted in the same way to an observed distribution of N = 9440 observations* for which $\sqrt{b_1} = 0.910$, $b_2 = 4.863$.

(c) An S_U and a Pearson Type IV curve fitted similarly to another distribution of N = 9440 observations* for which $\sqrt{b_1} = 0.441$, $b_2 = 3.654$.

* All three distributions were taken from Pretorius (1930).

If alternatively we make use of the Tables A1, A2, A3, A4 discussed below, we see that with $\sqrt{\beta_1} = \sqrt{b_1}$ and $\beta_2 = b_2$, the differences between all percentage points of Pearson and Johnson curves are ≤ 0.01 (i.e. 1/100 of the S.D.) in the stretch between and including the lower and upper 2.5% points for cases (a) and (b) and as far out as the lower and upper 1% points for case (c). This showed how similar the Pearson and Johnson curves are in these three examples, except towards the tails.

Merrington and Pearson (1958) introduced a further family of probability distributions into the comparison by examining how closely the distribution of non-central t could be represented by a Pearson curve of Type IV.

With the information from these rather diverse comparisons before them, Pearson and Tukey (1965) decided to explore the possibility of a different procedure, that of estimating the standard deviation of a distribution by applying factors to what they termed the "h% distances," i.e. the distances between the lower and upper h% points of a distribution which had not been standardized. The slowly changing values of these factors were shown by drawing systems of contours in the β_1, β_2 plane, see their Figs. 2, 3 and 4 for h = 5.0, 2.5 and 1.0, respectively. In the course of examining their problem they calculated afresh or collected from elsewhere the 0.5, 1.0, 2.5 and 5.0% points of 29 distributions selected from the Pearson, Johnson, log-normal, log χ^2 and non-central t distributions.

Similar comparison of standardized % points have been made elsewhere, e.g. in Pearson (1963) and in Pearson and Hartley's *Biometrika Tables for Statisticians*, Vol. 2 (1972, p. 76). The existence of the tables of standardized percentage points of several families of frequency distributions (see Appendix for references) as well as the availability of worked out computer programmes has made these comparisons much easier to carry out than formerly. For this reason it seemed to us that the time had come for systematizing and extending these comparisons, as well as filling in certain gaps.

3. THE SCOPE OF THE PROGRAMME UNDERTAKEN

3.1. The starting point.

It may be said that there are two aspects of the subject:

(a) Its interest as disclosing some rather unexpected properties of univariate frequency distributions. For instance, the two-decimal place comparison shown in Table 1 of standardized 5 and 1 percent points of members of five distinct families having beta values close to $\sqrt{\beta_1} = 0.8$, $\beta_2 = 4.2$, inevitably raises the question: over what area in the beta field does this degree of correspondence exist?

(b) What use can be made of our results in practical or theoretical research in mathematical statistics?

TABLE 1. Illustration of comparisons

Family	Shape parameters		Standardized percent points			
	$\sqrt{\beta_1}$	β_2	Lower 1	Lower 5	Upper 5	Upper 1
Pearson, Type VI	0.800	4.200	-1.80	-1.40	1.82	2.90
Johnson, S_U	.800	4.200	-1.80	-1.40	1.82	2.91
Non-central t	.780	4.229	-1.83	-1.42	1.81	2.89
Log-normal	.814	4.200	-1.78	-1.40	1.83	2.91
Log-χ^2	.780	4.188	-1.83	-1.41	1.81	2.90

Note. Published tables for Pearson and Johnson curves give values of the % points at exactly $\sqrt{\beta_1} = 0.8$, $\beta_2 = 4.2$; this beta-point is outside the non-central t area; no log-normal or log-χ^2 distributions have $\sqrt{\beta_1} = 0.8$, $\beta_2 = 4.2$ exactly.

Throughout the following analysis $\sqrt{\beta_1}$ rather than β_1 has been taken as the argument for skewness; this was because, to ease interpolation, the former has been used in a number of tables, e.g. of the standardized percentage points of Pearson curves. This advantage has to be balanced against certain disadvantages, e.g. (a) the regional boundaries of a chart like Fig. 1 cease to be linear or near-linear; (b) the Pearson Type I and S_B areas are cramped for space compared with those for Type IV and S_U. As a result, if calculations are made using a grid having equal intervals in terms of $\sqrt{\beta_1}$ it is impossible to follow changes in such detail in the Type I-S_B as in the Type IV-S_U area. Whether this matters, depends on where our interests are mainly focussed.

3.2. The Pearson and Johnson distribution comparisons.

The main comparison, set out in Tables A1-A10, has been made between distributions of the Pearson and the Johnson systems, both of which can be found at every beta point contained in the field of Fig. 1. The Johnson curves are divided into two types, S_U and S_B, their appropriate regions being separated by the log-normal line, defined by the parametric equations

$$\left.\begin{aligned} \beta_1 &= (\omega-1)(\omega+2)^2 \qquad (\sqrt{\beta_1} > 0) \\ \beta_2 &= \omega^4 + 2\omega^3 + 3\omega^2 - 3 \end{aligned}\right\} \qquad (1)$$

Because it was only necessary to read, without any interpolation, from existing tables of standardized percentage points of these systems, their comparison has been carried out, with certain exceptions, at a "grid" of beta points namely for $\sqrt{\beta_1} = 0.0(0.3)1.8,\ 2.0$ and $\beta_2 = 2.2(0.4)9.0,\ 9.8,\ 10.6,\ 11.8,\ 12.6,\ 13.8$.

The discussion of the comparisons given in the tables will be aided by a simultaneous study of Fig. 1. We did not carry the comparison far into the Pearson Type I J-curve area nor into the region at the bottom left hand corner of the diagram where the difference between the shape of the Pearson and Johnson curves had become so large that comparisons ceased to be useful. In Tables B as in A, the standardized percentage points for the Pearson distribution are given at every point of comparison to three decimal places, followed by the amount (in 0.001's) to *add* to these values to obtain those for other systems. This was done to economize space and not because the Pearson curve values were regarded as necessarily the most important. Also, so as not to overcrowd the diagram, certain of the grid points treated in Tables A and B have been omitted in Fig. 1.

3.3. Other distributions included.

These may be classed in two categories according to the restriction on the beta points.

(a) In the first, these points fall within certain *areas* in Fig. 1, and two independent shape parameters are involved: the Burr, the non-central t and non-central χ^2 distributions are in this category.

(b) In the second, the points lie on a line, involving only a single independent shape parameter: the log-normal, log χ^2 and Weibull distributions are in this category. Consider these restrictions in turn.

3.3.1. The Burr system.

The beta points available for this system cover a very broad area (Burr, 1973). See the appendix. The beta points chosen for this study correspond to most of the grid points selected for Pearson Types I, IV and VI (bell-shaped) and for Johnson S_B and S_U.

3.3.2. The non-central t system

The beta points fall in an area lying between the axis of β_2 where we have central t (or Student) distributions and a curved line along which the non-central parameter, δ, becomes infinite. This was shown by Merrington and Pearson, (1958), to be the line on which the beta points for the distribution of the reciprocal of χ fall. It lies just below the curve

$$\beta_1(\beta_2+3)^2 = 4(4\beta_2-3\beta_1)(2\beta_2-3\beta_1-6) \qquad (2)$$

along which fall the beta points of a Pearson Type V, i.e. of a distribution of the reciprocal of χ^2.

Here, because of the labour involved in deriving the two parameters ν and δ, given $\sqrt{\beta_1}$ and β_2, we did not use the "grid" points, but found standardized percentage points for eight fairly widely dispersed cases, some of which had already been derived in earlier papers (e.g. Pearson, 1963). (See Fig. 1 and Table B2.)

3.3.3. The non-central χ^2 system.

The beta points fall (Pearson, 1959, p. 364) in the region between the lines

$$\beta_2 - \frac{3}{2}\beta_1 - 3 = 0 \quad \text{and} \quad \beta_2 - \frac{4}{3}\beta_1 - 3 = 0 \ . \qquad (3)$$

We have chosen three points to examine in this area (Fig. 1 and Table B4).

3.3.4. The log-normal distribution

The parametric equation of the log-normal line has already been given in equation (1) above. As shown in Table B1, we picked out for study the ten distributions for which β_2 = 3.4, 3.8(0.8)8.6, 9.8, 10.6.

3.3.5. The log χ^2 distribution.

We have here taken three beta-points for which among others the parameters were given by Bartlett and Kendall (1946). The positions are shown in Fig. 1 and the standardized percentage points in Table B5. The distributions are negatively skew, and have been reversed in the table.

3.3.6. The Weibull distribution.

Harter and Dubey (1967) gave, as an Appendix to their Report, a table relating the parameter m (their M) to the first eight standardized cumulants of the distribution. We have examined nine distributions, those with

$$m = 1.1, 1.3, 1.5, 1.7, 2.0, 2.5, 3.6, 7.0, 10.0.$$

For $m = 3.6$, $\sqrt{\beta_1} = 0.000$, $\beta_2 = 2.717$, i.e. a normal distribution is not included in the system; for greater values of m the distribution is negatively skew, and it has therefore been reversed so that the beta points for $m = 7.0$ and 10.0 can be included in our field of study. The values of $\sqrt{\beta_1}$, β_2 associated with these nine values of m are shown with the standardized percentage points in Table B3 and the $(\sqrt{\beta_1}, \beta_2)$ points are plotted in Fig. 1.

4. DISCUSSION OF THE NUMERICAL COMPARISONS SHOWN IN TABLES A AND B

4.1. The "cross-over" points.

Tables A and B would provide if required,15 points on the standardized cumulative distribution curves of each of the large number of distributions considered. If these curves were to be drawn for each of the two - or three - distributions compared at a given (β_1, β_2) point, it would be found that they would have three, and sometimes four or even five cross-over points with the correspondong Pearson curve. The existence of these crosses results from tying down the distributions to have common first four moments. While we should have liked to give a diagrammatic illustration of this property, in even quite extreme cases of disagreement of the percentage points of, say, a Burr and a Pearson curve, the cumulative curves lie too closely together, having regard to the distances between the extreme upper and lower 0.25% points, for *visual* representation to be helpful.

The arithmetical results in the Tables, however, make it possible to assess with reasonable accuracy where the crosses occur. Take, for example, the case of $\beta_2 = 5.4$, $\sqrt{\beta_1} = 0.9$ given in Table A4: it will be seen that both the S_U and Burr curves have cross-overs with the Pearson Type IV, (a) near the median (P = 0.50), (b) between the lower 2.5% and 5% points (P = 0.025 and 0.05), and (c) between the upper 10% and 5% points (P = 0.90 and 0.95).

Though there is much variation in these cross-overs from case to case, probably it is the median and the two 5% points which most frequently occur in designating their location.

4.2. Comparison of the Pearson, log-normal and Johnson distributions (S_U and S_B)

Tables A have been arranged in pairs, facing each other, to make an extensive survey of changes as easy as possible. We can only refer to a few of the points suggested by this survey. The S_U and S_B regions are separated by the log-normal line of equation (1). One of the most striking results brought out in the present study is the closeness in agreement between the standardized percentage points of the log-normal and the corresponding Type VI distributions. This is shown by the differences given in Table B1; we have not succeeded in finding a mathematical explanation of this phenomenon. The differences gradually increase as we pass down the log-normal line, starting from the Normal point. If we take as an arbitrary but useful yard-stick a difference as great as ±0.010 or 1/100th of the standard deviation it is seen that this value is only exceeded: (a) at the *lower* 1% point when β_2 reaches 6.2, (b) at the lower 2.5% point when β_2 reaches 7.8 and (c) has only just got there at the lower 5% point where our table cuts off the line at $\beta_2 = 10.6$. For the corresponding *upper* percentage points a difference of 0.010 is not reached at all. As elsewhere, agreement is less satisfactory in the lower steep tails of the distributions than at the upper, drawn-out tails.

If we move "south-westwards" from the log-normal line into the S_U area we find that the differences S_U-PC gradually increase until they become really large, particularly in the lower tail. Using the same 0.010 difference yardstick, we find the results shown in Table 2.

TABLE 2. Absolute differences between standardized 5% points of S_U and corresponding Pearson curve exceeding 0.010.

When	At lower tail	At upper tail
$\sqrt{\beta_1}$ = 0.0	when β_2 > 7.8	β_2 > 7.8
0.3	> 7.0	> 8.6
0.6	> 5.6	> 9.0
0.9	> 5.8	> 9.6
1.2	> 7.4	> 9.8
1.5	> 10.4	>10.0
1.8	outside tables	

If these limiting points were inserted in Fig. 1 they would include a large part of the S_U area.

Moving "north-eastwards" from the log-normal line, the differences between the S_B and the Pearson curve distribution increase rapidly but as pointed out above the position would look rather different if the beta-points of the tabulation had increased by equal steps in β_1 rather than $\sqrt{\beta_1}$. Also, as the boundary for Pearson Type I J-curves is approached agreement in the lower tail is hardly to be expected. In the lower half of the distributions agreement is much better, e.g. for $\beta_2 = 3.8$, $\sqrt{\beta_1} = 0.9$ in Table A2.

4.3. Comparison of the Burr with the Pearson and Johnson distributions.

One of the most notable characteristics seen in Tables A is that the differences between the standardized percentage points of (a) Johnson and Pearson distributions and (b) Burr and Pearson distributions at a given beta point are of opposite sign. While it is not wise to generalize without detailed analysis, it seems that towards the Normal point and the log-normal line, the differences (b) are larger, often much larger, than the differences (a).

One interesting set of comparisons was made for us by Mr. N.W. Please at $\beta_2 = 10.8635$, $\sqrt{\beta_1} = 2.0$, a point lying on the log-normal line, but not included in Tables A or B1. Judging from the differences LN - PC when $\beta_2 = 10.6$, $\sqrt{\beta_1} = 1.969$ given in the latter table, we should expect fairly small differences at Please's beta point, except perhaps for the lower 2.5, 1.0, 0.5 and 0.25% points. He computed the standardized moments μ_r/σ^r for $r = 3(1)8$, with results shown in Table 3.

TABLE 3. Comparison of standardized moments, μ_r/σ^r, of three distributions having $\beta_2 = 10.86$, $\sqrt{\beta_1} = 2.00$.

r	Pearson Type IV	Log-normal	Burr*
3	2.00	2.00	2.00
4	10.8635	10.8635	10.8635
5	71.84	69.96	75.12
6	705.2	638.9	844.9
7	10,209.3	7,859.9	18,089.4
8	235,007.3	129,791.6	2,500,459.8

Note the way in which the moment ratios for the Type VI and log-normal keep relatively close together, while those for the Burr distribution shoot off in an opposite direction, as r increases.

Clearly there will be distributions, observational or theoretical, better fitted by Burr curves than by the perhaps more widely used Pearson and Johnson curves, but it is not known how far this matter has been explored.

* Figures derived by Mr. Please from Gruska *et al.* (1973).

4.4. Comparison of non-central t (say t') with Pearson Type IV distributions.

The eight comparisons made in Table B2 amply confirm Merrington and Pearson's (1958) finding of the close agreement between the distributions of t' and Type IV, having the same first four moments. With the exception of the single case where $\beta_2 = 12.219$, $\sqrt{\beta_1} = 1.732$ ($\nu = 6$, $\delta = 2.65$) the differences are surprisingly small, on the whole indeed smaller than those for the log-normal and Type VI distributions shown in Table B1. From the mathematical aspect Merrington and Pearson pointed out that the p.d.f. of t' contained the factor $(1 + t^2/\nu)^{-(\nu+1)/2}$ while that of Type IV contained the factor $(1 + x^2/a^2)^{-m}$.

4.5. Comparison of non-central χ^2 with Pearson Type I distributions.

Three comparisons are made in Table B4. For the first case where $\beta_2 = 3.296$, $\sqrt{\beta_1} = 0.468$ and the parameters $\nu = 6$, $\sqrt{\lambda} = 6$ are of medium size the differences (non-central χ^2-PC) are small; here, the beta-point is not far from the Normal point.

If only 3 beta points were to be included, looking back it is seen that the second and third cases were not very well chosen since with $\nu = 1$ and 2 the distributions of non-central χ^2 are very skew. For *central* χ^2, with $\lambda = 0$, the distributions are J-shaped, with $\beta_1 = 8.0$, $\beta_2 = 15.0$ when $\nu = 1$, and $\beta_1 = 4.0$, $\beta_2 = 9.0$ (the exponential) when $\nu = 2$. It is not surprising therefore that the differences (non-central χ^2-PC) are so large.

4.6. Comparison of Weibull with Pearson and S_B distributions.

Table B3 shows that the absolute values of the differences (W-PC) between the lower and upper 5% points do not exceed the yard stick 0.010 for the parameter $m \leq 2.0$ and when $m > 2.0$ the largest difference is 0.014.

What evidence there is suggests that a Weibull is closer than an S_B to a Pearson Type I distribution.

(a) Compare the differences (W-PC) for $\beta_2 = 3.772$, $\sqrt{\beta_1} = 0.865$ in Table B3 with those for (S_B-PC) at the neighbouring grid point given in Table A2.

(b) To test this further we have made the following special comparison between the standardized percentage points of the distributions: Weibull (m = 1.3) and Pearson curve, both with $\beta_2 = 5.432$, $\sqrt{\beta_1} = 1.346$ (see Table B3) and S_B with $\beta_2 = 5.40$, $\sqrt{\beta_1} = 1.35$. We find the figures in Table 4.

TABLE 4. Comparison of standardized percent points for Weibull, S_B and Pearson distributions, having moment ratios in the neighborhood of $\beta_2 = 5.4$, $\sqrt{\beta_1} = 1.35$. (Differences in 0.001's).

P	W	W-PC	W-S_B	P	W	W-PC	W-S_B
0.0025	-1.275	42	133	0.75	0.505	4	18
.005	-1.265	37	117	.90	1.362	2	7
.01	-1.248	30	88	.95	1.957	-2	-15
.025	-1.206	18	47				
0.05	-1.147	8	16	0.975	2.520	-8	-36
.10	-1.042	-2	-9	.99	3.229	-11	-48
.25	-0.754	-7	-23	.995	3.744	-10	-39
				.9975	4.243	-6	-11
0.50	-0.236	-1	-3				

In certain situations there are thought to be physical reasons suggesting that variation will be of Weibull form. However, when this is not the case, and we have neither simulation data nor knowledge of higher moment ratios, it would seem that we should make a choice from these three distributions, Weibull, S_B and Pearson Type I, according to simplicity in computation.

4.7. Comparison of log χ^2 with Pearson distributions.

This has been made in Table B5 at a selection of three beta-points, which are seen from Fig. 1 to lie very close to the curve dividing the Type VI and Type IV areas. Agreement is excellent when the degrees of freedom of χ^2 are $\nu = 10$ and 4. When $\nu = 2$ the correspondence deteriorates outside the 2.5% points. Note that the distributions of log χ^2 are negatively skew, and therefore the position of the tails has been reversed in the table.

5. CONCLUSION: ILLUSTRATIONS OF APPLICATIONS

5.1. The percentage points of the range (w) in samples from a Normal population.

On page 4 it was described how Pearson curves having approximately correct values of β_1,β_2 were used in 1932 to estimate the positions of certain percentage points of w. Had Johnson's S_B system been developed at that date it would have been realized that an S_B curve was a possible alternative, approximating distribution to use. Then, the percentage points of w, say at n = 3 to 12 could have been calculated and compared for both Type I and S_B systems. To two decimal places the differences might have been slight and this would perhaps have given increased confidence in whatever final values were adopted and used, e.g. in industrial quality control problems. It would however only have been possible to decide whether a Type I or S_B approximation (if they differed) was the more accurate, when the true values were derived by direct computation (Pearson and Hartley, 1942).

5.2. The distribution of $b_2 = m_4/s^4$ in samples from a Normal population.

This interesting problem on which a considerable amount of attention has been focussed for nearly 50 years shows how in spite of the knowledge of the true sampling moments of b_2 up to the sixth and the collection of literally tens of thousands of simulated samples, no completely acceptable answer has been found. The (β_1, β_2) points of b_2 for large values of n were plotted in a chart published by Pearson (1963, p. 106). These were largely derived from an unpublished PhD thesis by C.T. Hsu (1939). The numerical values for eight values of n are given in columns 2 and 3 of the accompanying Table 5. For some of these cases there are also available the 5th and/or 6th order moment or cumulant ratios. To aid the presentation, the $\sqrt{\beta_1}$, β_2 points are plotted in Fig. 2 and also some other points and bounding lines.

Without lengthy calculation which it is hardly worth undertaking, we cannot be sure, as n increases, exactly at what sample sizes the points $(\sqrt{\beta_1}(b_2), \beta_2(b_2))$ cross over from one "type region" to another. The critical boundaries are:

(i) the log-normal line, separating S_B from S_U;

(ii) the Type V line* (reciprocal of χ^2) separating Type VI from Type IV;

(iii) the reciprocal of χ line, the upper boundary of the non-central t area.

TABLE 5. Data regarding the distribution of b_2 for eight values of the sample size, n.

n	$\sqrt{\beta_1}$	β_2	μ_5/σ^5	μ_6/σ^6	κ_5/σ^5	κ_6/σ^6	Possible approximating systems	Source of data
25	1.75	8.90	46.1	309	28.7	175	Type IV, S_B	(a)
40	1.66	8.78	47.6	352	31.0	223	Type VI, S_U	(a)
50	1.5821	8.4164			29.30			(b)
60	1.51	8.03	42.0					(a)
75	1.4099	7.4933			23.48		Type IV, S_U,	(b)
100	1.2772	6.7740	31.4		18.66		Non-central t	(b)
150	1.0917	5.8258			12.51			(b)
200	0.9677	5.2487	18.7		9.04			(b)

References: (a) Pearson (1963) pp. 105-8, (b) Pearson (1965), p. 284.

*It has been established by computation that the beta-point for n = 50 falls just across this boundary, i.e. in the non-central t area.

On the basis of this diverse information and assisted by extensive simulation sampling, Pearson and D'Agostino (1973) proceeded as described on pp. 614-18 of their paper and produced the contour charts of probability levels for b_2 displayed on pp. 615, 616.

We have spent so much time on this illustration partly to warn the statistician that in spite of the rather elegant results displayed in Table 1, he must not hope too much from the 4-moment method of attack. The beta-point approach, with a study of our Tables A and B will undoubtedly often provide a method of entry to the process of finding an approximation to a mathematically unknown distribution. But the further the beta-point is from that of a Normal curve, the more difficult it will be to find a solution in which we can have confidence, particularly in the tails. The moment results need to be backed by an extensive simulation programme, the extent of which unfortunately may be found to be prohibitive.

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A great deal of computational work underlies the figures presented in Tables A and B. Some of this was carried out nearly 40 years ago and acknowledgement for help given has been made in the earlier papers. Most of the further calculations required to fill gaps in the rounded-off results now presented was carried out by the authors of the paper themselves, but we should like to thank for some recent help given us by Mr. Neil Please of University College, London and Mr. William Parr of the Southern Methodist University, Dallas, Texas.

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APPENDIX

Notes regarding the families of frequency curves compared in this paper and tables which have been useful to us in determining standardized percentage points. (N.B. B.T.S. 2 stands for Pearson and Hartley's *Biometrika Tables for Statisticians, Vol. 2* (1972).

Pearson curves.

The equations of the main curves are listed on p. 77 of the Introduction to B.T.S. 2, and the standardized percentage points are given in Table 32 of that volume to arguments $\sqrt{\beta_1}$, β_2. A rather fuller table of these points, used in checking and expanding this Table 32, was computed by Amos and Daniel, *Sandia Laboraories Report* (1971), No. SC-RR-71-0348.

Johnson S_U.

The distribution of X when

$$Z = \gamma + \delta \sinh^{-1}\{(X-\xi)/\lambda\} \qquad (\delta,\lambda > 0)$$

is a unit normal variable.

B.T.S. 2 gives values of $-\gamma$ (Table 34) and δ (Table 35) to arguments $\sqrt{\beta_1},\beta_2$. The second impression (1976) contains a corrected Table 34. A table of standardized percentage points of S_U to arguments $\sqrt{\beta_1},\beta_2$ (corresponding to B.T.S. 2 Table 32 for Pearson curves) was computed by N.L. Johnson and issued as No. 408 (1964) of the *Department of Statistics (UNC Chapel Hill) Mimeo Series.*

Johnson S_B.

The distribution of X when

$$Z = \gamma + \delta \log\{(X-\xi)/(\xi+\lambda-X)\} = \gamma + \delta \log\{y/(1-y)\} \quad (\delta,\lambda>0;\ \xi<X<\xi+\lambda)$$

is a unit normal variable.

B.T.S. 2, Table 36 gives values of γ, δ; $\mu_1'(y)$ and $\sigma(y)$ to arguments $\sqrt{\beta_1},\beta_2$.

Burr curves.

The cumulative distribution function of X is

$$F(x) = \Pr\{(X \le x)\} = 1 - (1+x^c)^{-k} \quad , \quad c,k,x > 0 \ .$$

For given c and k, one can find the moments μ, σ, $\sqrt{\beta_1}$, β_2. Then for any given value of x, the corresponding standardized variable is $y = (x-\mu)/\sigma$. $\sqrt{\beta_1}$ and β_2 can be found in terms of c and k, but c and k cannot be found explicitly in terms of desired $\sqrt{\beta_1}$ and β_2. Thus successive approximation is needed. Burr (1973) provides a wide coverage of c,k for given $\sqrt{\beta_1}$, β_2. Moreover further coverage into relatively low β_2's can be made through letting c be negative. Thus

$$G(x) = \Pr\{(X \le x)\} = (1+x^{-c})^{-k} \quad c,k,x > 0 \ .$$

These two families of distribution functions together cover an extremely wide area of $\sqrt{\beta_1}$, β_2 combinations.

Non-central t.

The distribution of

$$t' = \frac{(X+\lambda)\sqrt{\nu}}{\chi_\nu} = \frac{X+\lambda}{(\chi_\nu^2/\nu)^{\frac{1}{2}}} \quad (-\infty < t' < \infty, \ \nu > 0)$$

where X is a unit normal variable and χ_ν^2, independent of X, follows the standard χ^2 distribution, having ν degrees of freedom. Certain percentage points of t' (not standardized) may be obtained from B.T.S. 2 Table 26 and from (ii) Locks *et al.*'s *New Tables of the Noncentral t Distribution*, Aeronautical Research Laboratories Report (1963) No. A.R.L. 63-19, Tables III and VI.

Non-central χ^2.

The distribution of

$$(\chi')^2 = \sum_{i=1}^{\nu} (X_i+a_i)^2 \quad (0 < \chi'^2 < \infty, \ a_i > 0) \ ,$$

where X_i $(i = 1,2,\ldots,\nu)$ are ν independent unit normal variables and

$$\lambda = \sum_{i=1}^{\nu} a_i^2 .$$

Certain percentage points of χ' are given for arguments ν and $\sqrt{\lambda}$ in B.T.S. 2, Tables 24 and 29. This table was derived from a table of percentage points of non-central χ^2 distributions, computed by N.L. Johnson and issued as No. 568 (1968) of the *Department of Statistics (UNC Chapel Hill) Mimeo Series*.

Note that

$$\text{Mean}(\chi')^2 = \nu + \lambda, \ \text{Var}(\chi')^2 = 2(\nu + 2\lambda) .$$

Weibull curves.

The cumulative distribution function of x is

$$F(x|m) = \Pr\{X \geq x|m\} = 1 - \exp[-x^m] , \quad (m > 0, x > 0) .$$

For given values of m and $F(x|m)$ the percentage points of x can be found by inversion of this equation. Harter and Dubey (see main list of References for fuller details of their *Report*) have given in an appendix the mean, variance, $\sqrt{\beta_1}$, β_2 and the 5th up to the 8th standardized cumulants of x for $m = 1.1(0.1)10.0$.

TABLE A.1. Standardized % points for P.C. with $\beta_2, \sqrt{\beta_1}$ specified; also amounts to add to obtain Johnson and Burr values.

	$\beta_2=2.2$ $\sqrt{\beta_1}=0$			$\beta_2=2.2$ $\sqrt{\beta_1}=0.3$			$\beta_2=2.2$ $\sqrt{\beta_1}=0.6$		
P	Type II	S_B	Burr	Type I	S_B	Burr	Type I(J)	S_B	Burr
0.0025	-2.166	-20	109	-1.724	-62	181	-1.212	-73	35
.005	-2.101	-9	79	-1.703	-45	164	-1.211	-68	34
.01	-2.010	0	44	-1.668	-27	139	-1.210	-59	34
.025	-1.833	6	-3	-1.585	-6	89	-1.204	-40	31
.05	-1.636	6	-42	-1.474	7	38	-1.187	-20	25
.10	-1.354	3	-43	-1.287	11	-17	-1.137	4	11
.25	-0.766	-2	-4	-0.817	2	-51	-0.893	19	-15
0.50	0.000	0	47	-0.085	-6	17	-0.231	-9	-7
0.75	0.766	2	-6	0.744	2	44	0.749	1	25
.90	1.354	-3	-91	1.427	5	-51	1.557	10	-20
.95	1.636	-6	-92	1.763	-1	-99	1.898	-1	-57
.975	1.833	-6	-33	2.000	-7	-92	2.100	-12	-55
.99	2.010	0	22	2.211	-10	-1	2.243	-16	14
.995	2.101	9	189	2.318	-8	121	2.300	-14	102
.9975	2.166	20	494	2.395	-4	281	2.332	-8	212

	$\beta_2=2.6$ $\sqrt{\beta_1}=0.3$			$\beta_2=2.6$ $\sqrt{\beta_1}=0.6$			$\beta_2=2.6$ $\sqrt{\beta_1}=0.9$	
P	Type I	S_B	Burr	Type I	S_B	Burr	Type I(J)	S_B
0.0025	-2.085	-31	376	-1.490	-88	198	-0.974	-61
.005	-2.006	-18	309	-1.479	-70	188	-0.974	-59
.01	-1.904	-8	230	-1.459	-50	170	-0.974	-56
.025	-1.722	3	111	-1.408	-22	130	-0.973	-50
.05	-1.532	7	19	-1.333	-2	79	-0.972	-37
.10	-1.273	6	-56	-1.198	12	11	-0.962	-17
.25	-0.752	2	-68	-0.820	10	-67	-0.865	22
0.50	-0.064	-3	35	-0.161	-7	-9	-0.375	2
0.75	0.687	1	57	0.678	-3	69	0.671	-10
.90	1.366	4	-49	1.458	10	-24	1.651	18
.95	1.754	3	-112	1.882	10	-104	2.056	5
.975	2.070	0	-128	2.206	2	-134	2.281	-13
.99	2.409	-6	-72	2.521	-13	-81	2.426	-24
.995	2.619	-10	31	2.697	-25	27	2.478	-26
.9975	2.797	-14	183	2.832	-35	188	2.505	-23

	$\beta_2=3.0$ $\sqrt{\beta_1}=0$		$\beta_2=3.0$ $\sqrt{\beta_1}=0.3$			$\beta_2=3.0$ $\sqrt{\beta_1}=0.6$			$\beta_2=3.0$ $\sqrt{\beta_1}=0.9$	
P	Normal=$S_B S_U$	Burr	Type I	S_B	Burr	Type I	S_B	Burr	Type I(J)	S_B
0.0025	-2.807	71	-2.382	-6	96	-1.786	-62	382	-1.163	-100
.005	-2.576	30	-2.233	-3	57	-1.738	-44	336	-1.162	-90
.01	-2.326	1	-2.063	-1	24	-1.674	-27	278	-1.159	-77
.025	-1.960	-20	-1.797	1	-6	-1.550	-7	176	-1.149	-51
.05	-1.645	-22	-1.552	2	-17	-1.410	3	80	-1.127	-25
.10	-1.282	-14	-1.252	2	-17	-1.209	9	-20	-1.070	2
.25	-0.674	2	-0.710	1	-3	-0.766	4	-93	-0.837	22
0.50	0.000	10	-0.053	-1	9	-0.126	-5	4	-0.266	-3
0.75	0.674	0	0.653	-1	4	0.640	-5	82	0.624	-13
.90	1.282	-13	1.322	0	-10	1.390	4	-16	1.521	14
.95	1.645	-16	1.733	0	-16	1.844	8	-103	2.011	19
.975	1.960	-12	2.094	1	-16	2.231	8	-154	2.377	8
.99	2.326	4	2.519	-1	-7	2.664	0	-141	2.717	-18
.995	2.576	23	2.809	-2	9	2.944	-10	-66	2.897	-38
.9975	2.807	51	3.078	-4	33	3.190	-23	66	3.029	-56

P	$\beta_2=3.4$ $\sqrt{\beta_1}=0$ Type VII	S_U	Burr	$\beta_2=3.4$ $\sqrt{\beta_1}=0.3$ Type IV	S_U	Burr	$\beta_2=3.4$ $\sqrt{\beta_1}=0.6$ Type I	S_B	Burr	$\beta_2=3.4$ $\sqrt{\beta_1}=0.9$ Type I	S_B	Burr	$\beta_2=3.4$ $\sqrt{\beta_1}=1.2$ Type I(J)	S_B
0.0025	-3.002	-2	59	-2.608	-8	110	-2.052	-24	108	-1.377	-107	254	-0.865	-67
.005	-2.706	-3	14	-2.396	-5	57	-1.953	-16	73	-1.366	-88	243	-0.865	-65
.01	-2.402	-3	-15	-2.170	-1	16	-1.835	-8	42	-1.349	-64	227	-0.865	-63
.025	-1.980	-2	-29	-1.840	0	-17	-1.639	-2	7	-1.303	-33	184	-0.865	-57
.05	-1.636	0	-26	-1.558	1	-25	-1.449	2	-9	-1.238	-9	129	-0.864	-47
.10	-1.256	0	-14	-1.232	1	-21	-1.203	3	-16	-1.212	10	46	-0.860	-28
.25	-0.650	1	5	-0.681	0	-2	-0.728	2	-8	-0.795	16	-79	-0.803	20
0.50	0.000	0	11	-0.045	-1	11	-0.105	-2	7	-0.207	-3	-46	-0.433	19
0.75	0.650	-1	0	0.630	-1	6	0.616	-3	7	0.598	-10	87	0.537	-24
.90	1.256	0	-15	1.289	0	-11	1.342	0	-5	1.437	7	23	1.644	20
.95	1.636	0	-18	1.712	2	-19	1.808	2	-13	1.951	17	-84	2.190	23
.975	1.980	2	-15	2.101	3	-22	2.230	3	-18	2.384	19	-160	2.535	0
.99	2.402	3	1	2.583	3	-14	2.738	2	-15	2.858	4	-175	2.793	-33
.995	2.706	3	22	2.932	1	2	3.094	-1	-4	3.155	-16	-112	2.898	-49
.9975	3.002	2	49	3.273	-2	27	3.430	6	15	3.409	-43	18	2.960	-58

P	$\beta_2=3.8$ $\sqrt{\beta_1}=0$ Type VII	S_U	Burr	$\beta_2=3.8$ $\sqrt{\beta_1}=0.3$ Type IV	S_U	Burr	$\beta_2=3.8$ $\sqrt{\beta_1}=0.6$ Type IV	S_U	Burr	$\beta_2=3.8$ $\sqrt{\beta_1}=0.9$ Type I	S_B	Burr
0.0025	-3.145	-11	21	-2.780	-19	105	-2.271	-15	144	-1.602	-82	406
.005	-2.798	-12	-14	-2.515	-12	38	-2.119	-10	89	-1.567	-61	372
.01	-2.453	-10	-32	-2.243	-7	-2	-1.952	-4	45	-1.518	-41	324
.025	-1.990	-5	-34	-1.866	-1	-28	-1.697	0	1	-1.421	-17	234
.05	-1.626	-1	-24	-1.558	2	-30	-1.469	2	-18	-1.308	-2	138
.10	-1.236	2	-8	-1.216	4	-20	-1.194	2	-22	-1.141	8	24
.25	-0.633	3	10	-0.660	2	1	-0.700	1	-9	-0.759	9	-105
0.50	0.000	0	9	-0.040	-2	12	-0.091	-1	9	-0.172	-2	-38
0.75	0.633	-3	-5	0.614	-3	4	0.599	-1	10	0.581	-7	97
.90	1.236	-2	-15	1.263	1	-12	1.307	1	-5	1.381	1	31
.95	1.626	1	-14	1.694	4	-20	1.778	2	-16	1.900	11	-73
.975	1.990	5	-6	2.101	8	-21	2.221	3	-24	2.368	17	-158
.99	2.453	10	12	2.624	10	-12	2.779	3	-25	2.925	15	-204
.995	2.798	12	30	3.016	9	4	3.189	2	-15	3.307	5	-175
.9975	3.145	11	52	3.411	4	26	3.594	0	3	3.660	-11	-84

TABLE A.2. Standardized % points for P.C. with β_1, $\sqrt{\beta_2}$ specified; also amounts to add to obtain Johnson and Burr values.

TABLE A.3. Standardized % points for P.C. with β_2, $\sqrt{\beta_1}$ specified; also amounts to add to obtain Johnson and Burr values.

	$\beta_2=4.2$ $\sqrt{\beta_1}=0$		$\beta_2=4.2$ $\sqrt{\beta_1}=0.3$			$\beta_2=4.2$ $\sqrt{\beta_1}=0.6$			$\beta_2=4.2$ $\sqrt{\beta_1}=0.9$			$\beta_2=4.2$ $\sqrt{\beta_1}=1.2$			$\beta_2=4.2$ $\sqrt{\beta_1}=1.5$	
P	Type VII	S_U	Type IV	S_U	Burr	Type IV	S_U	Burr	Type I	S_U	Burr	Type I(J)	S_B	Burr	Type I(J)	S_B
0.0025	-3.254	-24	-2.912	-32	40	-2.446	-37	155	-1.814	-40	115	-1.148	-126	200	-0.721	-50
.005	-2.866	-23	-2.603	-24	-3	-2.247	-23	88	-1.743	-29	85	-1.145	-110	197	-.721	-50
.01	-2.488	-19	-2.296	-14	-27	-2.037	-11	35	-1.655	-19	55	-1.140	-91	192	-.721	-50
.025	-1.995	-10	-1.883	-3	-35	-1.736	0	-9	-1.504	-7	19	-1.123	-57	176	-.721	-48
.05	-1.617	-2	-1.556	3	-27	-1.479	4	-26	-1.352	0	-1	-1.091	-29	146	-.721	-45
.10	-1.220	4	-1.202	6	-12	-1.184	6	-26	-1.147	4	-18	-1.024	2	89	-.721	-36
.25	-0.620	6	-0.644	4	8	-0.680	4	-7	-0.731	4	-12	-0.789	25	-48	-.710	0
0.50	0.000	0	-0.037	-2	11	-0.082	-2	11	-0.149	0	-3	-0.274	5	-88	-0.520	39
0.75	0.620	-6	0.602	-6	-3	0.587	-5	10	0.569	-2	10	0.532	-18	75	0.379	-26
.90	1.220	-4	1.244	-1	-14	1.281	0	-6	1.340	1	1	1.445	0	67	1.686	18
.95	1.617	2	1.679	6	-15	1.754	6	-18	1.860	5	-10	2.029	21	-49	2.342	33
.975	1.995	10	2.098	13	-8	2.209	10	-26	2.347	10	-18	2.533	31	-155	2.734	4
.99	2.488	19	2.650	20	8	2.802	13	-27	2.957	12	-22	3.096	18	-216	2.999	-37
.995	2.866	23	3.075	21	25	3.253	11	-20	3.398	12	-17	3.453	-8	-181	3.096	-56
.9975	3.254	24	3.513	16	43	3.709	6	-3	3.827	7	-4	3.761	-47	-73	3.147	-65

	$\beta_2=4.6$ $\sqrt{\beta_1}=0$		$\beta_2=4.6$ $\sqrt{\beta_1}=0.3$		$\beta_2=4.6$ $\sqrt{\beta_1}=0.6$			$\beta_2=4.6$ $\sqrt{\beta_1}=0.9$			$\beta_2=4.6$ $\sqrt{\beta_1}=1.2$			$\beta_2=4.6$ $\sqrt{\beta_1}=1.5$	
P	Type VII	S_U	Type IV	S_U	Type IV	S_U	Burr	Type IV	S_U	Burr	Type I	S_B	Burr	Type I(J)	S_B
0.0025	-3.338	-41	-3.016	-48	-2.586	-55	133	-2.000	-24	161	-1.429	-120	314	-0.815	-87
.005	-2.917	-37	-2.671	-36	-2.346	-36	65	-1.890	-15	112	-1.395	-98	302	-.815	-85
.01	-2.514	-29	-2.335	-23	-2.101	-19	15	-1.763	-9	66	-1.352	-74	283	-.815	-81
.025	-1.997	-14	-1.894	-6	-1.762	-2	-23	-1.564	-2	17	-1.273	-40	239	-.815	-72
.05	-1.608	-3	-1.553	4	-1.485	7	-31	-1.379	2	-7	-1.184	-14	180	-.814	-59
.10	-1.207	6	-1.191	10	-1.175	10	-25	-1.147	3	-20	-1.053	8	87	-.810	-35
.25	-0.609	8	-0.632	8	-0.664	6	-3	-0.708	2	-15	-0.746	19	-79	-.755	19
0.50	0.000	0	-0.034	-2	-0.075	-3	13	-0.132	-1	4	-0.229	4	-90	-0.432	29
0.75	0.609	-8	0.592	-9	0.577	-7	8	0.560	-1	13	0.516	-13	88	0.426	-28
.90	1.207	-6	1.228	-3	1.260	-1	-8	1.310	0	1	1.389	-3	77	1.553	2
.95	1.608	3	1.666	6	1.734	8	-18	1.828	2	-11	1.984	13	-35	2.227	37
.975	1.997	14	2.094	18	2.198	16	-23	2.326	4	-22	2.526	26	-148	2.739	35
.99	2.514	29	2.668	31	2.816	24	-21	2.971	6	-30	3.162	28	-236	3.212	-9
.995	2.917	37	3.119	35	3.297	24	-12	3.456	5	-28	3.587	16	-236	3.454	-52
.9975	3.338	41	3.591	33	3.794	19	4	3.943	3	-19	3.966	-10	-169	3.626	-92

	$\beta_2=5.0$ $\sqrt{\beta_1}=0$		$\beta_2=5.0$ $\sqrt{\beta_1}=0.3$		$\beta_2=5.0$ $\sqrt{\beta_1}=0.6$			$\beta_2=5.0$ $\sqrt{\beta_1}=0.9$			$\beta_2=5.0$ $\sqrt{\beta_1}=1.2$			$\beta_2=5.0$ $\sqrt{\beta_1}=1.5$	
P	Type VII	S_U	Type IV	S_U	Type IV	S_U	Burr	Type IV	S_U	Burr	Type I	S_B	Burr	Type I(J)	S_B
0.0025	-3.405	-59	-3.099	-65	-2.700	-72	58	-2.158	-61	191	-1.475	-91	70	-0.913	-119
.005	-2.958	-51	-2.725	-49	-2.424	-49	17	-2.009	-39	125	-1.445	-71	57	-0.913	-112
.01	-2.534	-38	-2.365	-32	-2.150	-27	-22	-1.848	-21	68	-1.404	-50	43	-0.913	-102
.025	-1.998	-17	-1.901	-10	-1.782	-3	-35	-1.608	-3	12	-1.321	-24	22	-0.911	-81
.05	-1.601	-2	-1.549	4	-1.488	8	-29	-1.397	6	-14	-1.226	-6	8	-0.905	-57
.10	-1.196	10	-1.181	12	-1.167	13	-15	-1.144	9	-25	-1.081	7	-5	-0.885	-23
.25	-0.601	12	-0.622	11	-0.651	9	6	-0.691	6	-15	-0.743	12	-10	-0.767	26
0.50	0.000	0	-0.032	-2	-0.069	-4	11	-0.120	-3	6	-0.204	1	-1	-0.365	21
0.75	0.601	-12	0.584	-12	0.570	-11	-1	0.553	-5	15	0.526	-10	-3	0.449	-25
.90	1.196	-10	1.215	-6	1.244	-4	-14	1.286	-1	2	1.353	-5	3	1.471	-7
.95	1.601	2	1.655	6	1.718	8	-16	1.802	4	-12	1.925	5	-4	2.135	27
.975	1.998	17	2.090	22	2.188	21	-10	2.307	11	-24	2.467	15	-11	2.698	44
.99	2.534	38	2.681	41	2.825	35	5	2.978	16	-36	3.148	23	-16	3.302	25
.995	2.958	51	3.153	49	3.329	39	20	3.495	17	-36	3.641	22	-15	3.668	-14
.9975	3.405	59	3.652	51	3.858	36	37	4.025	13	-28	4.119	11	-8	3.968	-66

	$\beta_2=5.4$ $\sqrt{\beta_1}=0$		$\beta_2=5.4$ $\sqrt{\beta_1}=0.3$		$\beta_2=5.4$ $\sqrt{\beta}=0.6$		$\beta_2=5.4$ $\sqrt{\beta_1}=0.9$			$\beta_2=5.4$ $\sqrt{\beta_1}=1.2$			$\beta_2=5.4$ $\sqrt{\beta_1}=1.5$	
P	Type VII	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Burr	Type VI	S_B	Burr	Type I(J)	S_B
0.0025	-3.460	-79	-3.167	-84	-2.792	-91	-2.290	-89	200	-1.634	-53	119	-1.017	-139
.005	-2.990	-66	-2.718	-63	-2.488	-61	-2.106	-58	122	-1.580	-40	93	-1.016	-126
.01	-2.549	-47	-2.389	-41	-2.189	-35	-1.915	-30	0	-1.512	-27	65	-1.014	-108
.025	-1.999	-19	-1.907	-12	-1.796	-6	-1.641	-4	3	-1.391	-13	29	-1.005	-77
.05	-1.595	-1	-1.546	5	-1.489	9	-1.409	8	-21	-1.265	-4	6	-0.987	-46
.10	-1.187	13	-1.173	16	-1.160	17	-1.140	13	-29	-1.092	3	-10	-0.942	-11
.25	-0.594	15	-0.614	14	-0.640	12	-0.676	9	-14	-0.724	5	-14	-0.763	27
0.50	0.000	0	-0.030	-3	-0.065	-4	-0.111	-3	8	-0.183	1	-1	-0.315	15
0.75	0.594	-15	0.578	-16	0.564	-15	0.548	-10	14	0.522	-3	11	0.462	-22
.90	1.187	-13	1.205	-10	1.230	-6	1.267	-3	1	1.324	-1	5	1.415	-12
.95	1.595	1	1.645	6	1.704	8	1.780	7	-13	1.889	5	-4	2.064	16
.975	1.999	19	2.086	24	2.179	25	2.290	17	-25	2.438	11	-15	2.650	39
.99	2.549	47	2.691	50	2.830	45	2.980	27	-36	3.150	18	-25	3.334	42
.995	2.990	66	3.179	64	3.352	55	3.521	31	-36	3.684	21	-27	3.789	19
.9975	3.460	79	3.701	70	3.907	55	4.086	28	-29	4.217	20	-21	4.196	-25

TABLE A.4. Standardized % points for P.C. with $\beta_2,\sqrt{\beta_1}$ specified; also amounts to add to obtain Johnson and Burr values.

TABLE A.5. Standardized % points for P.C. with $\beta_2, \sqrt{\beta_1}$ specified; also amounts to add to obtain Johnson and Burr values.

	$\beta_2=5.8$ $\sqrt{\beta_1}=0$		$\beta_2=5.8$ $\sqrt{\beta_1}=0.3$		$\beta_2=5.8$ $\sqrt{\beta_1}=0.6$		$\beta_2=5.8$ $\sqrt{\beta_1}=0.9$		$\beta_2=5.8$ $\sqrt{\beta_1}=1.2$		$\beta_2=5.8$ $\sqrt{\beta_1}=1.5$		
P	Type VII	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Type I	S_B	Burr
0.0025	-3.505	-99	-3.224	-103	-2.870	-109	-2.401	-112	-1.780	-41	-1.129	-142	270
.005	-3.016	-80	-2.803	-78	-2.539	-75	-2.187	-72	-1.699	-27	-1.125	-122	266
.01	-2.561	-56	-2.408	-49	-2.221	-43	-1.968	-40	-1.603	-16	-1.116	-100	257
.025	-1.998	-22	-1.911	-15	-1.807	-9	-1.666	-6	-1.446	-5	-1.093	-63	234
.05	-1.589	0	-1.543	6	-1.490	11	-1.417	10	-1.294	1	-1.056	-32	198
.10	-1.179	16	-1.166	19	-1.154	21	-1.136	17	-1.097	4	-0.982	-2	129
.25	-0.588	18	-0.607	17	-0.632	16	-0.665	13	-0.708	4	-0.752	24	-41
0.50	0.000	0	-0.029	-2	-0.061	-5	-0.104	-3	-0.167	-1	-0.278	11	-130
0.75	0.588	-18	0.573	-19	0.559	-18	0.543	-13	0.520	-3	0.469	-17	59
.90	1.179	-16	1.196	-13	1.219	-9	1.252	-6	1.300	-1	1.375	-13	122
.95	1.589	0	1.637	4	1.692	8	1.762	7	1.860	1	2.010	7	16
.975	1.998	22	2.081	28	2.170	29	2.276	21	2.412	5	2.606	29	-117
.99	2.561	56	2.698	59	2.834	54	2.979	38	3.146	8	3.340	45	-254
.995	3.016	80	3.199	80	3.370	70	3.540	46	3.711	9	3.858	38	-295
.9975	3.505	99	3.740	91	3.947	74	4.132	46	4.287	9	4.347	11	-268

	$\beta_2=6.2$ $\sqrt{\beta_1}=0$		$\beta_2=6.2$ $\sqrt{\beta_1}=0.3$		$\beta_2=6.2$ $\sqrt{\beta_1}=0.6$		$\beta_2=6.2$ $\sqrt{\beta_1}=0.9$			$\beta_2=6.2$ $\sqrt{\beta_1}=1.2$			$\beta_2=6.2$ $\sqrt{\beta_1}=1.5$			$\beta_2=6.2$ $\sqrt{\beta_1}=1.8$	
P	Type VII	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Burr	Type IV	S_U	Burr	Type I	S_B	Burr	Type I(J)	S_B
0.0025	-3.543	-118	-3.271	-123	-2.934	-129	-2.495	-132	114	-1.911	-91	205	-1.246	-129	36	-0.787	-111
.005	-3.037	-95	-2.833	-91	-2.582	-89	-2.253	-87	47	-1.801	-60	146	-1.234	-107	31	-0.787	-107
.01	-2.571	-64	-2.424	-58	-2.246	-52	-2.012	-48	2	-1.678	-35	92	-1.215	-83	26	-0.787	-102
.025	-1.997	-24	-1.914	-17	-1.816	-11	-1.685	-9	-28	-1.488	-10	32	-1.170	-49	16	-0.786	-89
.05	-1.584	2	-1.540	8	-1.490	12	-1.423	12	-32	-1.315	4	-1	-1.110	-23	8	-0.785	-71
.10	-1.172	19	-1.160	22	-1.148	23	-1.132	21	-23	-1.100	11	-20	-1.009	1	0	-0.779	-42
.25	-0.583	21	-0.602	21	-0.625	19	-0.655	16	0	-0.695	9	-20	-0.739	18	-6	-0.719	19
0.50	0.000	0	-0.027	-3	-0.058	-5	-0.098	-4	12	-0.155	-1	1	-0.250	7	-2	-0.417	38
0.75	0.583	-21	0.568	-22	0.554	-21	0.539	-17	5	0.517	-7	15	0.474	-13	4	0.355	-23
.90	1.172	-19	1.188	-16	1.209	-12	1.239	-9	-9	1.282	-4	9	1.345	-13	3	1.447	-22
.95	1.584	-2	1.630	3	1.682	6	1.747	7	-16	1.836	2	-4	1.967	2	-1	2.192	24
.975	1.997	24	2.078	29	2.163	31	2.263	25	-18	2.390	11	-18	2.567	21	-5	2.837	56
.99	2.571	64	2.704	67	2.835	64	2.977	48	-11	3.140	19	36	3.335	39	-10	2.535	45
.995	3.037	95	3.216	94	3.385	83	3.553	61	-1	3.728	23	44	3.898	43	-11	3.959	-1
.9975	3.543	118	3.773	111	3.979	94	4.168	64	14	4.338	23	44	4.449	33	-9	4.308	-67

P	$\beta_2=6.6\ \sqrt{\beta_1}=0$ Type VII	S_U	$\beta_2=6.6\ \sqrt{\beta_1}=0.3$ Type IV	S_U	$\beta_2=6.6\ \sqrt{\beta_1}=0.6$ Type IV	S_U	$\beta_2=6.6\ \sqrt{\beta_1}=0.9$ Type IV	S_U	$\beta_2=6.6\ \sqrt{\beta_1}=1.2$ Type IV	S_U	$\beta_2=6.6\ \sqrt{\beta_1}=1.5$ Type VI	S_B	Burr	$\beta_2=6.6\ \sqrt{\beta_1}=1.8$ Type I(J)	S_B
0.0025	-3.575	-138	-3.312	-142	-2.990	-147	-2.574	-153	-2.025	-129	-1.365	-100	74	-0.858	-135
.005	-3.056	-107	-2.858	-105	-2.619	-102	-2.309	-101	-1.888	-85	-1.341	-79	63	-0.858	-128
.01	-2.579	-71	-2.437	-68	-2.268	-60	-2.048	-57	-1.741	-48	-1.306	-59	49	-0.857	-118
.025	-1.997	-25	-1.916	-19	-1.823	-13	-1.701	-11	-1.522	-13	-1.236	-32	28	-0.856	-95
.05	-1.580	4	-1.537	9	-1.489	13	-1.427	13	-1.330	6	-1.154	-13	12	-0.851	-79
.10	-1.167	24	-1.154	25	-1.143	27	-1.128	24	-1.100	15	-1.027	2	-2	-0.834	-32
.25	-0.579	25	-0.597	24	-0.618	21	-0.646	18	-0.683	13	-0.726	12	-11	-0.731	25
0.50	0.000	0	-0.026	-3	-0.056	-4	-0.093	-4	-0.145	-2	-0.228	3	-4	-0.373	31
0.75	0.579	-25	0.564	-25	0.551	-24	0.536	-21	0.515	-11	0.477	-12	7	0.382	-22
.90	1.167	-24	1.181	-20	1.201	-16	1.228	-12	1.266	-7	1.320	-14	6	1.402	-26
.95	1.580	-4	1.623	1	1.673	5	1.734	6	1.816	3	1.932	-6	0	2.121	11
.975	1.997	25	2.075	31	2.156	33	2.251	29	2.371	15	2.534	5	-8	2.778	47
.99	2.579	71	2.708	75	2.836	72	2.974	58	3.133	30	3.324	20	-17	3.545	61
.995	3.056	107	3.231	106	3.396	97	3.563	75	3.739	37	3.921	27	-21	4.053	35
.9975	3.575	138	3.801	130	4.005	113	4.197	83	4.376	39	4.520	26	-20	4.503	-19

P	$\beta_2=7.0\ \sqrt{\beta_1}=0$ Type VII	S_U	$\beta_2=7.0\ \sqrt{\beta_1}=0.3$ Type IV	S_U	$\beta_2=7.0\ \sqrt{\beta}=0.6$ Type IV	S_U	$\beta_2=7.0\ \sqrt{\beta}=0.9$ Type IV	S_U	$\beta_2=7.0\ \sqrt{\beta_1}=1.2$ Type IV	S_U	Burr	$\beta_2=7.0\ \sqrt{\beta_1}=1.5$ Type VI	S_B	Burr	$\beta_2=7.0\ \sqrt{\beta_1}=1.8$ Type I(J)	S_B
0.0025	-3.603	-157	-3.347	-162	-3.037	-166	-2.643	-172	-2.125	-160	244	-1.482	-61	117	-0.932	-151
.005	-3.071	-121	-2.879	-118	-2.650	-115	-2.357	-114	-1.963	-105	161	-1.441	-48	94	-0.932	-138
.01	-2.585	-79	-2.448	-73	-2.286	-67	-2.079	-64	-1.793	-60	91	-1.388	-34	70	-0.930	-122
.025	-1.996	-26	-1.918	-20	-1.828	-15	-1.714	-12	-1.550	-14	21	-1.292	-17	38	-0.925	-91
.05	-1.576	6	-1.534	10	-1.489	15	-1.430	14	-1.342	8	-13	-1.188	-6	14	-0.912	-60
.10	-1.161	26	-1.150	29	-1.139	30	-1.125	28	-1.100	20	-28	-1.040	3	-5	-0.879	-21
.25	-0.576	28	-0.592	26	-0.613	25	-0.639	22	-0.673	16	-21	-0.714	8	-15	-0.733	27
0.50	0.000	0	-0.026	-2	-0.054	-4	-0.089	-4	-0.137	-2	4	-0.211	3	-4	-0.337	25
0.75	0.576	-28	0.561	-28	0.547	-27	0.533	-24	0.513	-15	17	0.479	-4	9	0.400	-19
.90	1.161	-26	1.176	-24	1.194	-20	1.219	-16	1.253	-10	9	1.300	-3	10	1.368	-25
.95	1.576	-6	1.618	-1	1.665	3	1.723	4	1.798	3	-5	1.903	1	2	2.066	3
.975	1.996	26	2.071	32	2.150	34	2.241	31	2.354	19	-20	2.505	9	-9	2.726	37
.99	2.585	79	2.711	82	2.837	79	2.971	66	3.125	40	-38	3.311	19	-23	3.537	65
.995	3.071	121	3.242	119	3.405	110	3.570	89	3.746	51	-48	3.934	25	-31	4.105	58
.9975	3.603	157	3.824	149	4.027	132	4.220	112	4.405	57	-50	4.570	29	-32	4.634	23

TABLE A.6. Standardized % points of P.C. with $\beta_2,\sqrt{\beta_1}$ specified; also amounts to add to obtain Johnson and Burr values.

TABLE A.7 Standardized % points of P.C. with $\beta_2,\sqrt{\beta_1}$ specified; also amounts to add to obtain Johnson and Burr values.

	$\beta_2=7.4$ $\sqrt{\beta_1}=0$		$\beta_2=7.4$ $\sqrt{\beta_1}=0.3$		$\beta_2=7.4$ $\sqrt{\beta_1}=0.6$		$\beta_2=7.4$ $\sqrt{\beta_1}=0.9$		$\beta_2=7.4$ $\sqrt{\beta_1}=1.2$		$\beta_2=7.4$ $\sqrt{\beta_1}=1.5$		$\beta_2=7.4$ $\sqrt{\beta_1}=1.8$	
P	Type VII	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Type I(J)	S_B
0.0025	-3.627	-175	-3.377	-181	-3.079	-184	-2.703	-191	-2.212	-186	-1.592	-61	-1.010	-156
.005	-3.084	-133	-2.898	-130	-2.677	-127	-2.398	-127	-2.027	-123	-1.532	-44	-1.008	-139
.01	-2.591	-86	-2.457	-81	-2.301	-75	-2.105	-72	-1.838	-69	-1.460	-29	-1.004	-118
.025	-1.995	-27	-1.919	-22	-1.833	-17	-1.725	-14	-1.573	-16	-1.338	-11	-0.990	-83
.05	-1.572	8	-1.532	12	-1.488	16	-1.433	16	-1.352	10	-1.214	-2	-0.966	-50
.10	-1.157	30	-1.146	33	-1.135	33	-1.122	32	-1.099	24	-1.048	4	-0.914	-13
.25	-0.572	30	-0.589	30	-0.608	28	-0.633	25	-0.665	20	-0.703	6	-0.730	26
0.50	0.000	0	-0.025	-2	-0.052	-4	-0.085	-4	-0.130	-2	-0.197	1	-0.308	20
0.75	0.572	-30	0.558	-31	0.545	-30	0.530	-26	0.511	-19	0.480	-4	0.412	-14
.90	1.157	-30	1.171	-28	1.188	-24	1.211	-19	1.242	-13	1.284	-4	1.342	-23
.95	1.572	-8	1.613	-3	1.659	0	1.713	3	1.783	2	1.879	0	2.022	-2
.975	1.995	27	2.068	32	2.145	34	2.232	32	2.340	22	2.481	4	2.681	29
.99	2.591	86	2.714	88	2.837	85	2.968	73	3.118	49	3.298	11	3.521	61
.995	3.084	133	3.252	131	3.413	122	3.576	101	3.750	65	3.940	15	4.132	68
.9975	3.627	175	3.845	167	4.046	150	4.239	120	4.428	74	4.607	17	4.723	51

	$\beta_2=7.8$ $\sqrt{\beta_1}=0$		$\beta_2=7.8$ $\sqrt{\beta_1}=0.3$		$\beta_2=7.8$ $\sqrt{\beta_1}=0.6$		$\beta_2=7.8$ $\sqrt{\beta_1}=0.9$		$\beta_2=7.8$ $\sqrt{\beta_1}=1.2$			$\beta_2=7.8$ $\sqrt{\beta_1}=1.5$			$\beta_2=7.8$ $\sqrt{\beta_1}=1.8$		
P	Type VII	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Burr	Type IV	S_U	Burr	Type I	S_B	Burr
0.0025	-3.648	-193	-3.404	-199	-3.115	-203	-2.755	-209	-2.289	-209	212	-1.694	-123	197	-1.091	-149	-97
.005	-3.096	-144	-2.914	-143	-2.700	-140	-2.433	-141	-2.083	-138	124	-1.615	-85	149	-1.086	-128	-93
.01	-2.596	-92	-2.465	-88	-2.315	-82	-2.127	-79	-1.876	-78	56	-1.524	-52	103	-1.076	-105	-87
.025	-1.994	-28	-1.920	-23	-1.837	-18	-1.734	-15	-1.592	-18	-3	-1.376	-19	46	-1.051	-68	-74
.05	-1.569	10	-1.530	14	-1.487	17	-1.434	17	-1.359	12	-26	-1.235	-1	12	-1.013	-37	-58
.10	-1.153	34	-1.142	36	-1.131	6	-1.119	35	-1.098	28	-30	-1.054	10	-12	-0.941	-6	-36
.25	-0.570	34	-0.585	35	-0.604	3	-0.628	28	-0.657	23	-14	-0.694	12	-21	-0.724	23	4
0.50	0.000	0	-0.024	-2	-0.051	-3	-0.082	-4	-0.124	-2	8	-0.186	2	-4	-0.285	17	30
0.75	0.570	-34	0.556	-34	0.542	-32	0.528	-29	0.510	-22	14	0.480	-8	14	0.422	-12	17
.90	1.153	-34	1.166	-31	1.182	-27	1.204	-23	1.232	-17	2	1.270	-7	14	1.320	-21	-24
.95	1.569	-10	1.609	-6	1.653	-2	1.705	0	1.771	0	-11	1.859	-1	3	1.986	-7	-52
.975	1.994	28	2.065	33	2.140	35	2.224	33	2.327	24	-22	2.459	8	-11	2.643	16	-65
.99	2.596	92	2.716	95	2.837	91	2.965	80	3.110	58	-31	3.285	20	-31	3.502	48	-53
.995	3.096	144	3.261	142	3.419	134	3.580	114	3.752	79	-33	3.943	28	-44	4.146	61	-16
.9975	3.648	193	3.863	185	4.064	168	4.254	139	4.446	92	-29	4.634	32	-52	4.785	58	48

	$\beta_2=8.6$ $\sqrt{\beta_1}=0$		$\beta_2=8.6$ $\sqrt{\beta_1}=0.3$		$\beta_2=8.6$ $\sqrt{\beta_1}=0.6$		$\beta_2=8.6$ $\sqrt{\beta_1}=0.9$		$\beta_2=8.6$ $\sqrt{\beta_1}=1.2$		$\beta_2=8.6$ $\sqrt{\beta_1}=1.5$			$\beta_2=8.6$ $\sqrt{\beta_1}=1.8$		
P	Type VII	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Burr	Type VI	S_B	Burr
0.0025	-3.683	-228	-3.449	-233	-3.175	-239	-2.841	-245	-2.416	-250	-1.872	-209	255	-1.259	-107	71
.005	-3.115	-166	-2.941	-165	-2.739	-163	-2.492	-164	-2.175	-165	-1.754	-141	184	-1.240	-87	61
.01	-2.603	-104	-2.479	-99	-2.337	-95	-2.163	-94	-1.937	-94	-1.627	-84	119	-1.213	-66	50
.025	-1.992	-29	-1.922	-24	-1.843	-21	-1.748	-19	-1.621	-22	-1.436	-27	46	-1.156	-38	31
.05	-1.563	14	-1.526	18	-1.486	21	-1.437	20	-1.370	15	-1.267	3	6	-1.087	-17	16
.10	-1.146	40	-1.136	43	-1.125	42	-1.114	41	-1.095	34	-1.060	20	-20	-0.978	0	1
.25	-0.565	39	-0.580	41	-0.597	36	-0.619	34	-0.645	29	-0.678	19	-25	-0.710	15	-10
0.50	0.000	0	-0.023	-2	-0.048	-3	-0.078	-3	-0.115	-2	-0.168	1	-4	-0.249	10	-6
0.75	0.565	-39	0.551	-39	0.538	-38	0.524	-34	0.507	-28	0.481	-16	16	0.434	-8	5
.90	1.146	-40	1.158	-38	1.173	-34	1.192	-30	1.217	-24	1.248	-15	16	1.288	-15	8
.95	1.563	-14	1.601	-10	1.642	-7	1.690	-4	1.740	-3	1.826	-3	5	1.931	-11	3
.975	1.992	29	2.060	33	2.131	35	2.211	33	2.305	27	2.423	14	-11	2.582	1	-4
.99	2.603	104	2.720	105	2.836	102	2.959	92	3.097	71	3.261	38	-35	3.464	20	-15
.995	3.115	166	3.275	163	3.429	154	3.586	135	3.754	103	3.941	55	-50	4.151	33	-21
.9975	3.683	228	3.892	218	4.088	201	4.279	172	4.472	127	4.670	65	-63	4.858	42	-25

	$\beta_2=9.0$ $\sqrt{\beta_1}=0$		$\beta_2=9.0$ $\sqrt{\beta_1}=0.3$		$\beta_2=9.0$ $\sqrt{\beta_1}=0.6$		$\beta_2=9.0$ $\sqrt{\beta_1}=0.9$		$\beta_2=9.0$ $\sqrt{\beta_1}=1.2$		$\beta_2=9.0$ $\sqrt{\beta_1}=1.5$		$\beta_2=9.0$ $\sqrt{\beta_1}=1.8$	
P	Type VII	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Type IV	S_U	Type VI	S_B
0.0025	-3.697	-244	-3.468	-250	-3.201	-255	-2.877	-263	-2.470	-268	-1.948	-241	-1.342	-79
.005	-3.123	-176	-2.952	-176	-2.755	-175	-2.516	-176	-2.212	-179	-1.813	-162	-1.313	-64
.01	-2.606	-109	-2.484	-105	-2.346	-101	-2.178	-100	-1.962	-102	-1.669	-96	-1.274	-52
.025	-1.991	-29	-1.922	-25	-1.846	-21	-1.754	-20	-1.633	-23	-1.460	-29	-1.199	-28
.05	-1.561	17	-1.524	19	-1.485	22	-1.438	22	-1.374	16	-1.278	5	-1.115	-13
.10	-1.143	44	-1.133	46	-1.123	46	-1.111	43	-1.094	38	-1.062	24	-0.991	-1
.25	-0.563	42	-0.578	44	-0.595	40	-0.615	37	-0.640	32	-0.671	25	-0.703	7
0.50	0.000	0	-0.023	-1	-0.047	-3	-0.076	-3	-0.111	-2	-0.161	2	-0.235	4
0.75	0.563	-42	0.550	-42	0.537	-41	0.523	-38	0.506	-31	0.481	-19	0.438	-5
.90	1.143	-44	1.155	-41	1.169	-38	1.187	-33	1.210	-27	1.239	-18	1.275	-9
.95	1.561	-17	1.598	-13	1.638	-10	1.684	-7	1.740	-5	1.812	-4	1.909	-5
.975	1.991	29	2.058	33	2.128	34	2.205	33	2.295	28	2.408	16	2.557	3
.99	2.606	109	2.721	109	2.835	107	2.956	97	3.091	77	3.250	46	3.446	17
.995	3.123	176	3.281	180	3.433	182	3.588	145	3.754	114	3.939	67	4.148	28
.9975	3.697	244	3.904	247	4.098	253	4.289	187	4.482	143	4.682	81	4.880	37

TABLE A.8. Standardized % points of P.C. with $\beta_2,\sqrt{\beta_1}$ specified, also amounts to add to obtain Johnson and Burr values.

TABLE A.9. Standardized % points of P.C. with β_2, $\sqrt{\beta_1}$ specified; also amounts to add to obtain Johnson and Burr values.

	$\beta_2=9.8$ $\sqrt{\beta_1}=0.9$		$\beta_2=9.8$ $\sqrt{\beta_1}=1.2$		$\beta_2=9.8$ $\sqrt{\beta_1}=1.5$			$\beta_2=9.8$ $\sqrt{\beta_1}=1.8$			$\beta_2=9.8$ $\sqrt{\beta_1}=2.0$		
P	Type IV	S_U	Type IV	S_U	Type IV	S_U	Burr	Type VI	S_U	Burr	Type VI	S_B	Burr
0.0025	-2.939	-296	-2.560	-305	-2.080	-293	286	-1.500	-149	171	-1.128	-140	40
.005	-2.557	-199	-2.276	-202	-1.912	-196	191	-1.446	-107	137	-1.119	-118	36
.01	-2.203	-112	-2.004	-115	-1.739	-116	111	-1.381	-80	101	-1.103	-95	31
.025	-1.763	-22	-1.652	-26	-1.498	-34	29	-1.271	-29	55	-1.067	-61	21
.05	-1.439	24	-1.381	20	-1.296	8	-9	-1.159	-7	22	-1.018	-34	12
.10	-1.107	49	-1.091	44	-1.064	32	-28	-1.007	8	-5	-0.934	-8	3
.25	-0.609	42	-0.632	39	-0.659	31	-25	-0.690	14	-20	-0.704	16	-7
0.50	-0.072	-2	-0.105	-1	-0.149	1	0	-0.213	4	-9	-0.273	14	-5
0.75	0.520	-42	0.504	-36	0.481	-26	18	0.444	-8	10	0.402	-7	3
.90	1.179	-41	1.199	-35	1.224	-26	14	1.254	-11	16	1.277	-19	5
.95	1.673	-11	1.725	-10	1.790	-8	2	1.874	-6	9	1.946	-13	3
.975	2.194	33	2.279	28	2.383	18	-14	2.515	4	-2	2.629	4	-1
.99	2.951	105	3.080	87	3.230	59	-35	3.413	19	-23	3.561	34	-9
.995	3.591	163	3.753	133	3.932	90	-49	4.138	30	-28	4.290	56	-14
.9975	4.304	218	4.497	174	4.698	114	-60	4.907	40	-50	5.042	72	-16

	$\beta_2=10.6$ $\sqrt{\beta_1}=1.2$		$\beta_2=10.6$ $\sqrt{\beta_1}=1.5$			$\beta_2=10.6$ $\sqrt{\beta_1}=1.8$			$\beta_2=10.6$ $\sqrt{\beta_1}=2.0$		
P	Type IV	S_U	Type IV	S_U	Burr	Type IV	S_U	Burr	Type VI	S_B	Burr
0.0025	-2.635	-337	-2.189	-336	243	-1.641	-252	231	-1.261	-86	93
.005	-2.328	-224	-1.991	-225	148	-1.561	-174	178	-1.238	-70	80
.01	-2.037	-128	-1.795	-131	74	-1.469	-110	125	-1.206	-54	64
.025	-1.667	-28	-1.527	-37	6	-1.326	-42	62	-1.144	-31	41
.05	-1.385	22	-1.309	11	-22	-1.190	-6	21	-1.071	-15	21
.10	-1.089	50	-1.065	39	-30	-1.018	18	-9	-0.960	-2	3
.25	-0.625	44	-0.650	38	-17	-0.678	23	-25	-0.695	8	-12
0.50	-0.100	-1	-0.141	3	6	-0.197	6	-10	-0.249	4	-8
0.75	0.502	-41	0.481	-32	15	0.448	-15	13	0.413	-12	5
.90	1.190	-41	1.212	-33	6	1.238	-19	19	1.257	-23	11
.95	1.713	-15	1.772	-13	-6	1.846	-9	12	1.908	-23	8
.975	2.266	27	2.362	19	-17	2.482	7	-1	2.583	-19	0
.99	3.070	96	3.213	70	-30	3.385	33	-26	3.522	-9	-14
.995	3.751	151	3.925	110	-36	4.124	54	-44	4.274	0	-24
.9975	4.507	203	4.708	144	-37	4.921	70	-61	5.067	7	-33

	β_2=11.8 $\sqrt{\beta_1}$=1.5		β_2=11.8 $\sqrt{\beta_1}$=1.8			β_2=11.8 $\sqrt{\beta_1}$=2.0		
P	Type IV	S_U	Type IV	S_U	Burr	Type VI	S_U	Burr
0.0025	-2.318	-391	-1.819	-353	296	-1.448	-227	178
.005	-2.085	-261	-1.700	-241	217	-1.398	-162	144
.01	-1.859	-152	-1.573	-148	146	-1.337	-106	109
.025	-1.559	-41	-1.387	-52	63	-1.232	-47	61
.05	-1.323	16	-1.224	-1	16	-1.127	-12	28
.10	-1.064	47	-1.027	30	-17	-0.983	10	-1
.25	-0.639	46	-0.664	35	-29	-0.681	21	-20
0.50	-0.131	4	-0.179	7	-9	-0.221	8	-12
0.75	0.480	-38	0.452	-25	15	0.423	-11	9
.90	1.198	-43	1.219	-30	22	1.235	-18	19
.95	1.750	-20	1.814	-16	13	1.866	-11	13
.975	2.337	18	2.444	8	2	2.530	1	2
.99	3.192	82	3.350	49	-27	3.474	23	-18
.995	3.914	136	4.104	83	-48	4.247	42	-35
.9975	4.716	185	4.929	113	-68	5.080	58	-51

	β_2=12.6 $\sqrt{\beta_1}$-1.8		β_2=12.6 $\sqrt{\beta_1}$=2.0	
P	Type IV	S_U	Type IV	S_U
0.0025	-1.917	-404	-1.561	-319
.005	-1.775	-274	-1.490	-224
.01	-1.627	-166	-1.408	-143
.025	-1.418	-56	-1.278	-58
.05	-1.240	2	-1.154	-11
.10	-1.031	37	-0.993	18
.25	-0.651	42	-0.673	30
0.50	-0.170	8	-0.208	10
0.75	0.453	-29	0.428	-17
.90	1.210	-38	1.223	-26
.95	1.798	-22	1.844	-16
.975	2.423	8	2.502	2
.99	3.330	59	3.447	34
.995	4.092	100	4.230	60
.9975	4.931	139	5.081	85

	β_2=13.8 $\sqrt{\beta_1}$=1.8			β_2=13.8 $\sqrt{\beta_1}$=2.0		
P	Type IV	S_U	Burr	Type IV	S_U	Burr
0.0025	-2.040	-465	319	-1.707	-419	292
.005	-1.867	-314	214	-1.605	-290	220
.01	-1.693	-188	117	-1.495	-181	153
.025	-1.454	-60	40	-1.331	-68	74
.05	-1.257	6	-5	-1.184	-8	26
.10	-1.034	46	-28	-1.003	30	-10
.25	-0.647	50	-27	-0.662	41	-29
0.50	-0.159	9	-2	-0.192	11	-13
0.75	0.455	-37	17	0.433	-26	13
.90	1.198	-48	17	1.209	-37	24
.95	1.777	-28	8	1.819	-25	18
.975	2.398	6	-7	2.469	1	6
.99	3.306	68	-29	3.414	45	-20
.995	4.075	122	-46	4.206	85	-43
.9975	4.929	175	-61	5.077	123	-66

TABLE A.10. Standardized % points of P.C. with β_2, $\sqrt{\beta_1}$ specified, also amounts to add to obtain Johnson and Burr values.

TABLE B.1. Comparison of % points of log-normal distributions with those of Pearson curves (Type V)

	$\beta_2=3.4$ $\sqrt{\beta_1}=0.47$ $\omega=1.0244$		$\beta_2=3.8$ $\sqrt{\beta_1}=0.67$ $\omega=1.0478$		$\beta_2=4.6$ $\sqrt{\beta_1}=0.94$ $\omega=1.0918$		$\beta_2=5.4$ $\sqrt{\beta_1}=1.14$ $\omega=1.1408$		$\beta_2=6.2$ $\sqrt{\beta_1}=1.31$ $\omega=1.1707$	
P	PC	LN-PC	PC	LN-PC	PC	LN-PC	PC	LN-PC	PC	LN-PC
0.0025	-2.310	-1	-2.135	-3	-1.917	-9	-1.772	-16	-1.663	-23
.005	-2.161	0	-2.011	-2	-1.822	-6	-1.695	-11	-1.599	-17
.01	-1.994	0	-1.870	-1	-1.712	-3	-1.603	-8	-1.521	-11
.025	-1.736	0	-1.648	0	-1.532	-1	-1.451	-3	-1.388	-5
.05	-1.502	0	-1.442	0	-1.361	1	-1.302	0	-1.255	-1
.10	-1.218	0	-1.187	1	-1.141	1	-1.106	2	-1.077	2
.25	-0.706	0	-0.712	0	-0.715	1	-0.714	2	-0.712	3
0.50	-0.077	0	-0.105	-1	-0.142	0	-0.166	0	-0.184	1
0.75	0.622	0	0.596	0	0.558	-1	0.529	-1	0.505	-2
.90	1.316	0	1.320	0	1.318	-1	1.311	-1	1.302	-2
.95	1.764	0	1.801	1	1.842	0	1.864	0	1.877	0
.975	2.173	0	2.250	1	2.344	2	2.405	2	2.448	2
.99	2.675	0	2.812	1	2.991	2	3.114	4	3.208	5
.995	3.034	0	3.222	0	3.474	2	3.653	5	3.794	5
.9975	3.379	0	3.622	0	3.956	1	4.198	4	4.390	7

	$\beta_2=7.0$ $\sqrt{\beta_1}=1.46$ $\omega=1.2066$		$\beta_2=7.8$ $\sqrt{\beta_1}=1.59$ $\omega=1.2404$		$\beta_2=8.6$ $\sqrt{\beta_1}=1.71$ $\omega=1.2725$		$\beta_2=9.8$ $\sqrt{\beta_1}=1.87$ $\omega=1.3177$		$\beta_2=10.6$ $\sqrt{\beta_1}=1.969$ $\omega=1.3462$	
P	PC	LN-PC	PC	LN-PC	PC	LN-PC	PC	LN-PC	PC	LN-PC
0.0025	-1.576	-31	-1.503	-39	-1.441	-47	-1.362	-58	-1.318	-65
.005	-1.521	-23	-1.456	-30	-1.400	-36	-1.329	-46	-1.288	-48
.01	-1.453	-16	-1.397	-21	-1.348	-26	-1.285	-34	-1.249	-39
.025	-1.336	-8	-1.292	-10	-1.253	-14	-1.204	-18	-1.175	-21
.05	-1.216	-2	-1.183	-3	-1.153	-5	-1.115	-8	-1.092	-10
.10	-1.052	1	-1.031	2	-1.012	2	-0.986	0	-0.971	0
.25	-0.708	3	-0.705	4	-0.701	5	-0.696	6	-0.693	8
0.50	-0.198	1	-0.210	2	-0.220	3	-0.232	3	-0.240	5
0.75	0.485	-2	0.468	-3	0.452	-3	0.432	-4	0.419	-3
.90	1.293	-3	1.284	-4	1.275	-5	1.262	-6	1.254	-7
.95	1.886	-1	1.891	-1	1.895	-3	1.897	-4	1.898	-4
.975	2.481	2	2.507	2	2.528	1	2.553	0	2.566	-1
.99	3.282	7	3.344	8	3.397	8	3.462	9	3.499	9
.995	3.907	9	4.002	12	4.085	13	4.190	15	4.250	17
.9975	4.550	10	4.686	13	4.804	17	4.956	21	5.044	24

ν	6.00		5.00		6.38		8.27		6.00		6.00		9.59		20.30	
δ	0.500		0.612		1.88		1.95		1.98		2.65		5.50		8.55	
β_2	6.427		10.790		8.753		5.781		10.478		12.219		6.774		4.227	
$\sqrt{\beta_1}$	0.461		0.818		1.293		0.907		1.478		1.732		1.277		0.759	
P	PC	t'-PC	PC	t'-PC	PC	t'-PC	PC	t'-PC	PC	t'-PC	PC	t'-PC	PC	t'-PC	PC	t'-PC
0.0025			-3.085	2	-2.285	11	-2.384	0	-2.210	20	-1.989	44	-1.909	20	-2.145	5
.005	-2.722	0	-2.660	5	-2.074	11	-2.174	2	-2.008	17	-1.832	34	-1.794	13	-2.011	3
.01	-2.342	1	-2.271	6	-1.864	10	-1.960	3	-1.808	14	-1.671	24	-1.669	9	-1.861	1
.025	-1.866	3	-1.796	6	-1.577	5	-1.661	3	-1.535	8	-1.446	13	-1.477	3	-1.631	0
.05	-1.513	2	-1.452	4	-1.345	2	-1.415	2	-1.314	3	-1.257	6	-1.303	0	-1.423	-1
.10	-1.151	2	-1.106	2	-1.087	0	-1.135	0	-1.067	0	-1.039	1	-1.089	-2	-1.169	0
.25	-0.610	0	-0.597	-1	-0.653	-2	-0.666	-1	-0.649	-3	-0.655	-3	-0.690	-1	-0.704	1
0.50	-0.043	-1	-0.062	-2	-0.128	-1	-0.105	-1	-0.138	-2	-0.162	-2	-0.159	1	-0.111	-1
0.75	0.559	0	0.521	0	0.499	2	0.543	0	0.483	1	0.460	1	0.507	2	0.577	1
.90	1.195	1	1.166	2	1.222	4	1.254	1	1.212	4	1.209	3	1.272	1	1.306	1
.95	1.652	2	1.651	4	1.766	3	1.765	1	1.769	4	1.790	1	1.833	0	1.802	0
.975	2.120	2	2.164	4	2.334	1	2.280	-1	2.357	2	2.407	-3	2.400	-2	2.277	-1
.99	2.776	1	2.913	3	3.141	-2	2.982	-1	3.204	-3	3.302	-11	3.175	-5	2.885	-1
.995	3.315	1	3.552	1	3.809	-5	3.542	-2	3.913	-7	4.073	-38	3.789	-6	3.339	-1
.9975			4.268	-1	4.536	-9	4.133	-3	4.692	-13	4.881	-26	4.433	-6	3.793	-1

N.B. The columns are arranged according to increasing non-central parameter, δ, ν = degrees of freedom of t'.

TABLE B.2. Comparison of % points of non-central t (t') distributions with those of Pearson curves (Type IV).

TABLE B.3. Comparison of % points of Weibull distributions with those of Pearson curves (Type I).

	β_2=7.360 $\sqrt{\beta_1}$=1.734 m=1.1		β_2=5.432 $\sqrt{\beta_1}$=1.346 m=1.3		β_2=4.390 $\sqrt{\beta_1}$=1.072 m=1.5		β_2=3.772 $\sqrt{\beta_1}$=0.865 m=1.7		β_2=3.245 $\sqrt{\beta_1}$=0.631 m=2.0	
P	PC	W-PC	PC	W-PC	PC	W-PC	PC	W-PC	PC	W-PC
0.0025	-1.109	15	-1.317	42	-1.503	60	-1.668	71	-1.886	81
.005	-1.104	14	-1.302	37	-1.475	50	-1.625	56	-1.819	58
.01	-1.094	13	-1.278	30	-1.434	37	-1.566	38	-1.733	36
.025	-1.067	9	-1.224	18	-1.350	18	-1.455	16	-1.580	10
.05	-1.027	5	-1.155	8	-1.253	6	-1.330	1	-1.420	-5
.10	-0.952	0	-1.040	-2	-1.104	-5	-1.151	-8	-1.201	-12
.25	-0.729	-3	-0.747	-7	-0.753	-9	-0.754	-8	-0.748	-7
0.50	-0.281	-2	-0.235	-1	-0.195	0	-0.162	2	-0.120	4
0.75	0.432	2	0.501	4	0.549	7	0.585	7	0.623	6
.90	1.330	2	1.360	2	1.371	1	1.373	1	1.367	-4
.95	1.988	1	1.959	-2	1.923	-5	1.886	-8	1.834	-11
.975	2.634	2	2.528	-8	2.433	-11	2.350	-12	2.247	-14
.99	3.470	5	3.240	-11	3.056	-13	2.906	-12	2.729	-9
.995	4.091	5	3.754	-10	3.495	-10	3.290	-6	3.055	1
.9975	4.703	-4	4.249	-6	3.910	-1	3.649	6	3.354	17

	β_2=2.857 $\sqrt{\beta_1}$=0.359 m-2.5		β_2=2.717 $\sqrt{\beta_1}$=0.000 m=3.6		β_2=3.187 $\sqrt{\beta_1}$=0.463 m=7.0		β_2=3.570 $\sqrt{\beta_1}$=0.638 m=10.0	
P	PC	W-PC	PC	W-PC	PC	W-PC	PC	W-PC
0.0025	-2.178	80	-2.625	67	-3.271	24	-3.521	-9
.005	-2.073	52	-2.448	34	-2.966	1	-3.152	-10
.01	-1.946	27	-2.248	10	-2.640	-14	-2.777	-19
.025	-1.733	1	-1.934	-10	-2.169	-19	-2.244	-25
.05	-1.523	-12	-1.649	-15	-1.775	-14	-1.809	-12
.10	-1.253	-14	-1.304	-11	-1.334	-4	-1.334	-2
.25	-0.733	-4	-0.698	2	-0.633	7	0.606	8
0.50	-0.064	7	0.000	8	-0.081	5	0.107	4
0.75	0.661	4	0.698	-1	0.720	-5	0.722	-7
.90	1.348	-8	1.304	-11	1.227	-11	1.195	-10
.95	1.761	-12	1.649	-12	1.497	-7	1.442	-4
.975	2.115	-12	1.934	-7	1.712	3	1.637	6
.99	2.518	-3	2.248	7	1.940	22	1.840	27
.995	2.784	11	2.448	25	2.081	41	1.965	45
.9975	3.023	31	2.625	49	2.202	63	2.071	67

Note:

The last two Weibull distributions are negatively skew and have been reversed.

ν	6		1		2	
$\sqrt{\lambda}$	6		2		1	
β_2	3.296		5.519		7.500	
$\sqrt{\beta_1}$	0.468		1.362		1.768	
P	PC	χ'^2-PC	PC	χ'^2-PC	PC	χ'^2-PC
0.0025	-2.259	5	-1.313	135	-1.082	24
.005	-2.124	4	-1.297	119	-1.078	23
.01	-1.969	2	-1.273	96	-1.069	20
.025	-1.725	0	-1.220	47	-1.046	15
.05	-1.500	-1	-1.151	15	-1.011	10
.10	-1.222	-11	-1.038	-13	-0.942	3
.25	-0.713	0	-0.746	-17	-0.728	-5
0.50	-0.019	1	-0.235	-1	-0.288	-3
0.75	0.627	1	0.498	9	0.425	3
.90	1.324	-1	1.357	3	1.329	4
.95	1.769	0	1.959	-7	1.994	1
.975	2.172	-1	2.531	-14	2.646	-5
.99	2.659	-1	3.252	-18	3.490	-7
.995	3.002	0	3.771	-14	4.117	-9
.9975	3.329	1	4.274	-5	4.734	-10

TABLE B.4 Comparison of % points of non-central χ^2 (χ'^2) distribution (standardized) with those of Pearson curves (Type I)

ν	10		4		2	
β_2	3.437		4.188		5.400	
$\sqrt{\beta_1}$	-0.469		-0.780		-1.140	
P	PC	log χ^2-PC	PC	log χ^2-PC	PC	log χ^2-PC
0.0025	-3.393	0	-3.790	-5	-4.197	-23
.005	-3.041	-1	-3.344	-7	-3.653	-26
.01	-2.677	-1	-2.894	-7	-3.114	-23
.025	-2.171	0	-2.288	-4	-2.404	-12
.05	-1.760	0	-1.813	-2	-1.864	-2
.10	-1.312	0	-1.314	1	-1.310	5
.25	-0.620	0	-0.578	2	-0.529	8
0.50	0.075	0	0.118	0	0.166	-2
0.75	0.702	0	0.709	-2	0.714	-9
.90	1.217	0	1.167	-2	1.106	-6
.95	1.506	-1	1.412	0	1.302	4
.975	1.743	1	1.610	2	1.452	16
.99	2.008	1	1.824	7	1.605	36
.995	2.181	2	1.961	10	1.696	54
.9975	2.339	1	2.081	14	1.776	70

TABLE B.5 Comparison of standardized % points of log χ^2 distributions with those of Pearson curves (Types IV and VI)

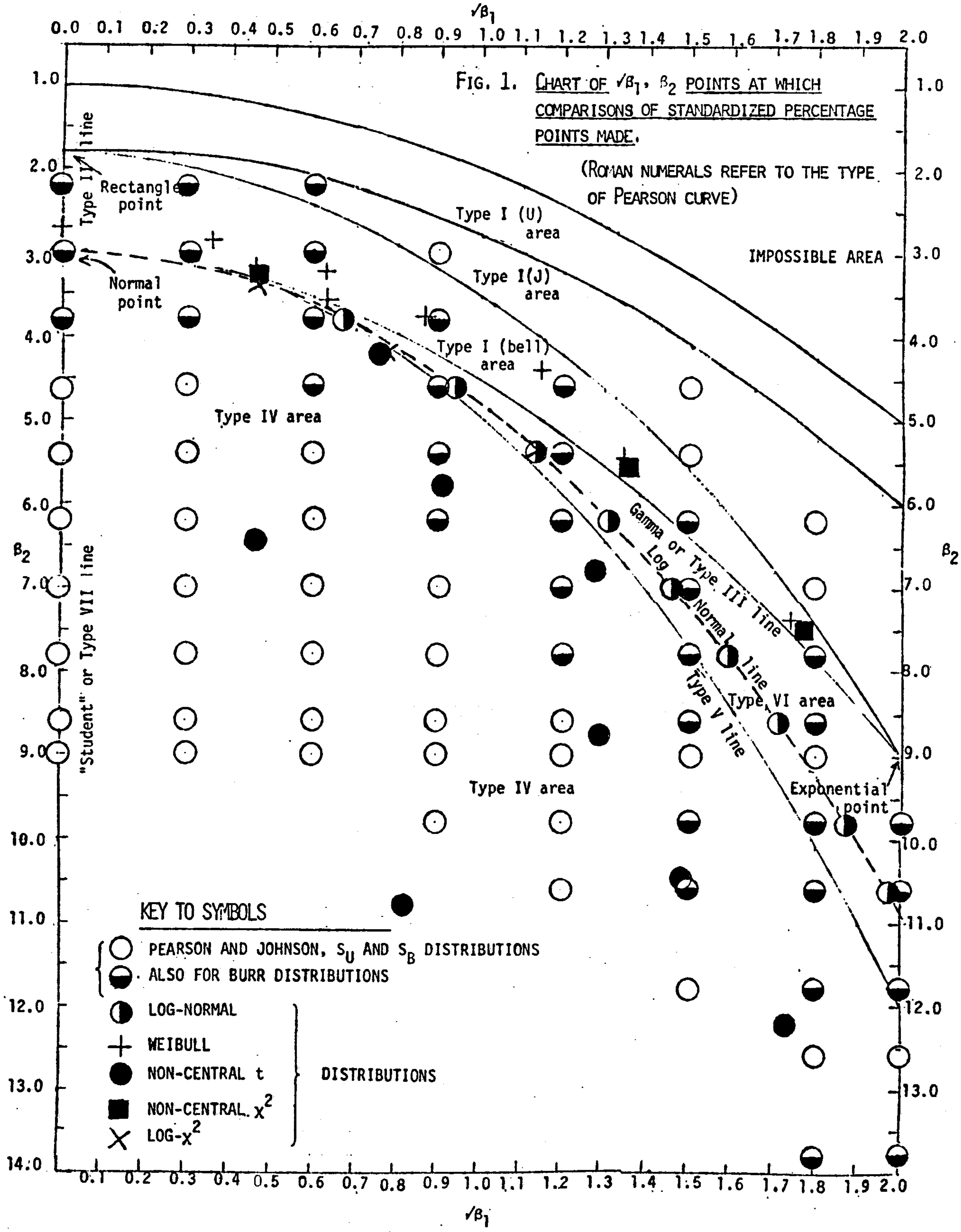
FIG. 1. CHART OF $\sqrt{\beta_1}$, β_2 POINTS AT WHICH COMPARISONS OF STANDARDIZED PERCENTAGE POINTS MADE.
(ROMAN NUMERALS REFER TO THE TYPE OF PEARSON CURVE)
$\sqrt{\beta_1}$
β_2
IMPOSSIBLE AREA
Type II line
Rectangle point
Normal point
Type I (U) area
Type I(J) area
Type I (bell) area
Type IV area
Gamma or Type III line
Log Normal line
Type V line
Type VI area
Exponential point
"Student" or Type VII line
KEY TO SYMBOLS
PEARSON AND JOHNSON, S_U AND S_B DISTRIBUTIONS
ALSO FOR BURR DISTRIBUTIONS
LOG-NORMAL
WEIBULL
NON-CENTRAL t
NON-CENTRAL χ^2
LOG-χ^2
DISTRIBUTIONS

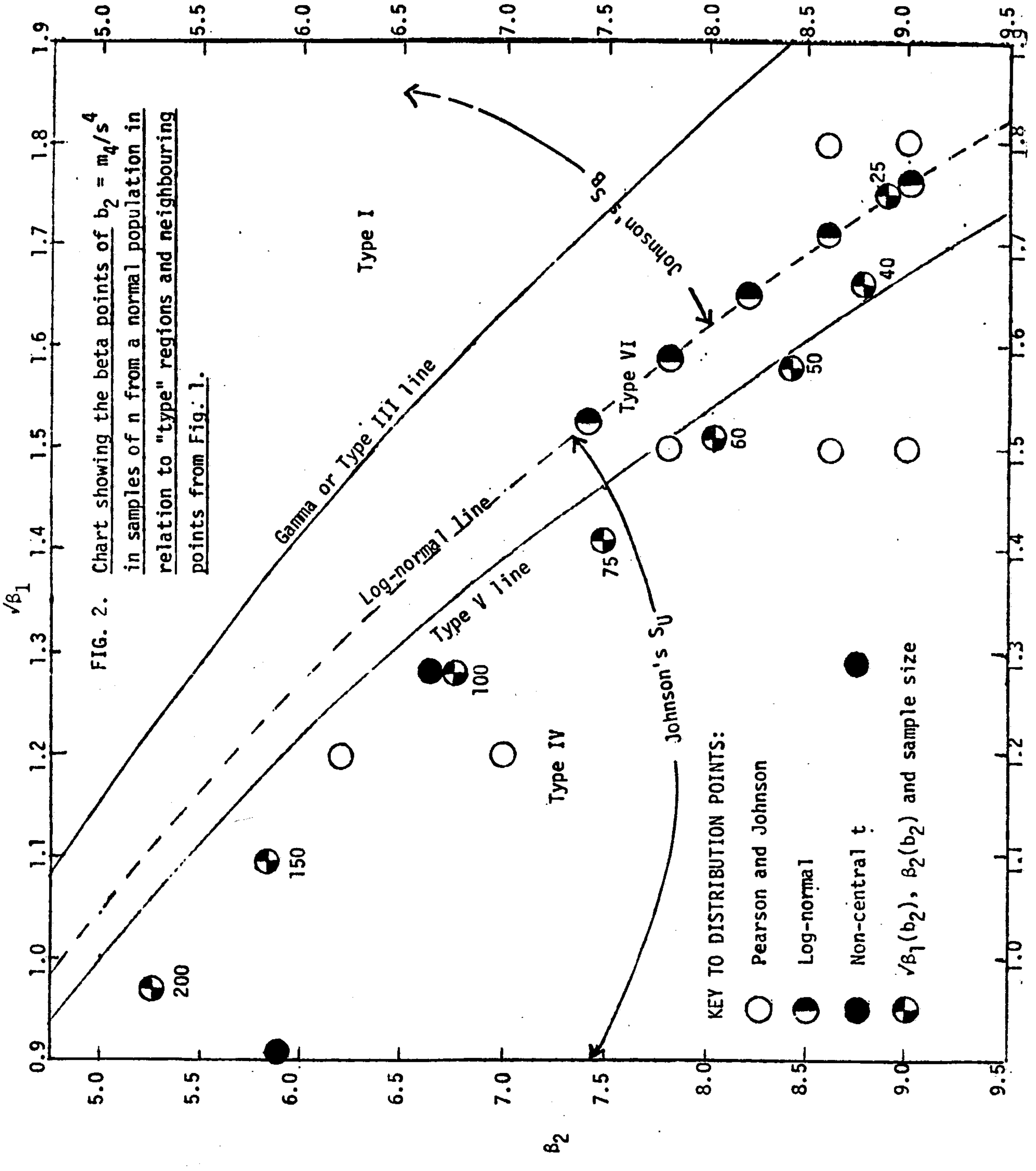

FIG. 2. Chart showing the beta points of $b_2 = m_4/s^4$ in samples of n from a normal population in relation to "type" regions and neighbouring points from Fig. 1.