



A review of simplified methods for the ratchetting assessment

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ABSTRACT

This paper is related to the assessment of the steady state of metallic structures subjected to the combination of constant primary and cyclic secondary loadings. In order to help the designers to choose among the available methods, an evaluation of the application domains of the main simplified methods existing up to now is carried out. That evaluation is performed comparing some test results available in the literature to the corresponding simplified methods forecasts. A number of structures and simplified methods categories was considered.

INTRODUCTION

A structure subjected to the combination of a field of controlled forces and a field of cyclic controlled displacements can undergo the ratchetting phenomenon. For a quick assessment of this risk, engineers generally make use of so-called simplified methods because of the relatively reduced means necessary to their application compared to the step by step analyses using complex models to describe the material behaviour up to the steady state.

Simplified methods can be classified into two categories according to their origins : theoretical or experimental. The first category is generally based on some assumptions (on the geometry, the loading and the behaviour of the material) which limit their validity domain. The methods of the second category are generally deducted from the analysis of a number of experimental results and consequently their validity is limited to the conditions which covered by the tests. In order to help the designers to choose from the available methods, it is necessary to make an evaluation of the application domains of the main simplified methods existing up to now : that is the goal of this paper. The best way for that evaluation is to use the experimental results as reference data. Various test results available in the literature have been considered. The experimental results are then compared to the forecasts of the majority of the simplified methods existing today.

In this paper, the first part is devoted to a short presentation of the chosen structures; the main constitutive equations concerning the considered simplified methods are given in the second part and, finally, a comparison between the experimental results and the simplified method forecasts are presented in the last part.

1. CHOSEN EXPERIMENTS

Four types of experiments are chosen; they differ particularly by the geometry of the structure (bars or cylinders), the type of the secondary loading (thermal gradient or prescribed displacement) and its evolution in the structure (constant or moving thermal gradient).

1.1 Three bars [1] and bitube [2] structures

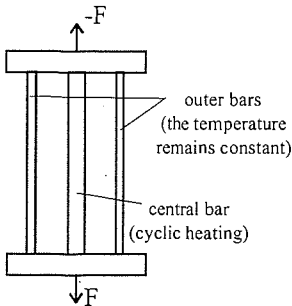


Figure 1 : three bars structure

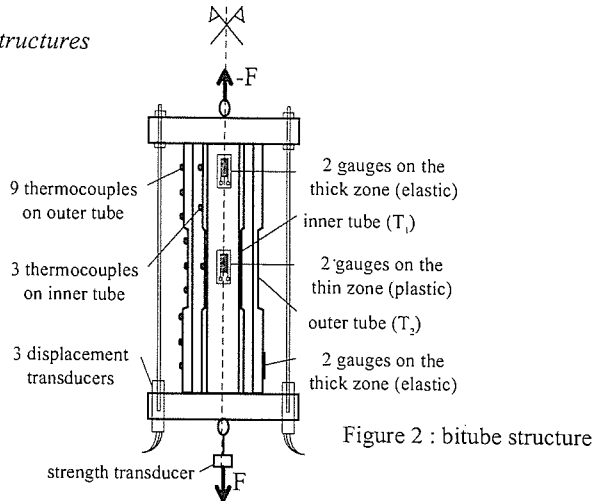


Figure 2 : bitube structure

The behaviour of the three bars [1] and the bitube [2] structures are similar; both primary and secondary stresses are membrane type. The bitube (three bars) structure is composed of two concentric cylinders (three bars) rigidly fixed at their ends in order to have the same equal global displacement at any time. The loading is a combination of a constant prescribed force applied to the tubes (the bars) and a cyclic heating of the outer tube (central bar) while the temperature of the inner one (outer bars) is maintained constant (figures 1 and 2). That loading is uniaxial and axisymmetric. The considered experiments concerning those structures have been carried out at the Institut National des Sciences Appliquées (INSA) of Lyon in France.

1.2 Tube subjected to constant tension and cyclic twisting [3-4]

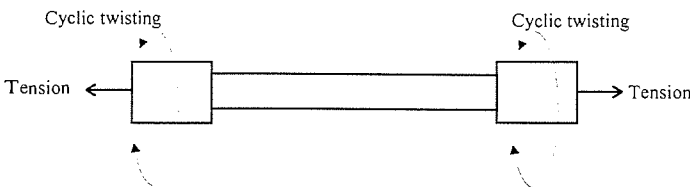


Figure 3 : tube subjected to constant tension and cyclic twisting

The applied loading is a combination of constant prescribed force (primary loading) and cyclic prescribed torsion (secondary loading) at constant temperature.

The considered test results have been carried out at the Commissariat à l'Énergie Atomique (CEA) of Saclay in France [3] and at Hiroshima University in Japan [4].

1.3 Cylinder subjected to constant tension and cyclic axial thermal gradient [5]

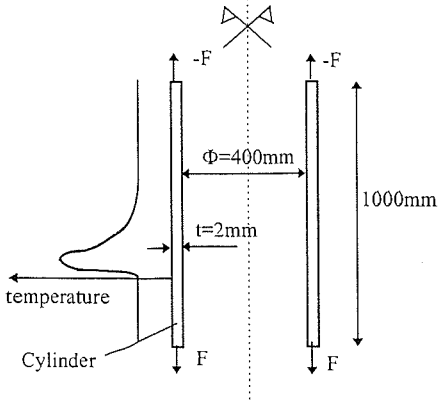


Figure 4 : cylinder subjected to constant tension and cyclic axial thermal gradient

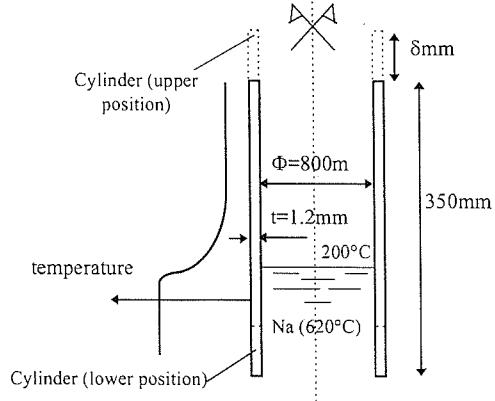


Figure 5 : cylinder subjected to axially moving thermal gradient

The structure is a cylinder subjected to an uniform constant prescribed tension (membrane primary stress) and a cyclic axial thermal gradient over a short length of the tube (membrane and bending secondary stress). The temperature variation through the thickness is not significant (figure 4). The test considered in this paper has been performed at the Institut National des Sciences Appliquées (INSA) of Lyon in France [5].

1.4 Cylinder subjected to axially moving thermal gradient [6]

That structure is called VINIL [6]; it is a cylinder only subjected to cyclic axial thermal gradient (axisymmetric) which moves axially. The temperature variation through the thickness is not significant (figure 5). The considered test has been carried out at the Commissariat à l'Energie Atomique (CEA) of Cadarache in France [6].

2. CONSIDERED SIMPLIFIED METHODS

The following notations are used :

- σ_p : maximum stress due to the prescribed force,
- ΔQ : maximum range of the stress, due to the applied thermal cycle, deduced from an elastic analysis,
- ε_1^{max} : maximum equivalent mechanical (elastic + plastic) deformation of the first cycle,
- σ_1^{max} : maximum equivalent stress obtained during the first cycle,
- Q^{max} : maximum equivalent stress, due to the applied thermal cycle, deduced from an elastic analysis and located at the same place that ε_1^{max} .
- $\sigma_{max} P_{eff}$: stress corresponding (on σ - ε curve) to the maximum equivalent mechanical deformation obtained at the steady state,
- $\sigma_{0.2\%}$: conventional yield stress corresponding to a plastic strain equal to 0.2%,
- $\sigma_{1\%}$: stress corresponding to a mechanical strain equal to 1% on the σ - ε curve,
- E : Young modulus.

The main constitutive equations of the considered methods are presented in the following.

2.1 The 3 S_m rule [7]

The loading is allowable if the following inequality is verified :

$$\sigma_p + \Delta Q \leq 3.S_m \quad (1) \quad \text{where } S_m \text{ is the allowable stress depending on the temperature.}$$

2.2 The Efficiency rule [7]

$$P_{eff}, \text{ related to the considered zone is given by : } P_{eff} = \sigma_p \quad \text{if} \quad \frac{\Delta Q}{\sigma_p} \leq 0.46 \quad (2)$$

$$P_{eff} = \frac{\sigma_p}{1.093 - 0.926 \cdot \frac{\Delta Q / \sigma_p}{(1 + (\Delta Q / \sigma_p)^2)^{1/2}}} \quad \text{if} \quad 0.46 < \frac{\Delta Q}{\sigma_p} < 4 \quad (3)$$

$$P_{eff} = \sigma_p \sqrt{\Delta Q / \sigma_p} \quad \text{if} \quad \frac{\Delta Q}{\sigma_p} \geq 4 \quad (4)$$

2.3 The modification of the efficiency rule proposed by Gatt [8]

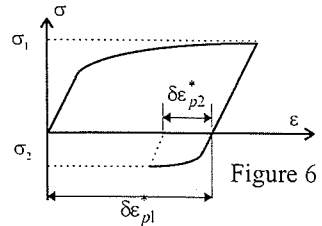
That modification [8] has two objectives : make the rule applicable when there is no controlled force and improve its forecasts when the secondary stress field has a component in the same direction that the primary one. The efficiency stress is given by equation (5) where P_G is the 'new primary' stress given by equation (6).

$$P_{eff} = P_G \cdot \left[1 + \left[\frac{\Delta Q}{P_G} \right]^2 \right]^{1/4} \quad (5),$$

$$P_G = \frac{1}{2} \frac{(\sigma_p + \sigma_1) * \delta \varepsilon_{p1}^* + (\sigma_p + \sigma_2) * \delta \varepsilon_{p2}^*}{\delta \varepsilon_{p1}^* + \delta \varepsilon_{p2}^*} \quad (6)$$

σ_i and $\delta \varepsilon_{pi}^*$ are membrane components deducted from

an elastoplastic analysis of the first cycle. Figure 6 specifies schematically those parameters.



2.4 The diagram proposed by Bree [9]

This diagram is a particular case of the one proposed by Ponter et al. (see 3.5).

2.5 The diagrams proposed by Ponter & al.[10-11]

Two main 'interaction' diagrams have been proposed depending on the secondary loading type (membrane or bending or both). The first one (fig. 7) is similar to the Bree diagram; it concerns the cases where the secondary loading is purely bending. The second one (fig. 8) is more severe, it concerns the cases where the secondary loading includes a membrane part [10]-[11]. These diagrams have two main domains : the allowable one and the one where freedom from ratchetting cannot be demonstrated. A procedure for checking the effects of creep when it is significant is also given [11]. Today the application of these diagrams is limited to the axisymmetric and near cylindrical thin shell structures. In figures 7 and 8 :

$$C = \frac{\max. \sigma_p}{K_1(\sigma_{0.2\%})_{\min}} \quad \text{and} \quad \Delta T = \frac{\max.(\Delta Q)}{K_1(\sigma_{0.2\%})_{\min}}$$

are respectively the non-dimensional primary and secondary (range) stresses. K_1 is a factor depending on the temperature and the material. For the 316L stainless steel: $K_1 = 0.75 + 0.0026(T - 20^\circ C)$ if $20^\circ C \leq T \leq 250^\circ C$ and $K_1 = 1.35$ if $T \geq 250^\circ C$. F is a non-dimensional factor which expresses the role of the membrane component of the secondary stress.

2.6 The method proposed by Kitade & al. [12]

The considered version of this method [12] concerns especially the cylindrical thin shell structures subjected to axial thermal gradient which moves axially with no primary loading

(like VINIL structure). The strain increment ($\Delta \epsilon_r$ = variation of the radius of the cylinder) at the cycle N is given by: $|\Delta \epsilon_r| = (\varphi + 1) \gamma_0^{N-1} (|\sigma_{\theta,el}| - \sigma_{0,2\%}) / E$ (7) where,

N is the number of cycle and $\sigma_{\theta,el}$ is the hoop-membrane stress deduced from an elastic analysis. φ and γ_0 are two parameters depending on the geometry, the material of the cylinder and the moving distance of axial thermal gradient (δ on figure 5).

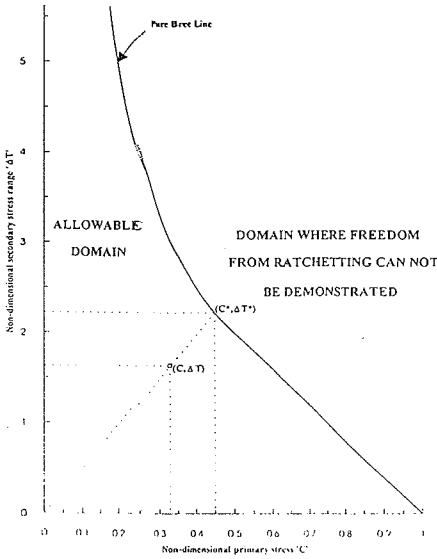


Figure 7 : Ratchetting interaction diagram : Pure Bending secondary loading

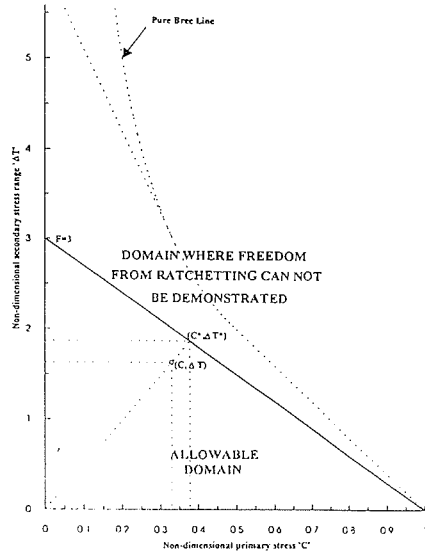


Figure 8 : Ratchetting interaction diagram : Membrane + Bending secondary loading

2.7 The extrapolation method [7]

The equivalent strain corresponding to the cycle N (ϵ_N) is extrapolated using the results of an elastoplastic analysis of the first n cycles ($n \geq 4$). Equations (8) and (9) correspond respectively to the cases where $N \leq 4n$ and $N \geq 4n$.

$$\epsilon_N = \epsilon_n + \frac{n \delta \epsilon_n}{m-1} \left[1 - \left(\frac{n}{N} \right)^{m-1} \right] \quad (8) \quad \epsilon_N = \epsilon_n + \frac{n \delta \epsilon_n}{m-1} \left[1 - 4^{1-m} \right] + \frac{\delta \epsilon_n}{4^m} [N - 4n] \quad (9)$$

Where $\delta \epsilon_i = \epsilon_i - \epsilon_{i-1}$, ϵ_i is the considered local deformation corresponding to the cycle i

and

$$m = 0.9 \min. \left[\frac{\text{Log} \frac{\delta \epsilon_n}{\delta \epsilon_{n-1}}}{\text{Log} \frac{n}{n-1}}, \frac{\text{Log} \frac{\delta \epsilon_n}{\delta \epsilon_{n-2}}}{\text{Log} \frac{n}{n-2}}, \frac{\text{Log} \frac{\delta \epsilon_{n-1}}{\delta \epsilon_{n-2}}}{\text{Log} \frac{n-1}{n-2}} \right]$$

2.8 The method proposed by the author [13]

For the application of this method, an elastoplastic analysis of the first cycle is necessary in order to avoid the problem concerning the classification of the stress into primary and secondary parts. In fact, this classification is not always easy especially when the elastic follow-up phenomenon is present and when the 'secondary' loading field has a significant membrane part in the same direction as the primary stress field. When these two phenomena are significant, the elastic analysis cannot generally be reliable. In general, a conservative value of the stress (σ_{max}) corresponding (on σ - ϵ curve) to the maximum equivalent mechanical deformation obtained at the steady state, is given by equation (10). When the elastic analysis

could be reliable and $\sigma_p \neq 0$ (see [13] for more details), both equations (10) and (11) could be considered to estimate σ_{\max} .

$$\sigma_{\max} = \left(\frac{E \cdot \varepsilon_1^{\max}}{P} \right)^{\frac{1}{2}} \cdot L n (1 + \alpha \cdot \beta \cdot \gamma) \cdot P \quad (10), \quad \text{or,} \quad \sigma_{\max} = \left(0.75 + \frac{\Delta Q}{\sigma_p} \right)^2 \cdot \sigma_p \quad (11)$$

where $\alpha = L n (1.75 + 2.8 \frac{\sigma_{0.2\%}}{E \varepsilon_{(1)}^{\max}})$, $\beta = L n (0.25 + 2.3 \frac{\sigma_{0.2\%}}{P})$, $\gamma = L n (2.1 + 4.5 \frac{\sigma_{1\%} - \sigma_{0.2\%}}{\sigma_{0.2\%}})$

$P = \sigma_p$ if $\sigma_p \neq 0$ and $P = (E \cdot \varepsilon_1^{\max} - Q^{\max})$ if $\sigma_p = 0$.

3. COMPARISON BETWEEN EXPERIMENTAL RESULTS AND SIMPLIFIED METHODS FORECASTS

For each structure one test is chosen. The following table summarises the comparison between the experimental results and the simplified methods forecasts :

- in the second line, some test conditions (material, temperature of the analysed point, total number of cycles carried out, deformation at the steady state and the corresponding stress on σ - ε curve) are briefly given. For certain tests the strain (stress) related to the steady state was extrapolated from the available results.

- for the methods which give an estimation of the deformation (stress) related to the steady state, the ratio ($(forecast - experiment) / experiment$) is given in % under brackets.

When thermal stress includes membrane component, certain methods specifies to consider all that component (efficiency rule, 3Sm rule for example) or only the axial part (Ponter & al.) as primary stress. The applications which take that recommendation into account are marked by (b). Conscious of the excessive conservatism generated by that choice, the cases where thermal stresses are entirely assumed secondary have also been considered (a).

The comparison between experimental results and simplified methods forecasts allows to make the following remarks :

- The diagram proposed by Bree seems to be very conservative for most of the considered cases except for VINIL structure type. Indeed in that case, the related point is in the ordinates axis because of $\sigma_p = 0$ and then the ratchetting phenomenon cannot occur whatever the secondary loading! which is not consistent with the experimental observations.

- 3 Sm rule is too conservative in all considered cases.

- The diagrams proposed by Ponter & al. are generally moderately conservative for the cases (a) for which they are applicable. For INSA tube structure, it could be noticed that a possible ratchetting is predicted which doesn't completely disagree with the experimental result. Indeed, according to [5], the steady state was not clearly observed and then the strain accumulation could probably exceed 2% within 500 cycles which are the criteria imposed by Ponter & al. to define the limit of the allowable domain. For bitube structure, in the case (b), this method is too conservative.

- The efficiency rule gives a conservative forecast for INSA tube structure; however it is not applicable for VINIL structure in the case (a) and too conservative in the case (b). For the structures where thermal stress includes a membrane part in the same direction as the controlled force (3 bars and bitube structures), the efficiency rule is not conservative in the case (a) and too conservative in the case (b). It seems also that this rule is too conservative for the CEA and Hiroshima tubes.

- The extrapolation method leads generally to an excessive deformation. Furthermore, although the strain growth decreases after each cycle, it doesn't cease for a reasonable number of cycles. The reported results concern the given experimental number of cycles.
- The method proposed by Kitade & al. is only applicable for VINIL structure; in that case it gives a correct forecast of the steady state. The slight negative difference could be linked to time effects which was significant for VINIL test but not taken into account by the method.
- The modification of the efficiency rule proposed by Gatt leads to some conservative forecasts for all considered tests. That conservatism is excessive for the CEA and Hiroshima considered tests for which there is almost no modification of the efficiency rule.
- The method proposed by the author gives also some conservative forecasts for all considered tests. This conservatism seems larger for the version based only on an elastic analysis. It could be remarked that for VINIL test, this method and the one proposed by Ponter & al. give almost the same forecasts.

Structure → Method ↓	Tube CEA	Tube Hiroshima	3 bars INSA	Bitube INSA	Tube INSA	VINIL CEA
Experiment	316L, 300°C 445 cycl. 253 MPa → 1.95%	304L, 650°C 3% → 171 MPa (estimated for 50 cycl.)	316L, 250°C Elas. shak. 35 cycles. 1.4% → 328 MPa	316L, 35°C Plastic shakedown 500 cycles. 0.9 % → 239 MPa	316L, 450°C Pl. shak. >10 cycles. >1.6 % → >250MPa	316L, 620°C pl. shak. 930 cycles. 0.43 % → 116 MPa
Bree	Ratchet	Ratchet	(a) Ratchet (b) $\sigma_p > \sigma_{0.2\%}$	(a) Pl. shak. (b) $\sigma_p > \sigma_{0.2\%}$	(a) Ratchet (b) $\sigma_p > \sigma_{0.2\%}$	(a) Pl. shak. (b) $\sigma_p > \sigma_{0.2\%}$
3 Sm	No el. shak.	No el. shak.	No el. shak.	No el. shak.	No el. shak.	No el. shak.
Ponter & al.	Not applicable	Not applicable	Not applicable	(a) Ratchetting is possible. (b) $\sigma_p > \sigma_{0.2\%}$	Ratchetting is possible	0.74% → 124MPa (+7%) no ratchet
Efficiency rule	353 MPa (+40%)	264 MPa (+55%)	(a) 304MPa (-7%) (b) $\sigma_p > \sigma_{0.2\%}$	(a) 193MPa (-19%) (b) $\sigma_p > \sigma_{0.2\%}$	(a) 267MPa (+7%) (b) $\sigma_p > \sigma_{0.2\%}$	Not applicable (b) $\sigma_p > \sigma_{0.2\%}$
Extrapolation	28.6%	4.65%	2.42%	1.84%	?	?
Kitade & al.	Not applicable	Not applicable	Not applicable	Not applicable	Not applicable	0.36% → 114MPa (-2%)
Gatt	354 MPa (+40%)	265 MPa (+55%)	345 MPa (+5%)	273 MPa (+14%)	272 MPa (+9%)	136 MPa (+17%)
Taleb (elastic)	322 MPa (+27%)	235 MPa (+37%)	Not applicable	Not applicable	290 MPa (+16%)	Not applicable
Taleb (plastic)	265 MPa (+5%)	193 MPa (+13%)	360 MPa (+10%)	261 MPa (+9%)	305 MPa (+22%)	123 MPa (+6%)

CONCLUSION

In this paper a comparison of the forecasts of certain simplified methods, related to the ratchetting assessment, with the experimental results is carried out. Two main observations could be point out. First, it is difficult to compare the forecasts of the considered methods because of the criteria to define the allowable domain are not the same for each method.

Those based on the shakedown theorem (3 Sm for example) are generally more restrictive compared to the others where a moderate strain (stress) limit is fixed (Ponter for example). This imposed conservatism could be linked especially to the absence of a sufficient number of studies. In our point of view, the works recently performed cover a larger variety of structures and loadings and then allow to be less restrictive. Secondly, it seems that the classical simplified methods (3Sm, Bree diagram, efficiency rule) still present in the designing codes are not suitable for the structures where thermal stress includes a membrane part. On one hand if that part is taken into account as primary stress, these methods become generally extremely conservative. On the other hand, if thermal stress is entirely considered as secondary, these methods could be unconservative (efficiency rule for example). The methods proposed respectively by Gatt and the author could be an alternative in order to avoid this problem.

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