Transactions, **SMiRT-25**
Charlotte, NC, USA, August 4-9, 2019
Division V

Equivalent linear calculation of dynamic civil structure response: a new efficient and robust approach

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ABSTRACT

This paper provides the main outcomes of the investigations carried out to evaluate the potential of use of an equivalent linear method applied on civil structures. The approach has been applied on macrostructural elements by capturing their damage through a set of kinematic modes. The equivalent linearization is carried out separately on different parts of the structure.
Indeed, this method basically consists in mimicking the geodynamics approach. It consists in degrading the mechanical properties of a macro-element, based on a macrostructural damage indicator. This indicator is linked to a very specific mode of response.
This macrostructural damage indicator is the driving parameter of an empiric constitutive equation for a very specific mode of response of the macro-structural element. For this purpose a broad set of reference tests is required to capture as most as it can be a large validity domain in terms of amplitude of the response and amplitude of the reinforcement, aspect ratio …
Shear mode has been the first one to be looked at in order to set up a first proof of concept. The main basic elements were accessible by means of the SAFE testing campaign: magnitude of loading compared to the design value, population of the type of shear walls.
The method has been applied first through a transient response based on modal basis. Then the approach has been applied to response spectrum method.
Both models are compared to an experimental reference for validation purpose: the SMART reinforced concrete mockup.

INTRODUCTION

Earthquake engineering methods use elastic modulus reduction methods that present often discussed elements. Typically, the approach that is most commonly found is a division by 2 of the module. Moreover, the engineering practice integrates only a part of the evolution of the dynamic properties through the degradation of the modulus of elasticity by not taking into account any evolution of the equivalent critical damping rate.

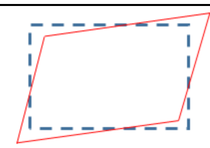
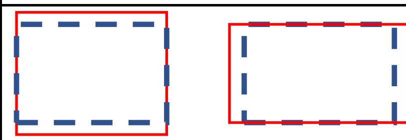
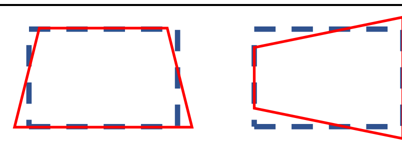
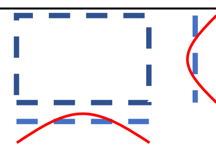
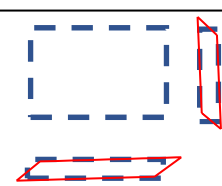
The main idea is to upgrade the engineering approach by taking inspiration from the equivalent linear method of soil dynamics to apply it to the structural elements of a reinforced concrete nuclear structure. This consists of carrying out this philosophy on structural elements (slabs, walls, …) by updating the dynamic properties of groups of meshes based on a damage indicator that homogenizes the degradations undergone by the structural element. By inscribing itself in this elastic framework of degraded rigidity with respect to the cracking, one nevertheless comes to bring an element less imprecise than the cracked qualification by considering a quantity of interest evaluated from the results of computation.

As the nuclear structures are reinforced concrete structures essentially braced by shear walls, it has been prioritized to characterize first the membrane behavior of these elements and more specifically the shear stiffness.
Two set of calculations have been carried out with the in-house EDF's FEM code, code_ASTER; transient calculations and response spectrum method calculations.
The SAFE testing campaign has been considered to set up empiric constitutive equations (see Pégon, P., & Molina, J. (1998)) based on the approach proposed in Pégon, P., & Molina, J. (1998).
The SMART testing campaign has been considered as a reference to benchmark the method when applied to an overall structure in order to validate the methode (see Richard, B., et al. (February 2016)).
Time histories in between calculations and tests were compared using Anderson criteria (see Anderson, J. (August 1-6, 2004)).

MECHANICAL ASSUMPTIONS

As exposed in introduction the idea is degrade globally macrostructural properties.
Thus we need to link a macrostructural kinematic mode with the most relevant associated damage indicator.
We considered the following modes:

Table 1: Potentially considered modes.

Family	Name	Figure	Number of modes
-	In-plane shear		1
In-plane elasticity	In-plane dilation		2
-	In-plane bending		2
Out-of-plane elasticity	Out-of-plane bending		2
-	Out-of-plane shear		2

Those different modes should in theory guide the different orthotropic stiffness matrices:

- In-plane:

$$H_L = h \begin{bmatrix} \frac{E_L}{1-\nu_{LT}\nu_{TL}} & \nu_{TL}\frac{E_L}{1-\nu_{LT}\nu_{TL}} & 0 \\ \nu_{LT}\frac{E_T}{1-\nu_{LT}\nu_{TL}} & \frac{E_T}{1-\nu_{LT}\nu_{TL}} & 0 \\ 0 & 0 & G_{LT} \end{bmatrix} \quad (1)$$

- Bending:

$$H_f = \frac{h^3}{12}\begin{bmatrix} \frac{E_L}{1-\nu_{LT}\nu_{TL}} & \nu_{TL}\frac{E_L}{1-\nu_{LT}\nu_{TL}} & 0 \\ \nu_{LT}\frac{E_T}{1-\nu_{LT}\nu_{TL}} & \frac{E_T}{1-\nu_{LT}\nu_{TL}} & 0 \\ 0 & 0 & G_{LT} \end{bmatrix} \quad (2)$$

- Out-of-plane shear:

$$H_{ct} = kh\begin{bmatrix} G_{LZ} = G_{LT} & 0 \\ 0 & G_{TZ} \end{bmatrix} \quad (3)$$

The parameters E_L, E_T et ν_{LT} have to be linked to damage indicator related to the dilation and curvature. Those indicator should be dimensionless and with respect to their respective orthotropic directions e_L et e_T.
In our first approach, among all those modes, we decided to simplify the problem and deal with isotropic model and considering only in-plane shear degradation because it is the most relevant in nuclear civil structures to predict the horizontal response. The parameter G_{LT} in red in the matrices is then the first one that we chose to consider. We linked it to the drift d.

The matrices are then written as follows:

- In-plane:

$$H_m = \frac{E_c h}{1-\nu_c{}^2}\begin{bmatrix} 1 & \nu_c & 0 \\ \nu_c & 1 & 0 \\ 0 & 0 & \frac{1-\nu_c}{2} \end{bmatrix} = h\begin{bmatrix} \frac{E_c}{1-\nu_c{}^2} & \nu_c\frac{E_c}{1-\nu_c{}^2} & 0 \\ \nu_c\frac{E_c}{1-\nu_c{}^2} & \frac{E_c}{1-\nu_c{}^2} & 0 \\ 0 & 0 & G_c(d) \end{bmatrix} \quad (4)$$

- Bending:

$$H_f = \frac{E_c h^3}{12(1-\nu_c{}^2)}\begin{bmatrix} 1 & \nu_c & 0 \\ \nu_c & 1 & 0 \\ 0 & 0 & \frac{1-\nu_c}{2} \end{bmatrix} = \frac{h^3}{12}\begin{bmatrix} \frac{E_c}{1-\nu_c{}^2} & \nu_c\frac{E_c}{1-\nu_c{}^2} & 0 \\ \frac{E_c}{1-\nu_c{}^2} & \frac{E_c}{1-\nu_c{}^2} & 0 \\ 0 & 0 & G_c(d) \end{bmatrix} \quad (5)$$

- Out-of-plane shear:

$$H_{ct} = \frac{kE_c h}{2(1+\nu_c)}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = kG_c(d)h\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (6)$$

- With:
  - k as the correction factor of the out-of-plane shear,
  - E_c as the concrete Young's modulus,
  - G_c as the concrete shear modulus,
  - ν_c as the concrete Poisson ratio,
  - h the plate stiffness,
  - d the drift – damage indicator.

Finally, as code_ASTER doesn't allow us in homogenous isotropic plate constitutive equations to define G_c solely, E_c has been degraded and the Poisson's ratio has remained unchanged.

- In-plane:

$$H_m = E_c(d)\frac{h}{1-{\nu_c}^2}\begin{bmatrix}1 & \nu_c & 0\\ \nu_c & 1 & 0\\ 0 & 0 & \frac{1-\nu_c}{2}\end{bmatrix} \quad (7)$$

- Bending:

$$H_f = E_c(d)\frac{h^3}{12(1-{\nu_c}^2)}\begin{bmatrix}1 & \nu_c & 0\\ \nu_c & 1 & 0\\ 0 & 0 & \frac{1-\nu_c}{2}\end{bmatrix} \quad (8)$$

- Out-of-plane shear:

$$H_{ct} = E_c(d)\frac{kh}{2(1+\nu_c)}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad (9)$$

SETTING UP THE EMPIRIC CONSTITUTIVE EQUATIONS

As exposed in introduction we needed to build an empiric degradation of the mechanical considering a damage indicator – the drift – . We considered the approach proposed by Pégon, P., & Molina, J. (1998). We used the spatial method to build a set of data based on the SAFE campaign with a 100 instants windowing for the stiffness and a global window for the damping (see Hocine et al. 2014).
In order to identify an appropriate empiric constitutive equation we considered the work carried out by T.-A. Nguyen in his PhD Thesis (see Nguyen T.A. (2017)).

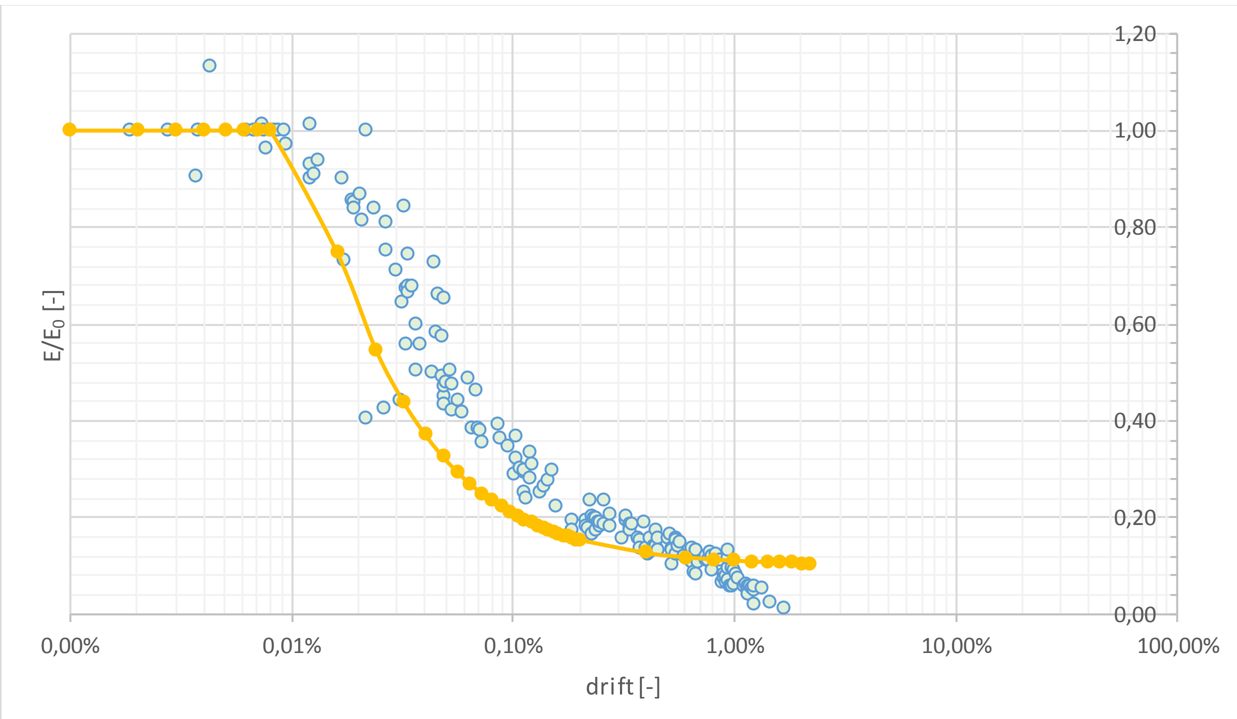


Figure 1. Elastic modulus degradation

Following his systematic approach we identified the linear equivalent response system he identified based on a damage constitutive equation without plasticity with a hardening coefficient of 10% when the first damage is appearing.
Thus we selected the law he proposed:

$$E(d)/E_0 = min\left(\left[\sqrt{\alpha_p} + \frac{1-\sqrt{\alpha_p}}{1+(\frac{\frac{d}{d_e}-1}{\mu_0-1})^{d\_}}\right]^2 ; 1\right) \quad (10)$$

With the following parameters :

$$\begin{cases} \alpha_p = 0.1 \\ d_e = 0.00011 \\ \mu_0 = 2.96 \\ d\_ = 0.96 \end{cases}$$

We choose to make the damping ratio evolve as follow, strictly based on the post-processing of SAFE tests:

$$\xi(d) = \max[5\%; \eta + \zeta . d + \kappa . e^{\lambda . d + \mu}] \quad (11)$$

ξ being expressed in percent in the previous formula, with the following parameters

$$\begin{cases} \eta = -28.23 \\ \zeta = -1374.14 \\ \kappa = 2.92 \\ \lambda = 39.14 \\ \mu = 2.43 \end{cases}$$

Leading to the following curve.

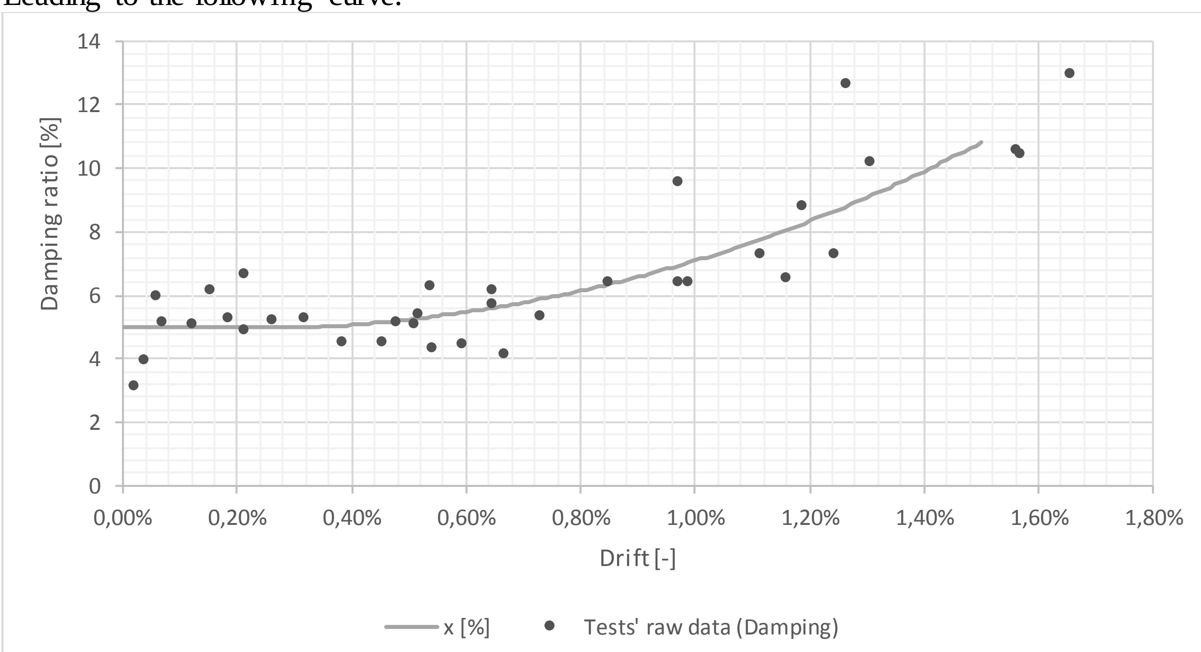


Figure 2. Damping ratio evolution.

These empiric laws are associated to a validity domain that is:

- A slenderness of 0.4,
- A reinforcement ratio from 0.11% up to 0.8% of the concrete section.

CALCULATING THE DAMAGE INDICATOR OF EACH PANEL

Let's consider the global coordinate system (O, x, y).

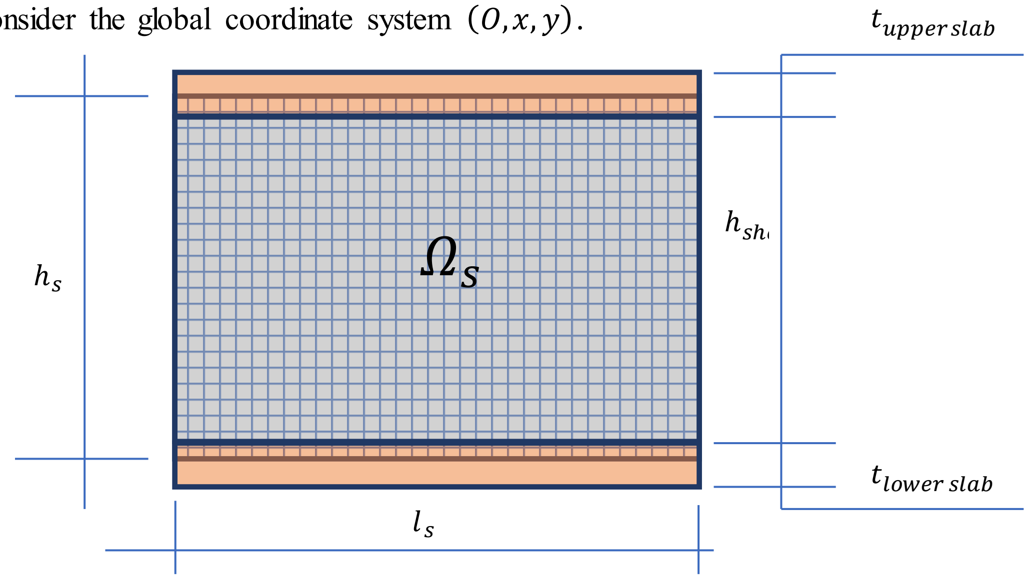


Figure 3. General view of the elements and the macro-element.

For this panel we calculate the drift considering the figure 4

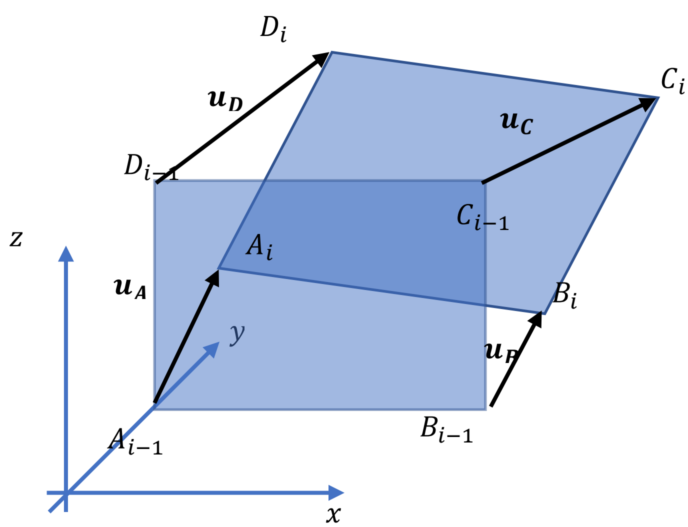


Figure 3. Calculation of the drift.

Let's consider the in-plane displacement vector based on the degrees of freedom of the plate. We get rid of the out of plane DOF by using the normal vector of the panel. Thus the considered vector is a 2 components array $\boldsymbol{u_A}$ oriented by x and z in the considered example.

$$\boldsymbol{u_A} = \begin{Bmatrix} u_{Ax} \\ u_{Az} \end{Bmatrix}$$

The scalar drift value of the panel Ω_s for an iteration k, is based on the drift of each macrostructural element.

$$d_s = \frac{1}{h_s}\left[\frac{u_{Dx} + u_{Cx}}{2} - \frac{u_{Ax} + u_{Bx}}{2} - h_s \frac{u_{Az} - u_{Bz}}{l_s}\right]$$

GENERAL ALGORITHMS FOR MODAL TRANSIENT CALCULATION AND SPECTRUM RESPONSE METHOD

For transient calculation the method is easily applicable on transient calculation considering the modal basis.

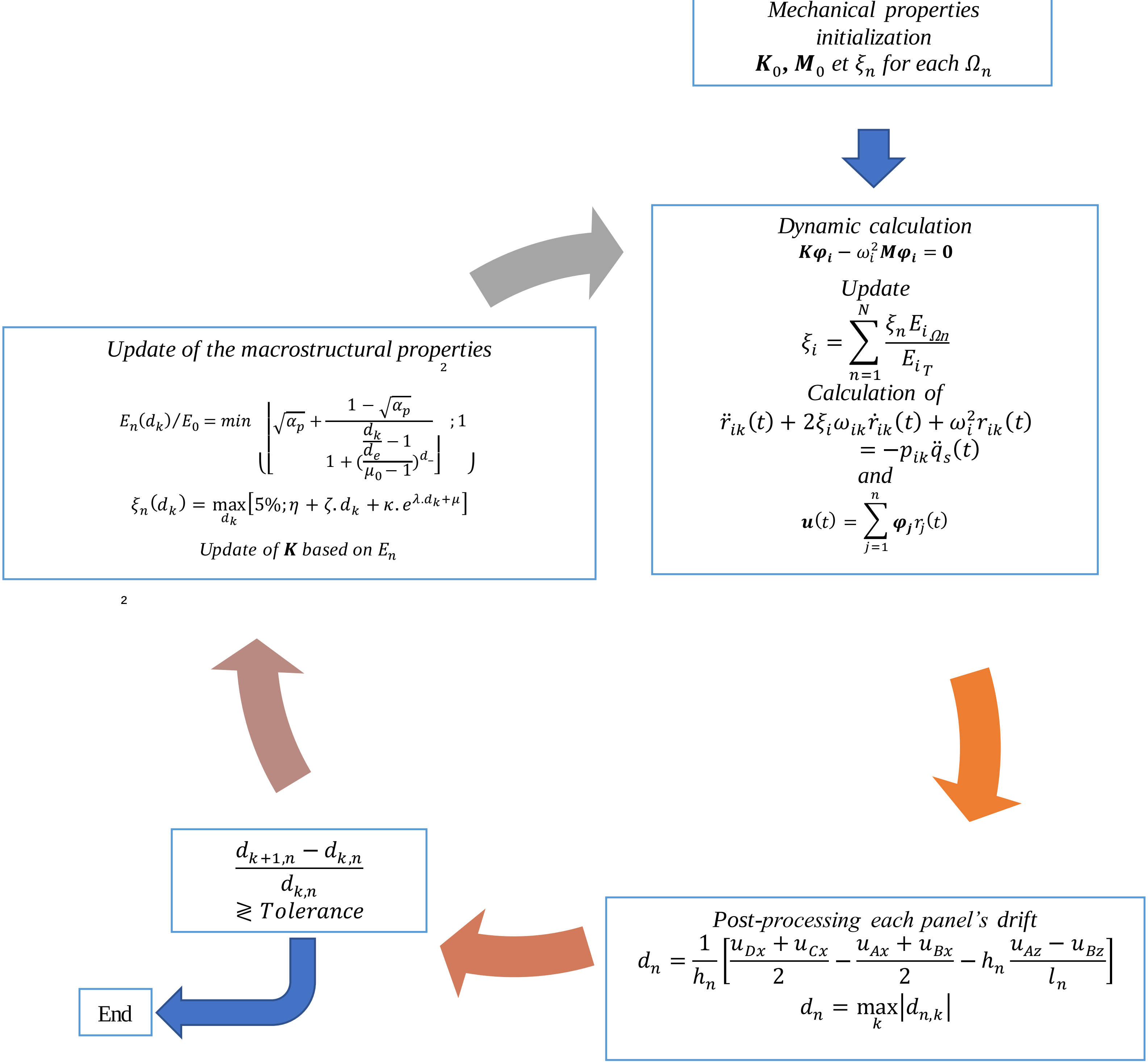


Figure 4. Algorithm for transient analysis.

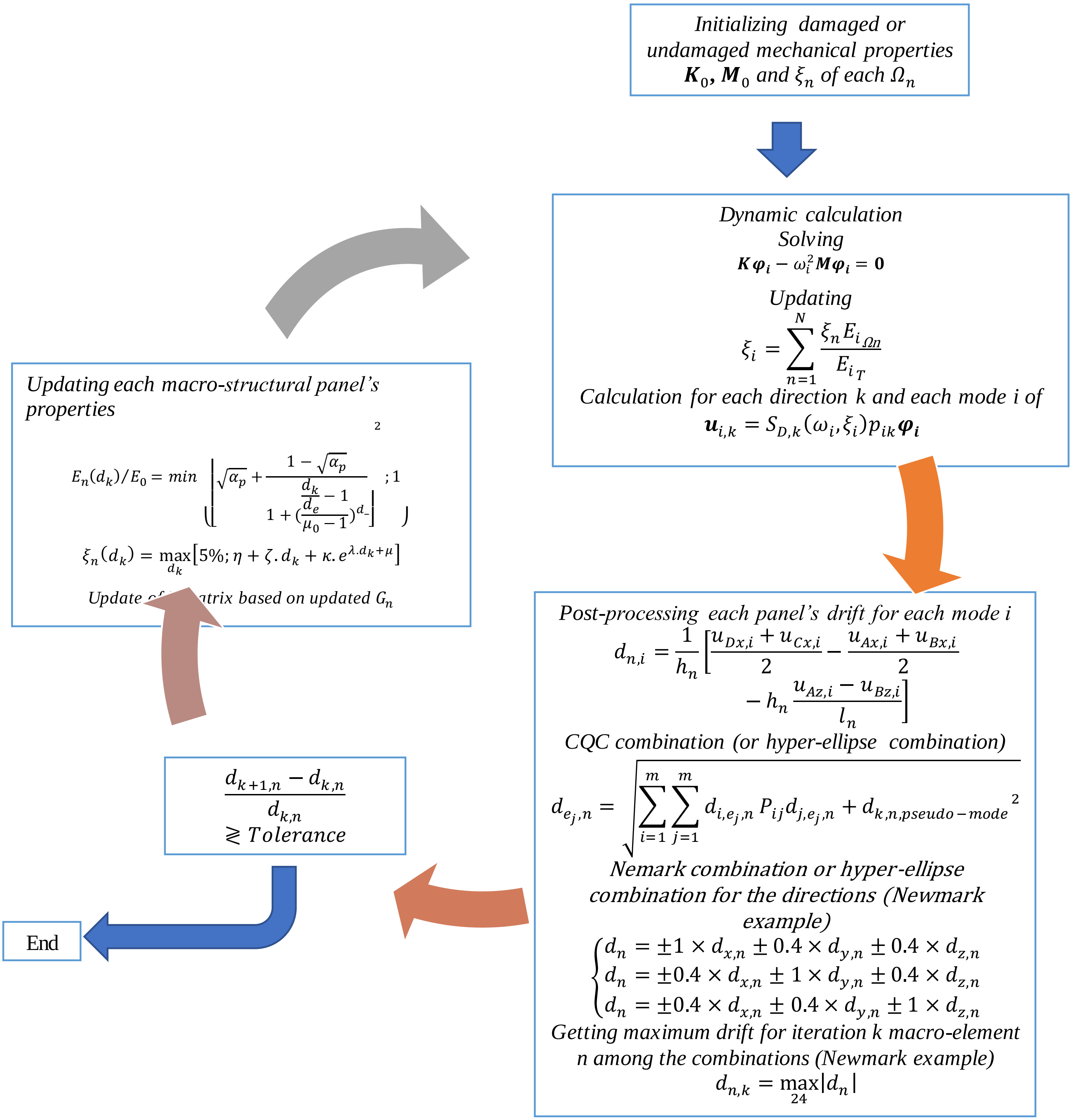


Figure 5. Algorithm for response spectrum method.

With:

- E_{i_T} being the total potential energy for mode i: $E_{i_T} = \frac{1}{2}\boldsymbol{\varphi}_i^T \boldsymbol{K} \boldsymbol{\varphi}_i$
- $E_{i_{\Omega_j}}$ being the potential energy for mode i for a structural element Ω_j whose damping ratio is ξ_j
- p_i being the participation coefficient i:

$$p_{i,k} = \frac{\boldsymbol{\varphi}_{i,k}^T \boldsymbol{M} \boldsymbol{\Delta}}{\boldsymbol{\varphi}_{i,k}^T \boldsymbol{M} \boldsymbol{\varphi}_{i,k}}$$

- $\boldsymbol{\Delta}$ being the loading direction vector corresponding to the DOFs that are moved by the acceleration of the earthquake $\ddot{q}_s(t)$.

For response spectrum method the algorithm is slightly more complicated involving some more steps.

COMPARISON WITH EXPERIMENT: SMART MOCKUP

SMART mockup and the shaking table AZALEE consists in our real reference for the analyses.

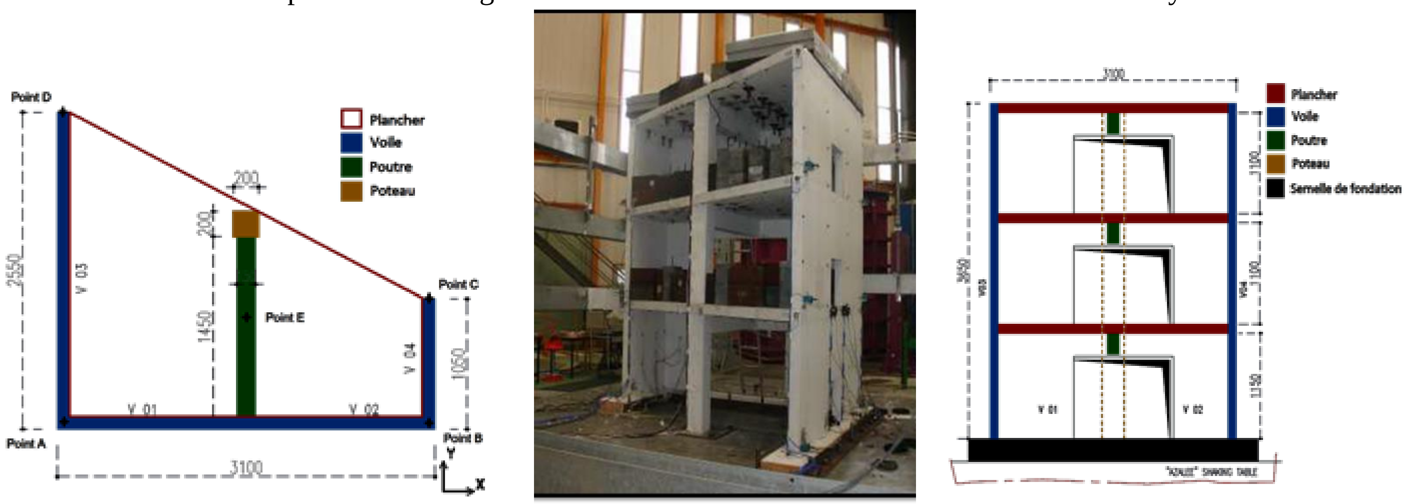


Figure 6. General overview of SMART mockup.

SMART means "Seismic design and best-estimate Methods Assessment for Reinforced concrete buildings subjected to Torsion and non-linear effects".

All the details of this mockup depicting an auxialary building can be found in (Richard, B et al. 2016).

The considered run is 19th, whose testing spectrum and associated time history are exposed in figure . Le niveau d'accélération de dimensionnement exprimé sous la forme de l'accélération asymptotique était de 0.22g. It is the 7th earthquake motion applied on the SMART2013 mockup. The motion is the Northridge earthquake at 100% of its nominal level.

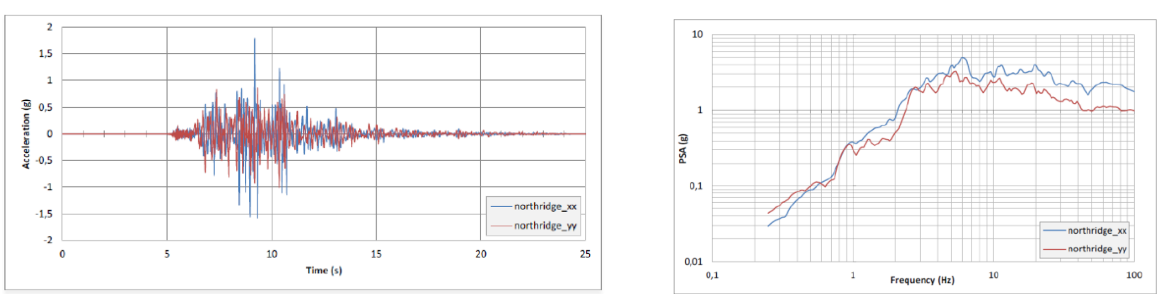


Figure 7. Time history and corresponding response spectrum.

The considered mesh is exposed in figure 8 . 12 points have been monitored, the motions have been compared with the experimental motions by means of Anderson criteria (see Anderson, J. (August 1-6, 2004). On this mesh we created some groups of meshes that figure out the macro-elements whose properties will evolve during the calculations.

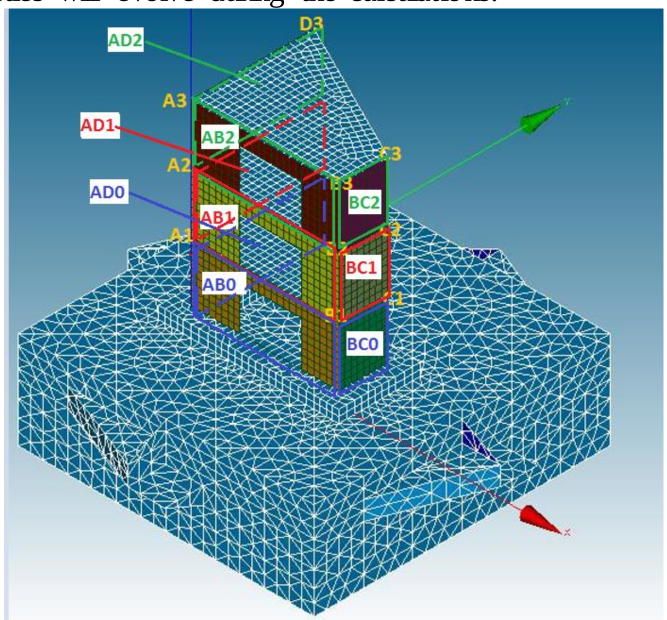


Figure 8. Groups of meshes.

The iterations were limited to 12. The calculation cost is very small (lower than 5 minutes for each iteration). Figure 9 exposes the evolution of the drift criterion and Figure 10 exposes the evolution of the mechanical properties.

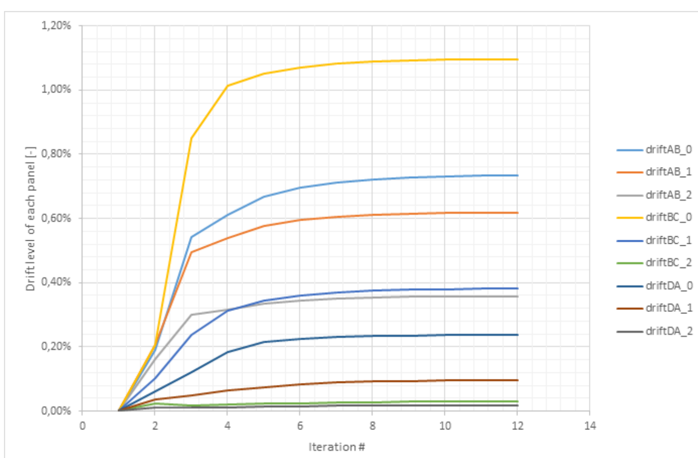


Figure 9. Convergence curves.

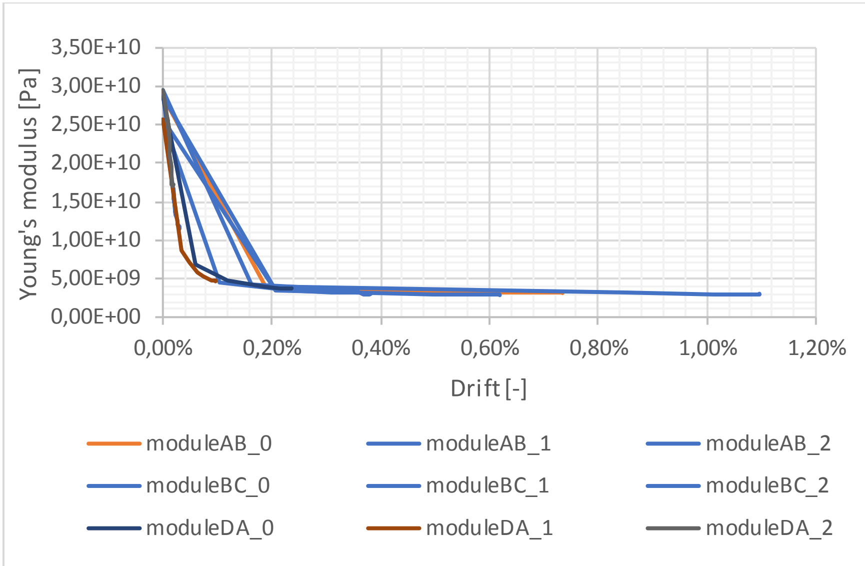


Figure 10. Softening of the Young's moduli.

The damping ratio has never exceed 5%.

The rating of the Anderson criteria reveals that the results are quite good considering the very cheap numerical approach that has been used.

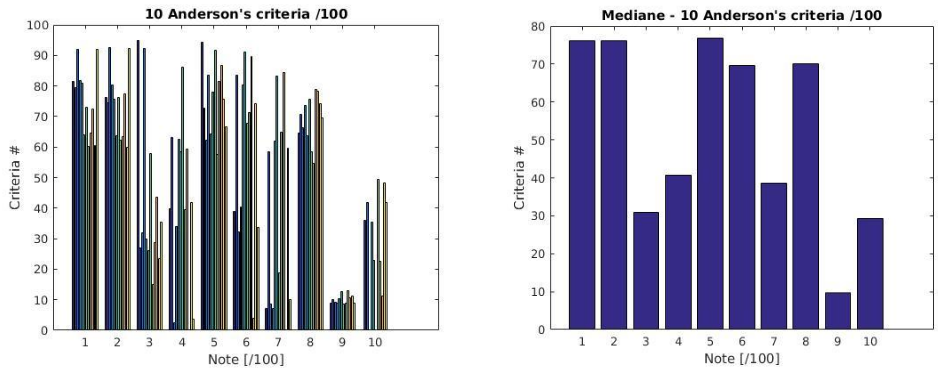


Figure 11. Anderson criteria rating for the considered set of points.

The damage indicator distribution is also very consistent with the one observed during the test (see (Richard B. et al. 2016) as the figure 12 exposes it.

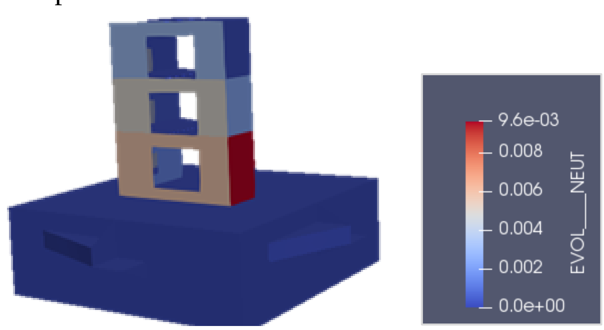


Figure 12. Damage indicator mapping.

The stiffness degradation mapping can also be post-processed as it is exposed in figure 13.

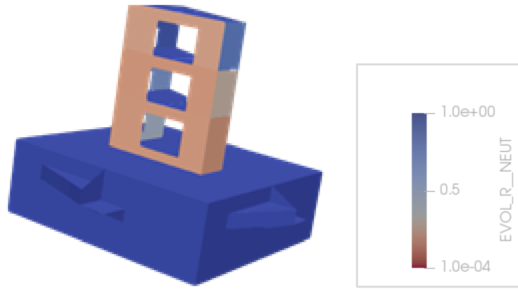


Figure 13. Young's modulus degradation.

CONCLUSIONS

The presented method is a very cheap approach to get a good approximation of the level of damage of a structure. At the design stage, it can easily embark randomness and uncertainty and then help the designer to appreciate the level of margin of a considered design without running non linear calculations whose level of refinement is not necessary on the same line as the knowledge of the project at this stage may be. As it linear equivalent approach response spectrum method is fully applicable. It is reducing the duration of the calculations and also reduces the amount of calculations that have to be run (example at least a minimum amount of 5 for many standards when considering non linearities).

To be complete this method requires to investigate the other modes of degradation and also to identify the interaction in between different failure modes. Its domain of validity has to be intensively documented as it may be very dependent on it.

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