Instability of a circumferential crack in a piping system subject to bending and tensile loadings

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1 INTRODUCTION

The problem of the instability of crack growth in a pipe fabricated from a ductile material has received extensive consideration, both theoretically and experimentally, during the last few years. This consideration has been motivated, to a large extent, by the technological problems posed by the intergranular stress corrosion cracking of Type 304 stainless steel piping systems in Boiling Water Reactors. The cracks have been observed in the heat-affected zones of girth welds and they form at the inner surface of a pipe; they are oriented circumferentially and are able to grow slowly in service by an environmentaly assisted mechanism. From a safety perspective, an important question is whether a severe overload, due to for example severe earthquake, will convert a part-through stress corrosion crack into a through-wall crack, which will then spread unstably around the pipe circumference and so cause complete pipe severance (guillotine rupture). Instability can be predicted by assuming that failure follows a net-section stress criterion, using as input a knowledge of the critcal net-section stress value and the anticipated loading[1]. Application of this criterion defines the conditions for failure for load-control deformation.

However, there will be a safety margin which stems from the fact that a dominant component of any accident condition load is not load-control, but rather displacement control. It is for such loading states that Paris and co-workers[2] have developed a tearing modulus methodology for deriving the crack instability criterion. With the aid of this methodology Tada, Paris and Gamble[3] examined the stability of a through-wall crack in an initially straight pipe that is subject to equal but opposite rotations at the pipe ends, together with a constant axial tensile load. The general conclusion arising from their analysis is that, because of the high non-environmentally assisted crack growth resistance of 304 stainless steel, unstable crack growth should not occur in service, since the pipe length required for instability is much greater than the typical pipe run lengths in actual piping systems. The axial tensile load simulated the effect of internal pressure in a reactor system, and at this level, the axial load did not have a marked adverse effect on crack stability. The key assumptions in in the Tada-Paris-Gamble approach are: (a) the material is non-work-hardening, (b) plasticity is

confined to the fully yielded cracked cross-section, with the plasticity being represented solely by a plastic rotation about a neutral axis, (c) the remainder of the pipe deforms elastically.

Since Tada, Paris and Gamble's initial analysis, several subsequent analyses, based on their model approach, have had as their general objective the exploration of factors that might possibly erode the large safety margin predicted by their simple analysis. Thus, for the case of a straight pipe segment, the author has examined the effects of combined tensile and bending loadings, produced by arbitrary imposed displacements[4,5]. In these analyses, the author allowed for a plastic tensile displacement as well as a plastic rotation; however these analyses have limitations in that the relative contributions from the rotational and tensile contributions are indeterminate, and simplifying assumptions must be made in order to proceed with the analyses. More recent analyses[6,7], again for a straight pipe segment, have assumed that plasticity is represented solely by a plastic rotation about a neutral axis at the cracked section, though allowing for the axial plastic displacement induced by this rotation. If the imposed deformations at the built-in ends generate an axial displacement, the instability criterion depends on the details of these imposed deformations, but if the built-in ends are subjected to deformations that generate only bending deformation, the instability criterion is independent of the details of the imposed deformations.

This paper is a further contribution to the understanding of the effect of complex loadings on crack instability, in that it presents the results of analyses for a particular pipe configuration which contains pipe bends. The ends of the system are subjected to fixed deformations that generate, in the general case, a combination of bending and tensile loadings at the cracked section. By varying the configuration's geometrical parameters, it is possible to assess the effect of a wide range of combinations of bending and tensile loadings on the instability criterion. The instability criterion is also compared with that obtained by a simple procedure[8] that is currently used for piping system integrity evaluations.

2 THEORETICAL ANALYSIS

The model of the cracked cross-section of the circular cylindrical pipe, thickness t and radius $R \gg t$, is shown in Figure 1. As in the original Tada-Paris-Gamble model[3], it is assumed that the material is non-work-hardening with plastic deformation being confined to the cracked cross-section, which is fully yielded, the remainder of the pipe deforming elastically. The cracked cross-section is therefore subject to a moment Mp about an axis that passes through the centre of the section and a tensile force P, acting perpendicular to the section and again passing through its centre. It is presumed, again as in the original Tada-Paris-Gamble analysis, that the plastic deformation is entirely in the form of a rotation about the neutral axis. If the numerical magnitude of the tensile stress σ (representative of the material's yield stress) above the neutral axis is the same as that of the compressive stress acting below the axis, equilibrium at the cracked section requires that

$$(1) \qquad \frac{P}{2\sigma Rt} = 2\alpha - \theta$$

$$(2) \qquad \frac{M_p}{4\sigma R^2 t} = \cos\alpha - \frac{1}{2} \quad \sin\theta$$

where the angle α defines the position of the neutral axis; these relations are valid provided the neutral axis lies below the crack tips, i.e. provided $\alpha + (\pi/2) > \theta$.

The crack tip opening displacement D_{TIP} is given by the expression

$$(3) \qquad D_{TIP} = R\Phi_{PL}\,(\sin\alpha + \cos\theta)$$

where Φ_{PL} is the plastic rotation about the neutral axis, whereupon with the J integral expressed as the product of D_{TIP} and the yield stress σ, it follows that

$$(4) \qquad J = \sigma R\Phi_{PL}\,(\sin\alpha + \cos\theta)$$

The tearing modulus methodology for predicting crack instability, due to Paris and co-workers[2], is based on the premise that the shape of the J-crack growth resistance curve is specific for a given material, with the material tearing modulus being related to the initial slope

$$(5) \qquad T_{MAT} = \frac{E}{\sigma^2}\,\frac{dJ_{MAT}}{da}$$

where E is Young's modulus and δa is an increment of crack extension. There is a corresponding expression for T_{APP}, the applied tearing modulus, and instability is assumed to occur if $T_{APP} > T_{MAT}$. T_{APP} can be obtained from relation (4) noting that $\delta a = R\delta\theta$ for circumferential crack growth. Thus if J_{IC} is the value of J at the onset of crack extension, the instability criterion is

$$(6) \quad T_{APP} = \frac{EJ_{IC}}{\sigma^2 R} \cdot \frac{\left[\cos\alpha\,\frac{d\alpha}{d\theta} - \sin\theta\right]}{(\sin\alpha + \cos\theta)} + \frac{E}{\sigma}\,(\sin\alpha + \cos\theta)\frac{d\Phi_{PL}}{d\theta} > T_{MAT}$$

Remembering that the concern is with the behaviour of a very ductile material, in the sense that its crack growth resistance is high, and a conservative value of T_{MAT} for 304 stainless steel is 200, then following Tada, Paris and Gamble[1], the term involving J_{IC} in expression (6) for T_{APP} can be neglected, whereby the instability criterion can be expressed in the form

$$(7) \qquad T_{APP} = \frac{E}{\sigma}\,(\sin\alpha + \cos\theta)\,\frac{d\Phi_{PL}}{d\theta} > T_{MAT}$$

with α and $d\Phi_{PL}/d\theta$, which depend on the particular problem being considered, having yet to be determined.

In examining the stability of the crack in Figure 1 with regards to circumferential growth, consider first of all the case where a straight segment of pipe with length L contains a crack at its mid-length position, and the ends are rigidly linked to segments of length h, the free ends of these segments being subject to fixed displacements D, which are associated with a tensile force P (Figure 2). Since the horizontal and vertical pipe segments are rigidly connected, and noting that $M_p = Ph$, simple beam theory gives the displacement D in the form

$$(8) \qquad D = \left[\frac{PL}{4\pi ERt}\right] + \left[\frac{R\,\Phi_{PL}}{2}\sin\alpha\right]$$

$$+ \left[\frac{h\,\Phi_{PL}}{2} + \frac{MphL}{2EI}\right] + \left[\frac{Ph^3}{3EI}\right]$$

where $I \equiv \pi R^3 t$ is the second moment of area of the pipe cross-section (the vertical and horizontal segments of the system are assumed to have the same R and t values). In expression (8), the first term on the right-hand side is due to the elastic extension of the horizontal segment, the second term is due to the plastic extension of the horizontal segment induced by the plastic rotation ϕ_{PL}, the third term is due to the rotation at the ends of the horizontal segements, and the fourth term is due to the relative displacement at the ends of the vertical segments. With D fixed, it follows from relations (1), (2), (7) and (8), after ignoring a term involving ϕ_{PL} in comparison with the term involving $d\phi_{PL}/d\theta$ (i.e. similar to the arguments leading from (6) to (7)), that the instability criterion can be written in the form

$$(9) \qquad T_{APP} = \frac{2}{\pi R}\left[L + \frac{2h}{3} + \frac{LR^2}{2h^2}\right] \frac{(\sin\alpha + \cos\theta)^2}{\left[1 + \frac{R\sin\alpha}{h}\right]^2} > T_{MAT}$$

where, because $M_P = Ph$, α is given from equations (1) and (2), by the equation

$$(10) \qquad \frac{R}{h} = \frac{\left[\alpha - \frac{\theta}{2}\right]}{\left[\cos\alpha - \frac{1}{2}\sin\theta\right]}$$

Inspection of expressions (9) and (10) immediately shows that the instability criterion depends on the manner in which the moment and tensile loadings at the cracked section combine. R/h = 0 corresponds to the case where there is no tensile load at the cracked section, whereas R/h = ∞ corresponds to the case where the cracked section is subject only to a tensile load. For the case R/h = 0, expression (9) simplifies to

$$(11) \qquad T_{APP} \equiv \frac{2}{\pi R}\left[L + \frac{2h}{3}\right]\left[\sin\frac{\theta}{2} + \cos\theta\right]^2 > T_{MAT}$$

At the other extreme where R/h = ∞, and the configuration degenerates into a single straight pipe segment that is subjected to only an applied tensile force, expression (9) simplifies to

$$(12) \qquad T_{APP} \equiv \frac{L}{\pi R}\left[1 + \frac{\cos\theta}{\sin\left\{\cos^{-1}\left[\frac{1}{2}\sin\theta\right]\right\}}\right]^2 > T_{MAT}$$

Intermediate values of R/h can be considered by reference to relations (9) and (10), thereby giving a complete gradation from the situation where the cracked section is subject to pure tensile loading to the situation where it is subject to pure moment loading.

Probably a more realistic simulation of a piping system is that shown in Figure 3 where the ends of the system are built-in, which is not the case with the system in Figure 2; otherwise the piping configuration remains the same. The lower built-in ends of the vertical segments are subjected to transverse displacements D and rotations θ , these being associated with forces P and moments M. With ϕ_{PL} again being the anti-clockwise plastic rotation at the cracked section, simple beam theory shows that the horizontal displacement D_L and anti-clockwise rotation θ_L at the right-hand linkage are given by respectively the expressions

(13) $$D_L = \frac{PL}{4\pi ERt} + \frac{R\Phi_{PL}}{2}\sin\alpha$$

(14) and $$\theta_L = \frac{M_p L}{2EI} + \frac{\Phi_{PL}}{2}$$

Simple beam theory applied to the right-hand vertical segment shows that

(15) $$D = \frac{Ph^3}{3EI} + \frac{Mh^2}{2EI} + D_L + h\theta_L$$

(16) and $$\theta_* = \frac{Ph^2}{2EI} + \frac{Mh}{EI} + \theta_L$$

Noting that $M_p = M + Ph$, equations (13) to (16) combine together to give

(17) $$D = P\left[\frac{L}{4\pi ERt} - \frac{h^3}{6EI}\right] + \frac{M_p h}{2EI}(L+h) + \frac{\Phi_{PL}}{2}(h + R\sin\alpha)$$

(18) and $$\theta_* = -\frac{Ph^2}{2EI} + \frac{M_p}{EI}\left[h + \frac{L}{2}\right] + \frac{\Phi_{PL}}{2}$$

With D and θ both being fixed, it follows from equations (1), (2), (7), (17) and (18), after ignoring a term involving Φ_{PL} in comparison with a term involving $d\Phi_{PL}/d\theta$, that the instability criterion becomes

(19) $$T_{APP} = \frac{2}{\pi}(\sin\alpha + \cos\theta)^2 X$$

$$\frac{\left[\frac{2Lh^3}{3R^4} + \frac{h^4}{3R^4} + \frac{hL}{R^2} + \frac{L^2}{2R^2}\right]}{\left[\frac{2h^3}{3R^3} + \frac{L}{2R}\right] + \frac{2h^2}{R^2}\sin\alpha + 2\left[\frac{h}{R} + \frac{L}{2R}\right]\sin^2\alpha} > T_{MAT}$$

Supposing that θ is related to D via the expression $\theta = kD$ where k is a constant, and with the crack tip displacement D_{TIP} equal to δ_{IC} at the onset of crack extension, relations (1), (2) (3), (17) and (18) show that

(20) $$k = \frac{\left[\frac{2\sigma R}{\pi E}\left[\frac{L}{4R} - \frac{h^3}{3R^3}\right](2\alpha-\theta) + \frac{2\sigma h}{\pi ER}(L+h)(\cos\alpha - \frac{1}{2}\sin\theta) + \frac{\delta_{IC}(h+R\sin\alpha)}{2R(\sin\alpha + \cos\theta)}\right]}{\left[-\frac{\sigma h^2}{\pi ER^2}(2\alpha-\theta) + \frac{4\sigma}{\pi ER}\left[h + \frac{L}{2}\right]\left[\cos\alpha - \frac{1}{2}\sin\theta\right] + \frac{\delta_{IC}}{2R(\sin\alpha + \cos\theta)}\right]}$$

Inspection of expressions (19) and (20) immediately shows that, in general, the instability criterion depends on the details of the imposed deformations, i.e. on the magnitude of k; this is because α depends on k (see expression (20)) and T_{APP} depends on α (see expression (19)). However, for the special case where h is very large, relation (20) shows that $\alpha = \theta/2$ irrespective of the value of k. The deformation at the cracked section then proceeds entirely by bending and the instability criterion (19) simplifies to

$$(21) \quad T_{APP} = \frac{2}{\pi}\left[\sin\frac{\theta}{2} + \cos\theta\right]^2\left[\frac{h}{2R}\right] > T_{MAT}$$

3 DISCUSSION

The preceding section has analysed the behaviour of a piping system in which there are bends in the system, with results being presented for two sets of boundary conditions. The second set, i.e. where the ends of the system are built-in and are subjected to applied rotations and displacements, is probably more relevant to the behaviour of an actual piping system subjected to accident loadings. In this case it has been shown that, in general, the criterion for the instability of growth of a circumferential crack depends on the precise form of the imposed boundary conditions, a result that is in accord with results for a straight pipe segment subjected to a combination of bending and tensile deformation[7]. It is only for the extreme case where the system geometry is such that there is no overall tensile force at the cracked cross-section that the instability condition is independent of the details of the imposed boundary conditions at the built-in ends, i.e. independent of the way in which the imposed rotations and displacements are related to each other.

This conclusion has important implications with regards to a simple procedure that is currently used[8] to determine the instability criterion for a crack in a piping system. This procedure involves a separation of the complete piping system into two elastic parts at the cracked section, with the system ends remaining fixed, the application of equal and opposite moments M_A to the cut faces and the correlation of M_A with the rotational discontinuity ϕ generated at this section, whereupon the instability criterion evolves immediately from this correlation. It is[8]

$$(22) \quad T_{APP} = \frac{2L_{EFF}}{\pi R}\left[\sin\frac{\theta}{2} + \cos\theta\right]^2 > T_{MAT}$$

$$(23) \quad \text{where} \quad L_{EFF} = \frac{EI\phi}{M_A}$$

If this procedure is used for this paper's model, and if P_A is the tensile force at the cracked section, simple bending theory shows that the rotational discontinuity ϕ due to the moment M_A and the tensile force P_A is given by the equation

$$(24) \quad \phi = \frac{M_A}{EI}[L + 2h] - \frac{P_A h^2}{EI}$$

while to maintain compatibility in displacement at the cracked section

$$(25)\quad \frac{P_A h^3}{3EI} + \frac{P_A L}{4\pi RtE} - \frac{M_A h^2}{2EI} = 0$$

It follows from relations (22)-(25) that the instability criterion, as given by the simple procedure

$$(26)\quad T_{APP} = \frac{2}{\pi}\left[\sin\frac{\Theta}{2} + \cos\Theta\right]^2 X \frac{\left[\frac{2Lh^3}{3R^4} + \frac{h^4}{3R^4} + \frac{hL}{R^2} + \frac{L^2}{2R^2}\right]}{\left[\frac{2h^3}{3R^3} + \frac{L}{2R}\right]} > T_{MAT}$$

This expression differs from expression (19), in that it does not include terms involving α; the expressions are identical only in the extreme case where h is very large, when there is no overall tensile force at the cracked section. This result underscores the view that the simple procedure[8] gives strictly accurate instability predictions only for the limiting case where there is no overall tensile force acting at the cracked section, in other words when the deformation at the cracked section is primarily bending.

Finally, it is worth emphasizing that the analyses in this paper have been based on the assumption that the horizontal and vertical pipe segments are rigidly linked, whereas in an actual piping system this will not be the case. Flexibility at the bends can, in principle, be incorporated within the analyses by using appropriate flexibility factors. However this would complicate the analyses, and it is not anticipated that the conclusions would be changed if this factor is taken into account.

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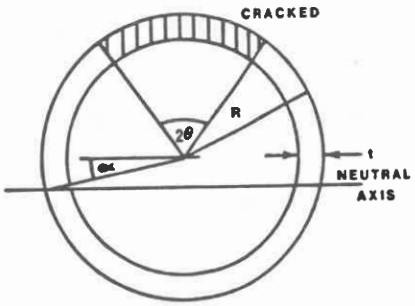

FIG. 1.

A cross-section of a pipe.

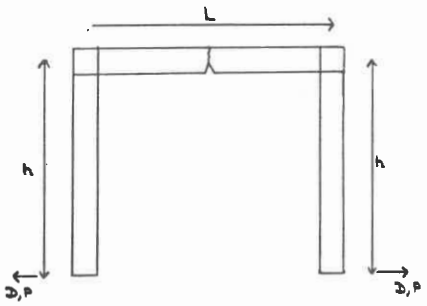

FIG. 2

The symmetrical piping configuration showing the first set of boundary conditions.

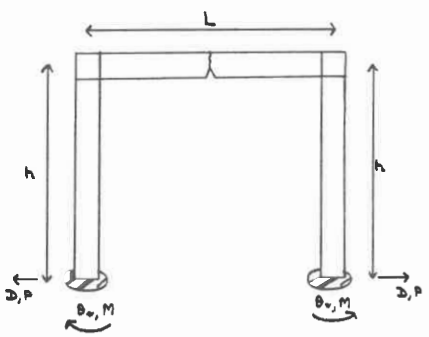

FIG. 3

The symmetrical piping configuration showing the second set of boundary conditions.