

## COUPLING PHENOMENA IN THERMOPLASTICITY

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### Abstract

The paper is restricted to a phenomenological theory of inelastic solid bodies with special regard to metals. The bodies are considered as classical continua underlying thermo-mechanical irreversible processes including solid phase transformations, recrystallization, recovery etc.

After some remarks on mechanical and thermodynamical foundations of such processes special emphasis is given to the general structure of the constitutive equations of plastic and visco-plastic materials. From these considerations some conclusions on coupling phenomena between mechanical, thermal, and internal processes are drawn. The coupling phenomena can be divided into two classes: the first kind of coupling phenomena appears already in homogeneous processes as, for instance, the interaction between hardening due to plastic deformations and softening caused by recrystallization and recovery, the second kind of coupling phenomena occurs only in non-homogeneous process like the generation of residual stresses in quenching.

## 1. Introduction

We deal with non-isothermic deformations of inelastic solid bodies with special regard to metals within the frame of a phenomenological theory. Solid phase transformations as well as recrystallization and recovery phenomena may be included. Processes at the microscale are not considered in detail; they are regarded as the physical background of the phenomenological theory which assume the bodies as classical continua.

The thermo-mechanical processes under consideration are in general irreversible processes. Whereas the mechanical foundations of such processes seem to be fairly clear (chapter 2) the thermodynamical foundations of such process are widely under discussion (chapter 3). We shall adopt a position which seems to be suitable and sufficient to deal with technical problems in a wide range. This frame allows for a consistent formulation of constitutive laws of the materials (chapter 4) from which some conclusions can be drawn with respect to coupling phenomena between mechanical, thermal and internal processes (chapter 5).

## 2. Mechanical foundations

In order to describe the deformations of a classical continuum, we can follow different concepts which can be separated into two groups. The one group is based on the consideration of the motion of the material points against a fixed reference space. The other group takes the changes of the metric of a body-fixed coordinate system as the starting point for the description of the deformations. Different aspects of both of these possibilities are discussed extensively in [1-5].

We adopt the second way and choose the Almansi strain tensor

$$\epsilon_{ik}(\xi^r, t) = \frac{1}{2} [g_{ik}(\xi^r, t) - g_{ik}^0(\xi^r, t)] \quad (1)$$

as a suitable strain measure, where the superscribed 0 indicates the initial undeformed state.  $\epsilon_{ik}$  is related to the base of the current deformed state as are all other quantities to be introduced.  $\epsilon_{ik}$  can be decomposed additively into a recoverable (elastic) part and an inelastic part according to

$$\epsilon_{ik} = \underset{(e)}{\epsilon}_{ik} + \underset{(i)}{\epsilon}_{ik} = \frac{1}{2} (g_{ik} - g_{ik}^*) + \frac{1}{2} (g_{ik}^* - g_{ik}^0), \quad (2)$$

where the superscribed \* denotes an imaginary intermediate state. The corresponding decomposition of the strain rate is

$$\begin{aligned} d_{ik} = \dot{\epsilon}_{ik} &= \underset{(e)}{\dot{\epsilon}}_{ik} + \underset{(i)}{\dot{\epsilon}}_{ik} \\ &= \underset{(e)}{d}_{ik} + \underset{(i)}{d}_{ik} \end{aligned} \quad (3)$$

In the same way the specific mechanical work rate  $\dot{w}$  can be decomposed into

$$\begin{aligned} \dot{w} &= \frac{1}{\rho} \sigma^{jk} \dot{\epsilon}_{;ik} = \frac{1}{\rho} \sigma^{jk} \underset{(e)}{\dot{\epsilon}_{;jk}} + \frac{1}{\rho} \sigma^{jk} \underset{(i)}{\dot{\epsilon}_{;jk}} \\ &= \underset{(e)}{\dot{w}} + \underset{(i)}{\dot{w}}. \end{aligned} \quad (4a)$$

Herein  $\sigma^{jk}$  denotes the Cauchy stress tensor and  $\rho$  the mass density. Eq. (4a) can also be written as

$$\rho \dot{w} = s^{ik} \dot{\epsilon}_{;ik} = s^{ik} \underset{(e)}{\dot{\epsilon}_{;ik}} + s^{ik} \underset{(i)}{\dot{\epsilon}_{;ik}}, \quad (4b)$$

where

$$s^{ik} = \frac{\rho}{\rho} \sigma^{ik} \quad (5)$$

represents the weighted Cauchy stress tensor.

The inelastic work rate has to be split once more into one part  $\underset{(d)}{\dot{w}}$ , which is dissipated immediately, and another part  $\underset{(h)}{\dot{w}}$ , which is involved in the changing of the internal material structure. This leads to

$$\rho \dot{w} = \underset{(e)}{\rho \dot{w}} + \underset{(h)}{\rho \dot{w}} + \underset{(d)}{\rho \dot{w}}. \quad (6)$$

The mechanical aspect of the processes under consideration is governed by the balance equation for linear momentum

$$\sigma^{ik}{}_{;i} + \rho(f^k - a^k) = 0. \quad (7)$$

Herein  $f^k$  represents the specific body forces and  $a^k$  denotes the acceleration against the reference space. Eq. (7) can be transformed into an equation for the weighted stress  $s^{ik}$  or, if necessary, for the weighted stress rate  $s^{ik}{}_{|0}$ , where  $|_0$  denotes the covariant (Zaremba-Jaumann) differentiation with respect to time [3].

### 3. Thermodynamic foundations

The thermodynamics of irreversible (non-equilibrium) processes are widely under discussion; there are nearly as many approaches as authors in this field [6, 7]. For our purposes we adopt an approach based on the assumption that each material element of the body has the properties of a local thermodynamical system whose state is defined uniquely by a set of (external and internal) state variables. This assumption allows the introduction of specific free energy and specific entropy as unique state functions. This approach, of course, is limited to processes not too far from equilibrium.

In such an approach, the balance equation for the specific internal energy  $u$ , i.e. the first law of thermodynamics, reads

$$\dot{u} = \dot{w} - \frac{1}{\rho} q^i |i + r - \frac{1}{\rho} p^i |i. \quad (8)$$

In this equation,  $q^i$  is the heat flux,  $r$  denotes the energy sources due to absorption of radiation etc., and  $p^i$  represents the energy flux apart from applied mechanical work and heat flux. One example for such an energy flux is the migration of dislocations, which is not necessarily accompanied by macroscopic deformations, and does not represent heat flux. The balance for the specific entropy yields

$$T\dot{s} = \dot{w} - \frac{1}{\rho} q^i |i + r - \frac{1}{\rho} p^i |i + \dot{\eta}. \quad (9)$$

Herein  $T$  denotes the absolute temperature,  $r$  the dissipated part of the energy supplied from sources,  $p^i$  the dissipated part of  $p^i$ , and  $\dot{\eta}$  the energy dissipated due to changes of the internal material structure. Concerning the specific entropy production, the second law of thermodynamics states (in the form of Clausius-Duhem inequality)

$$\begin{aligned} T\dot{s} &= T\dot{s} - \frac{T}{\rho} \left( \frac{q^i}{T} \right) |i \\ &= \dot{w} - \frac{1}{\rho T} q^i T |i + r - \frac{1}{\rho} p^i |i + \dot{\eta} \geq 0. \end{aligned} \quad (10)$$

Under our assumptions, the specific free energy (Helmholtz function)

$$\varphi = u - Ts \quad (11)$$

must be expressible by the thermodynamic state variables. As such state variables we may introduce  $\varepsilon_{(e)ik}$ ,  $T$ , and a set of scalar and tensor valued internal variables. For brevity we introduce only one scalar valued variable,  $b$ , and one tensor valued variable,  $\beta_k^i$ , representing the whole set of internal variables. Then we obtain

$$\dot{\varphi} = \dot{u} - T\dot{s} - s\dot{T} = -s\dot{T} + \dot{w} + \frac{1}{\rho} (p^i - p^i) |i - \dot{\eta} \quad (12a)$$

$$= \frac{\partial \varphi}{\partial T} \dot{T} + \frac{\partial \varphi}{\partial \varepsilon_{(e)ik}} \dot{\varepsilon}_{(e)ik} + \frac{\partial \varphi}{\partial b} \dot{b} + \frac{\partial \varphi}{\partial \beta_k^i} \dot{\beta}_k^i. \quad (12b)$$

The comparison between eq. (12a) and (12b) yields

$$\text{the caloric state equation} \quad s = - \frac{\partial \varphi}{\partial T}, \quad (13a)$$

$$\text{the thermic state equation} \quad \frac{1}{\rho} s^{ik} = \frac{\partial \varphi}{\partial e^{ik}} \quad (13b)$$

$$\text{and} \quad \dot{w} + r - r + \frac{1}{\rho} (\dot{p}^i - \dot{p}^i) | i - \dot{n}^i = \frac{\partial \varphi}{\partial b} \dot{b} + \frac{\partial \varphi}{\partial \beta_k^i} \dot{\beta}_k^i | o. \quad (13c)$$

From eq. (13c) we conclude that the evolution laws for the internal variables contain in general not only source terms but also the divergence of fluxes. Neglecting these energy fluxes, as usually done, we obtain a simplified theory with evolution laws containing only source terms. In the following we restrict ourselves to such a simplified theory. It may be mentioned that in homogeneous processes the fluxes  $p^i$  are actually vanishing; but in other cases they also remain obviously small due to the fact that we are dealing with solid bodies only.

#### 4. Constitutive equations

The constitutive equations for the material elements comprehend

- a) the evolution laws for the dependent external process variables (conjugated to the chosen independent external process variables);
- b) the evolution laws for the internal variables;
- c) the evolution laws for the entropy production.

The formulation of these evolution laws has to satisfy the thermodynamical requirements given by the equations (13) and (10). Possible satisfying formulations cannot be discussed here in detail. We must restrict ourselves to some general remarks (for more details see [3]).

In the frame of our simplified theory and with the usual assumptions concerning smoothness, the constitutive equations can be written as ordinary first order differential equations with respect to time, with one exception, the law for applied heat, which is represented by a partial differential equation. However, only in the simplified theory does this law take such an exceptional position.

Concerning the other evolution laws we can distinguish three different types:

- (A) the equilibrium type (rate type), describing a quasistatic sequence of equilibrium states characterized by the fact that changes of the considered dependent variable are always related to increments of the independent process variables;
- (B) the non-equilibrium type (flow type), characterized by the fact that the evolution of the considered variable depends only on the actual state;
- (C) the mixed type.

The evolution law for the strain of elastic-plastic material is, for instance, of equilibrium type (A), whereas for elastic-viscoplastic material, this law is of the

mixed type. The evolution laws for solid phase transformations, recrystallization, and recovery are of the non-equilibrium type. The same is true for the law of heat flux, since heat flux depends on spatial non-equilibrium of temperature. If and only if all the evolution laws entering the process description are of the equilibrium type (A) the thermo-mechanical process is rate insensitive.

##### 5. Some coupling phenomena in thermoplasticity

Let us start with the consideration of simple homogeneous adiabatic elastic-plastic deformations with given strains. In this case no field equations enter the problem. The stresses and the temperature represent dependent external process variables. The temperature is influenced on the one hand by the elastic volume changes and on the other hand by the dissipated part of inelastic work. The first effect is very small and in the most cases negligible. The rising temperature due to dissipation counteracts the hardening of the material caused by plastic deformations. However, the process remains rate insensitive, so long as nothing else happens. At higher temperature, solid phase transformations, recrystallization and recovery processes may occur. The thermo-mechanical process then becomes rate sensitive, since the corresponding evolution laws are of the non-equilibrium type. At the same time we obtain an additional interaction between hardening due to plastic deformations and softening as the result of these internal processes. This interaction may support the occurrence of bifurcation processes, leading to non-homogeneous deformations and temperature distributions connected with the generation of residual stresses. In such cases we have to take into account the (general and constitutive) field equations, even when the mechanical and thermal boundary conditions allow for homogeneous processes.

We obtain non-homogeneous processes (not necessarily combined with bifurcation problems), too, when we remove the adiabatic constraint at the surface of the body but retain the mechanical boundary conditions so that homogeneous deformations remain possible. In this case the temperature distribution becomes non-homogeneous, leading to differences in the hardening state inside the body and therefore to non-homogeneous deformations connected with the generation of residual stresses. This effect can be amplified when solid phase transformations are involved in the thermo-mechanical process [8].

The same coupling phenomena appear in elastic-viscoplastic bodies; the only difference is that the evolution law for the total strain is of the mixed type (C). Therefore thermo-mechanical processes in such bodies are always rate sensitive. Hence, in such bodies creep deformations and relaxation processes may occur even at constant temperature, whereas in elastic-plastic bodies such effects can occur only in connection with temperature changes.

The possible coupling phenomena between mechanical, thermal, and internal processes cannot be exhausted here. This paper shows only some examples derived from general statements.

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