# Updating Fuzzy Models for Seismic Risk Assessment

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## ABSTRACT

In this paper we approach some problems of updating fuzzy models for seismic risk assessment (SRA) with its subfields – seismic damage assessment and seismic fragility estimation. A short review of recent proposals for introducing fuzzy concepts and methods in structural reliability and seismic risk studies is given in the Introduction. Main concepts related to fuzzy sets and fuzzy logical inference are presented in the next section. The problem of Bayesian updating of fragility models is developed in the next two sections. We thus continue a research started with a paper of 1998. Once adopted a fuzzy model, it has to be possibly updated after acquisition of new data of seismically induced damages. This updating regards the central points for fuzzy parameters, the shape of fuzzy distributions, and also changing the prior membership functions by means of maximum likelihood method. Two numerical examples and a couple of final remarks close the paper.

## INTRODUCTION

The methods and models based on the Theory of Probabilities and Random Processes for structural system reliability studies and seismic risk assessment have a long history. The inherent randomness of structural and load variables make such concepts to be unavoidable. However, another important source of uncertainty comes from the probabilistic / stochastic models selected in a particular reliability analysis problem. This uncertainty is explicitly taken into account in seismic fragility models developed since the early 80's, mainly in connection with the US NRC projects for the PSA (Probabilistic Safety Assessment) of NPPs but also applied to civil engineering structures by ATC, in the 90's. The need for incorporating fuzzy concepts and methods in reliability and seismic risk studies occurred not very late after 1965 when the Fuzzy Set Theory was founded by L.A. Zadeh. One of the first approaches by fuzzy models to the prediction of structural accidents was due to D.I.Blockley [1], and the same author published an article entitled "The role of fuzzy sets in civil engineering", in the specialized journal *Fuzzy Sets and Systems* of 1979 [2]. C.B. Brown and J.T.P. Yao continued along this way ; Ref. [3] is only an example.

The necessity of passing from probabilistic / statistical to fuzzy models is concisely formulated by Rao & Sawyer [4]. If the system parameters are treated as random variables with known probability distributions, the performance or output of the system can be determined using the theory of probability. If the system parameters are imprecise, fuzzy theory can be used. In fact, the probability distributions cannot ever be considered as known. They are always assumed, i.e., they are hypothetical, and the observation data processed by statistical methods, including advanced sampling and simulation methods, can check to what extent the distributional assumptions were correct. The posterior acquired data can be used for changing the prior assumed distribution for a posterior one by means of Bayes' theorem. This approach needs numerical data, but it is often the case when the state of a structural system cannot be numerically quantified but only evaluated in terms of qualitative states like "slightly damaged", or "severely damaged", etc. In spite of recent advances in instrumental testing and nondestructive inspections of systems, the visual field inspections have not been entirely abandoned. Their results are usually stated by "linguistic" variables that describe the level of deterioration / degrading of a given system. Such variables were adopted in the fuzzy models for structural reliability analysis, and expressed by suitable fuzzy membership functions over the interval of possible values of a parameter considered to be most relevant. They can incorporate what is called the subjective information, what does not exclude the objective information, i.e., quantified data from a complex analysis. The idea of combining the fuzzy models with Bayesian updating is quite natural. The only references we found that presents effective fuzzy-Bayesian approaches to reliability of existing structures are due to V.P. Savchuk [5] and K. Chou & J. Yuan [6]. We have tried to extend and combine some ideas from the latter reference with methods of Bayesian updating for seismic fragility models – as those of [7,8] – in our paper [9]. The same line is further developed in the present paper, in which we try to incorporate more refined methods of Bayesian updating and some (older or newer) results on applications of fuzzy sets to seismic damage prediction as those due to authors from Chinese school in fuzzy models and to M.A.Ali [10] who proposes a fuzzy logic methodology for the seismic risk assessment. The next section presents some general concepts on fuzzy sets, fuzzy relations and fuzzy logic rules. The next two include the basic steps of a Bayesian updating procedure, and applications of fuzzy theory concepts to the Bayesian updating of seismic risk measures. Two numerical examples and a couple of remarks on possibilities of further developing such studies close the paper. The main conclusion is: the fuzzy-based approaches cannot replace the probabilistic models in seismic risk studies but they can enrich them and broaden their applicability.

## FUZZY REPRESENTATION OF DAMAGE SEVERITY

### Fuzzy Sets and Fuzzy Logic Rules

The original definition of fuzzy (sub)sets emerged from the idea of extending the definition of the characteristic (or indicator) function of a subset. Thus, if U is a set and $\mathcal{P}(\mathrm{U})$ is the set of its subsets, then the membership of an element $x$ in U to a subset $S \subseteq \mathrm{U}$ can be characterized by means of a function $\chi_S : \mathrm{U} \longrightarrow \{0,1\}$ as follows :

$$\chi_S(x) = \begin{cases} 1 & \text{if } x \in S, \\ 0 & \text{if } x \notin S. \end{cases} \tag{1}$$

Thus, the set of all subsets of U is in 1-to-1 correspondence with all the 0 - 1 valued functions of the form in Eq. (1). The fuzzy subsets of U are, in principle, defined by means of membership functions taking values in the whole interval $[0,1]$ and not only at the ends of this interval. Such functions are usually denoted by $\mu_S$ ; hence a fuzzy membership function is a mapping of the form

$$\mu_S : \mathrm{U} \longrightarrow [0,1]. \tag{2}$$

For any $x$ in U the real number $\mu_S(x)$ may be regarded as the degree of membership of $x$ to the subset $S$. A more general definition of the fuzzy sets is obtained [11] by taking a lattice $\mathcal{L}$ with 0, 1 as the minimum, respectively maximum element, instead of the interval $[0,1]$. A Boolean algebra $\mathcal{B}$ (with the distributivity of $\vee, \wedge$ operations with respect to each other, with the complementary operation defined by $a \vee \bar{a} = 1,\ a \wedge \bar{a} = 0,\ \bar{1} = 0,\ \bar{0} = 1$) instead of a lattice is considered in some references. For any element $a$ in a Boolean algebra $\mathcal{B}$ it is satisfied the inequality $0 \le a \le 1$ ; thus $\mathcal{B}$ is a generalization of the real interval $[0,1]$ with the linear order induced by the linear (or total) order relation on $\mathbb{R}$. A fuzzy (sub)set of U defined by means of a membership function as in Eq. (2), with $[0,1]$ replaced by $\mathcal{B}$, is called in [10] a $\mathcal{B}$-fuzzy set. The choice of the membership function is – up to a point – a matter of subjective choice.

In practical applications of fuzzy models to problems like the state of damage evaluation for a structural system, a damage parameter is considered whose range is a set U of real numbers. In most cases this is just an interval, and if the damage functional is normalized then it is just the unit interval $[0,1]$. Then a number in U may be regarded as a fuzzy number if it is a central point in a fuzzy set. An example is given in [10] for the strength capacity of a structure estimated to be (most likely) equal to 20000 kN. This would be a "crisp" number, but it is more realistic to consider it as the most probable value of a fuzzy number taking values in the interval [19000 , 21000]. The fuzzy representations of this fuzzy / crisp number are plotted in Fig. 1 - a) & b), respectively.

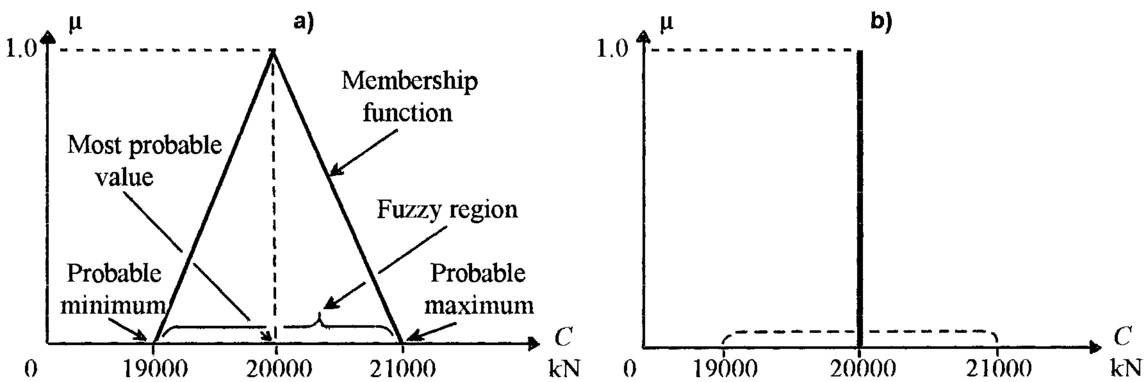


**Fig.1 Typical representations of a fuzzy and a crisp number by fuzzy membership functions**

The fuzzy subsets are often assimilated to fuzzy numbers and a specific "fuzzy arithmetic" is formulated to allow for operations with fuzzy numbers, and thus with fuzzy sets. If a fuzzy $S$ set is represented by a polygonal line defining its membership function $\mu_S$ (the triangular representation in Fig. 1 gives an example) and $x_i$ are the points where this line changes its slope then $S$ can be represented as a formal sum of the form

$$S = \sum_i \mu(x_i) \,|\, x_i. \tag{3}$$

The fuzzy set in Fig. 1 will thus be represented by $C = 0\,|\,19000 + 1\,|\,20000 + 0\,|\,21000$.

If $S$ and $T$ are fuzzy sets with their membership functions $\mu_S$ & $\mu_T$ then their fuzzy union and intersection are defined [11] by means of the membership function as follows:

$$S \cup T : \; \mu_{S \cup T}(x) = \mu_S(x) \vee \mu_T(x), \; x \in \mathrm{U}, \tag{4}$$

$$S \cap T : \; \mu_{S \cap T}(x) = \mu_S(x) \wedge \mu_T(x), \; x \in \mathrm{U}, \tag{5}$$

where $\vee$ and $\wedge$ are (respectively) the max and min operators. The algebraic product $S \cdot T$ and the algebraic sum $S + T$ of two fuzzy subsets of $\mathrm{U} \subseteq \mathbb{R}$ are respectively defined by the corresponding membership functions

$$\mu_{S \cdot T}(x) = \mu_S(x) \cdot \mu_T(x), \;\; \mu_{S+T}(x) = \mu_S(x) + \mu_T(x) - \mu_S(x) \cdot \mu_T(x). \tag{6}$$

The similarity of the rules in Eqs. (6) with the rules giving the probability of the intersection of two (independent) events and the union of two events is obvious.

Let now $f$ be a mapping from U to V ($f : \mathrm{U} \longrightarrow \mathrm{V}$) and $S$ a fuzzy set with the membership function $\mu_S$. Then the image of $S \subseteq \mathrm{U}$ through $f$ is the set with the membership function

$$\mu_{f(S)}(x) = \begin{cases} \sup\limits_{x \in f_{-1}(y)} & \text{if } f_{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise}. \end{cases} \tag{7}$$

If $T \subseteq \mathrm{V}$ is a fuzzy set with the membership function $\mu_T$ then its counterimage through $f$ is the fuzzy subset of U with

$$\mu_{f_{-1}(T)}(y) = \mu_T(f(x)). \tag{8}$$

We use $-1$ as a subscript (and not as a superscript) since $f$ needs not be invertible, in general.

The term of "logical intersection" is used in [10] for the intersection of two fuzzy sets defined by Eq. (5). We illustrate, in Fig. 2, the union and the intersection of two fuzzy sets.

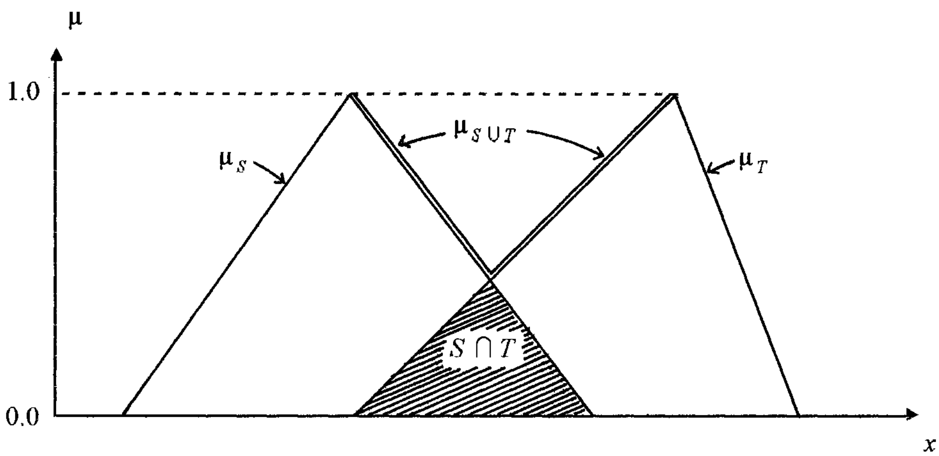


**Fig. 2 The union and the intersection of two fuzzy subsets**

This simple example shows that the graph of a fuzzy membership should not be always triangular. In general, it is a polygonal line. This is the case with $\mu_{S \cup T}$ in Fig.2. Moreover, not only the resulting $\mu$ - functions for fuzzy sets obtained through certain operations may have other than triangular shapes but just the initially adopted functions may be, for instance, trapezoidal ; this is the case with the $\mu$ - functions that describe the small and large peak ground accelerations in [12]. We may only state two general conditions on the shape of the graphs of membership functions : they should be univalued polygonal lines included in the rectangle $I \times [0\,,\,1]$ with $I$ = a finite interval included in $\mathrm{U} \subseteq \mathbb{R}$. The height of a specific $\mu$ - function should not be necessarily $= 1$ but only $\leq 1$. There exists a degree of subjectivity in selecting the shape of the membership functions to describe certain fuzzy subsets, and just this requires to take into account the possibility to (suitably) update the prior shapes of these functions after newly acquired observation data on the parameter which is analyzed.

Fuzzy logical (or inference) rules were also formulated in several papers related to seismic risk assessment. We do not intend to give a presentation of a Fuzzy Logic. There exist various ways to introduce fuzzy logical rules. Let us mention the approach of [12]. Starting from the necessity to evaluate the spectral intensity of ground motion in terms of "linguistic" values (from *Small* to *Large*), a general form of a fuzzy inference rule is proposed ; it is expressed in terms of several variables and – in fact – it consists of a set of $m$ rules :

$$\text{IF } (x_1 \text{ is } A_{i,1}) \text{ and } (x_2 \text{ is } A_{i,2}) \text{ and } \dots \text{ and } (x_n \text{ is } A_{i,n}) \text{ THEN } (y \text{ is } B_i),\ i = \overline{1,m}. \tag{9}$$

The variables $x_i$ are observed values of the ground motion parameters like the PGA $a_g$ or spectrum intensity $SI$ while $y$ is a damage measure. After estimating local damage measures, these are assembled into global fuzzy sets by means of weighted sums.

In a more general approach, a fuzzy logical system is built on a set $\mathcal{V} = \{v_1, v_2, \dots, v_n\}$ of logical values, and a truth function is defined on $\mathcal{V}$, $\tau : \mathcal{V} \to [0, 1]$. The set $\mathfrak{I}$ of formulas of the fuzzy logic is built up in a classical way : (*i*) any variable is a formula, (*ii*) if $F$ is a formula then non-$F$ is a formula, (*iii*) if $F, F'$ are in $\mathfrak{I}$ then $F \vee F'$ and $F \cdot F'$ are also in $\mathfrak{I}$, (*iv*) $\mathfrak{I}$ consists of the formulas obtained by (*i*), (*ii*) and (*iii*) only. Here $\vee$ is the logical disjunction, $\cdot$ denotes the logical conjunction and non-$F$ is also denoted by $\overline{F}$. The truth function is extended from logical values to formulas by

$$\begin{cases} F = v_i \Rightarrow \tau(F) = \tau(v_i), \\ \tau(F \cdot F') = \min\{\tau(F), \tau(F')\}, \\ \tau(F \vee F') = \max\{\tau(F), \tau(F')\}, \\ \tau(\overline{F}) = 1 - \tau(F). \end{cases} \tag{10}$$

A logical formula $F$ is said to be valid / inconsistent, if $\tau(F) \geq ½$, respectively if $\tau(F) < ½$. The two complementary properties are similarly defined for logical propositions or sentences.

**Fuzzy Representation of Seismic Damage States**

The evaluation of the damage level for a given structure is often expressed in terms of qualitative (or linguistic) variables. For instance, a five level scale is (no damage or insignificant damage, minor damage, moderate damage, severe damage, collapse). If we conventionally represent these five levels by integers $(0, 1, \dots, 4)$ and if the damage states are evaluated in terms of a damage indicator (or damage measure) $D$ taking values $d \in I$, where $I$ is an interval called the support of $D$, then a fuzzy model can be built by defining a membership function $\mu : I \to [0, 1]$. For each damage $\ell$ $(0 \leq \ell \leq 4)$, it has to be selected a specific interval is selected,

$$I_\ell = [\delta_\ell^{\inf}, \delta_\ell^{\sup}],\ \ell = \overline{0,4} \ \text{ such that } I = \bigcup_{\ell=0}^{4} [\delta_\ell^{\inf}, \delta_\ell^{\sup}]. \tag{11}$$

Two natural conditions of the endpoints of the intervals that occur in the union of Eq. (11) is

$$\delta_\ell^{\inf} \leq \delta_\ell^{\sup},\ \ell = \overline{0,4} \ \ \& \ \ \delta_{\ell+1}^{\inf} \leq \delta_\ell^{\sup},\ \ell = \overline{0,3}. \tag{12}$$

These conditions are implicitly accepted in the most references on fuzzy models for damage evaluation, although not explicitly stated. Let us remark that the first inequality in Eqs. (12) may be an equality, that is the corresponding interval may be reduced to a point, when the respective value is accepted as a crisp number. The second inequality in Eqs. (12) allows the intervals in the union of Eq. (11) to overlap. In other words, some values $d \in I = [a, b]$ may correspond to different (but neighbor) damage levels. The membership function $\mu$ is analytically defined on each interval $I_\ell$ so that its graph is a polygonal line. The most usual shapes are triangular or trapezoidal (as we have earlier mentioned). If the height of each of these triangles or trapeziums is taken $= 1$, then the general analytic expression of $\mu$ over $I_\ell$ can be written as

$$\mu(x) = \begin{cases} 0 & \text{for } x \leq x_\ell^{\inf} \text{ or } x \geq x_\ell^{\sup}, \\ (x - x_\ell^{\inf}) / (\underline{x}_\ell - x_\ell^{\inf}) & \text{for } x \in [x_\ell^{\inf}, \underline{x}_\ell], \\ 1 & \text{for } x \in [\underline{x}_\ell, \overline{x}_\ell], \\ 1 - (x - \overline{x}_\ell) / (x_\ell^{\sup} - \overline{x}) & \text{for } x \in [\overline{x}_\ell, x_\ell^{\sup}]. \end{cases} \tag{13}$$

Regarding this analytical expression of a $\mu$ - function, a couple of remarks are necessary. **1°** We have written it for a generic fuzzy variable $x$ instead of a damage parameter $d$, since a fuzzy model may be accepted for other parameters like the ground motion intensity (magnitude, spectral acceleration, etc.); we let the subscript $\ell$ to take values in {0, 1, . . . , $m$ }, thus allowing for $m+1$ intervals in the a union of the form in Eq.(11). **2°** The underlined / overlined values are the points inside the interval $I_\ell$ where the polygonal (trapezoidal) line changes its slope : from a positive one to 0 {for the value 1) and from the latter to a negative slope. The general inequality between the four values in an interval $I_\ell$ is

$$a \le x_\ell^{\inf} \le \underline{x}_\ell \le \overline{x}_\ell \le x_\ell^{\sup} \le b. \tag{14}$$

**3°** Then, it is easy to see that the second line in the right-hand side of Eq. (13) should be deleted if $\ell = 0$, when $x_0^{\inf} = \underline{x}_0 = a$ ; similarly, the fourth line will not appear if $\ell = m$, when $\overline{x}_m = x_m^{\sup} = b$. We present, in Fig.3 that follows, a fuzzy model with five fuzzy sets that cover the interval [0 , 1]. Two of them are trapezoidal, two of them are triangular and we also considered a crisp number.

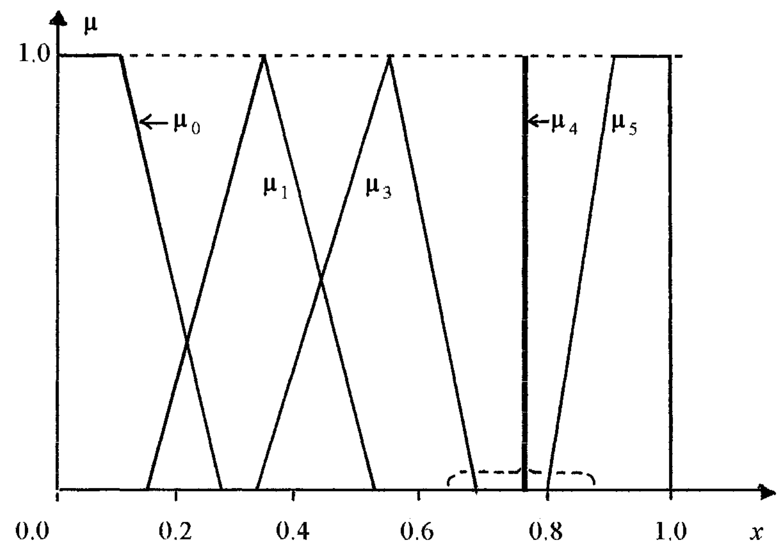


**Fig. 3 Five fuzzy subsets of various shapes over [0,1]**

A fuzzy membership function can be considered as an approximation to a probability distribution function. For instance, the *pdf* of a normal random variate $X \in \mathrm{N}(m, \sigma)$ can be approximated by a triangular $\mu$ - function if its standard deviation $\sigma$ is small while a trapezoidal shape would result in a better approximation for a larger $\sigma$. In both cases, the graph of $\mu$ should be symmetric with respect to the vertical line of equation $x = m =$ the expectation of $X$.

As we have earlier mentioned, fuzzy models may be employed both for the representation of the structural damage level and for the ground motion parameters. For instance, the natural period $T$ (in sec) of deep soft soil is modeled by a fuzzy number in [11]. In general, if $X$ denotes an input (seismic motion parameter) and $Y$ is a response or damage parameter then four possible approaches are, in principle, possible :

$$(i): [\mathrm{P} \mid \mathrm{P}]; \quad (ii): [\mathrm{P} \mid \mathrm{F}]; \quad (iii): [\mathrm{F} \mid \mathrm{P}]; \quad (iv): [\mathrm{F} \mid \mathrm{F}]. \tag{15}$$

We have use the conditioning line for suggesting that the damage level reached by a system is conditional on the intensity of the ground motion. The "classical" seismic damage fragility / seismic vulnerability models are purely probabilistic, meaning that they evaluate the probabilities of events of the form $(Y \ge y \mid X = x)$. An approach of type ($iv$) could be said to be purely fuzzy. Such a double fuzzy model for the seismic damage prediction is proposed in [14].

The approximation of pdf $f$ by a fuzzy membership function $\mu$ should follow some restrictions. The interval over which $\mu > 0$ is necessarily finite while a probability distribution like the Gaussian is $> 0$ over the whole real axis. As regards the characteristic property of a *pdf* that the area under its graph is $= 1$ cannot be satisfied by a $\mu$ - function. However, a condition of approximately equal areas may be stated for a scaled *pdf* to the [0 , 1] interval, that is for $f / M$ where $M$ is the (absolute) maximum value of $f$. Such conditions may be stated as follows :

$$f(x)/M \approx \mu(x) \ne 0 \text{ for } x \in I = [q_{0.05}, q_{0.95}], \quad \frac{1}{M}\int_{\mathbb{R}} f \approx \int_I \mu \tag{16}$$

where $q_{0.05}$ & $q_{0.95}$ are the corresponding percentiles of the distribution with the *pdf* $f$. Certainly, the particular shape of $\mu$ will be chosen so that the corresponding polygonal line be as close as possible to the graph of $f$.

The fuzzy representation of the seismically induced damage state of a structure or – more relevant – to a class of similar structures located in a seismologically homogeneous area has to follow the main steps presented below.

Building a [ F | F ] model for seismic damage assessment

① Selection of a vector $\mathbf{x} = (x_1, x_2, \ldots, x_k)$ of structural, ground motion and soil type parameters which are considered as relevant to the damage state assessment.

② On the basis of seismic hazard analysis, a fuzzy distribution is established for each component $x_j$ of $\mathbf{x}$ in terms of a specific membership function $\mu_j$, $1 \le j \le k$.

③ Selection of a damage state indicator $d$ and its modeling by a fuzzy distribution $\boldsymbol{\mu}_D$ over a scale of $m$ damage severity levels.

④ Statement of a set **R** of fuzzy logic rules of the form in Eqs. (9) to assess the damage level for possible fuzzy (or crisp) values of input parameters.

⑤ Final assessment of most likely damage states induced by possible earthquakes of specific intensity classes.

A couple of remarks are necessary regarding these steps. 1) Some of the parameters in **x** may be crisp or statistically described by a mean (expected) value and a standard deviation. For example, the yield strength of typical RC members has been extensively studied and probabilistic evaluations are available. If a purely fuzzy approach is adopted then the respective probabilistic distributions can be rather easily modeled by fuzzy membership functions (as we have suggested - see Eq. (16)). 2) The fuzzy distribution of the damage severity depends on the number $m+1$ of damage levels adopted. The five level scale is most often used. 3) The selection of the fuzzy logic rules in **R** is the key step of a fuzzy approach to seismic damage assessment. In fact, this would correspond to the conditional distributions used in seismic fragility and vulnerability models. It is not possible to propose a general pattern for selecting these logical rules. Basically, they should be derived from the mechanical model adopted for the structural response to seismic actions. In many studies, hysteretic models are used for structural damage evaluation. Formulas as those in Eqs. (7) and (8) can be used for deriving analytical expressions for the fuzzy distribution of the output (damage) parameter $d$.

## UPDATING FUZZY MODELS FOR SEISMIC RISK ASSESSMENT

### Updating Fuzzy Membership Functions and Cumulative Functions

Let us assume that a [ F | F ] model has been built, according to the guideline just presented. The behavior of a structure in a certain class / of a certain type under site specific predicted earthquake motions is described in terms of what can be called a fuzzy conditional distribution of the form

$$[\boldsymbol{\mu}_D(d) \mid \boldsymbol{\mu}_E(\mathbf{x}_G)], \tag{17}$$

where $\boldsymbol{\mu}_D$ denotes the (prior) vector of membership functions defined over the $m+1$ intervals over the damage range, $\boldsymbol{\mu}_E$ is the fuzzy distribution that models the earthquake characteristics, and its arguments in $\mathbf{x}_G$ are the relevant parameters of the ground motion, including those related to the soil characteristics in the site zone ; for instance, two components of $\mathbf{x}_G$ may be the PGA and the local attenuation factor. The conditioning is modeled in terms of the fuzzy logic rules **R** (see step ④ of above). The model in Eq. (17) is build on the basis of statistical evidence consisting of damage data on specific types of structures from past / simulated earthquakes. The updating of this prior fuzzy model may become necessary after new data become available, that is on the basis of a richer statistical evidence. For instance, a component $\mu$ - function $\mu_\ell$ $(1 \le \ell \le m)$, initially triangular, may need to be turned to a trapezoidal shape if the newly acquired statistical data are more scattered. We could say – in such a case – that the fuzzy variance (or standard variation) around the central point $c_\ell$ should be enlarged. In the other case, if the prior function $\mu_\ell$ was trapezium-shaped but the updated data are more concentrated around $c_\ell$ then the shape of $\mu_\ell$ may be changed for a triangular one or (at least) the length of the interval over which $\mu_\ell$ was maximum may be reduced. If the "gravity center" of the cloud of newly acquired data points in the $\mathbf{x}_G - d$ relationship is significantly remote from $c_\ell$ then just this central point has to be shifted and the whole (graph of) function $\mu_\ell$ has to be shifted to left / right. A solution to the problem of updating the prior fuzzy distributions is proposed in [5]. It is based on an approach by means of a reliability function whose argument is a fuzzy number. We modify it according to the [ F | F ] model of Eq. (17):

$$R = \zeta(r) = \sup_j \{ \mu_{\ell | j}(d) : \Psi(d \mid \mathbf{x}_j \in E_j) = r \}. \tag{18}$$

In this Eq. (18), $\Psi$ is a conditional *cdf* (cumulative distribution function) assembled from the cumulative fuzzy functions

$F_{\mu;\ell|j}$ defined by Eq. (19) that follows, and $E_j$ is the range of parameter vector $\mathbf{x}_G$ for a ground motion of class $j$;

$$F_{\mu;\ell,j}(d) = \int_{\delta_\ell^{\inf}}^{d} \mu_{\ell|j}(y)\,dy. \tag{19}$$

The *cdf* $\Psi$ considered in [5] is the standard Gaussian *cdf* $\Phi$; although, this assumption cannot be accepted in a case when $\mu_{\ell|j}$ is strongly asymmetric, for instance. Since the function $\zeta$ in Eq. (18) is monotonically increasing it can be inverted. If this function is $d = \eta(r)$ and a normal-type distribution $\mathrm{N}(d_{j,0}, \Delta)$ is assumed for $R$ with $d_{j,0}$ = the mean point of the newly acquired data and $\Delta$ is a measure of their fuzziness, then the updated $\mu$ function for ground motions of class $j$ can be obtained

$$\mu_j(d) = \exp\left[-\frac{1}{2}\left(\frac{d - d_{j,0}}{\Delta}\right)^2\right]. \tag{20}$$

A weighted sum of $\mu$ - functions of the form in Eq. (20), over the possible earthquake categories j, will result in an updated fuzzy model for the seismic damage of the considered class of structures.

In the next examples we present two cumulative fuzzy functions corresponding to a triangular and a trapezoidal fuzzy sets, and their updated shapes that follow from additional statistical evidence.

**Example 1**

Let $S$ be a triangular fuzzy set described by $s = 0\,|\,0.2 + 1\,|\,0.4 + 0\,|\,0.6$, and $T$ a trapezoidal fuzzy set with its fuzzy number description $t = 0\,|\,0.2 + 1\,|\,[0.4, 0.6] + 0\,|\,0.8$. The corresponding $\mu$ - functions $\mu_S$, $\mu_T$ and the cumulative fuzzy functions are plotted in Fig. 4 - a) and b), respectively.

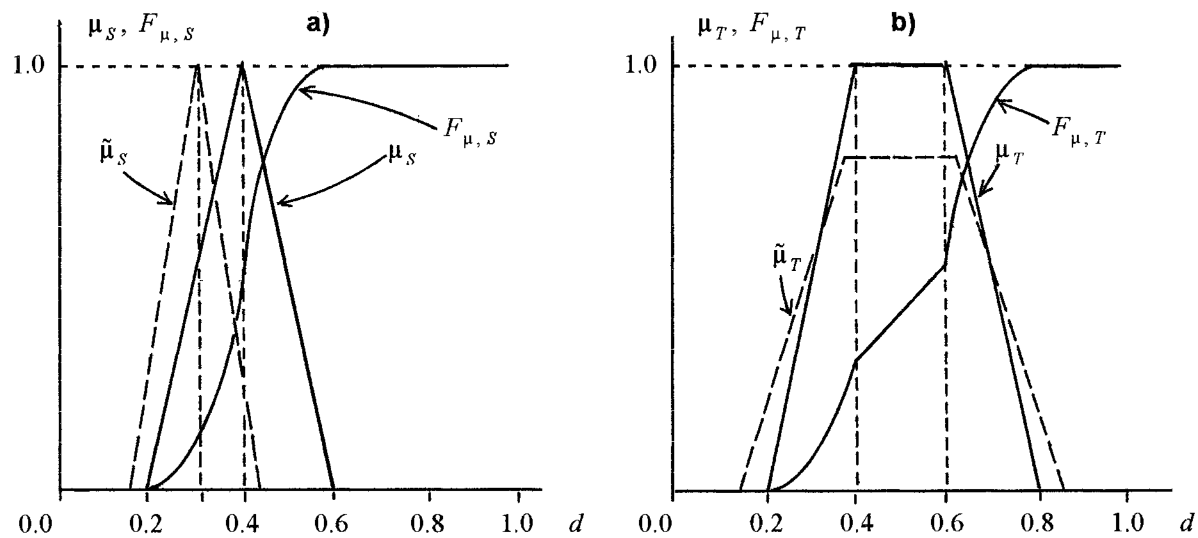


**Fig. 4 Two fuzzy μ - functions and the corresponding cumulative functions**

The cumulative fuzzy functions are obtained by integrals of the form in Eq. (19), but divided by the area $A$ (which is always $< 1$) under the graph of $\mu$ for normalizing them. Clearly, they are quadratic (plotted by arcs of parabolas) over the intervals where the $\mu$ - functions are linear, and $F_{\mu,T}$ is linearly growing over the interval where $\mu_T = 1$. Similar cumulative fuzzy functions as depending on a synthetic ground motion parameter $x_G$ could be defined and considered as fuzzy versions of seismic fragility / vulnerability models.

**Example 2**

We consider the case when additional statistical evidence becomes available for both sets $S$ and $T$. The average for the data on $S$ is $= 0.3$, and they are less scattered for about 25%. As regards $T$, the group of most probable data are more scattered for about 30%.

Consequently, the two fuzzy models should be respectively updated. We denote the updated $\mu$ - functions by $\tilde{\mu}_S$ and

$\tilde{\mu}_T$ (respectively) and we present them in terms of the corresponding fuzzy numbers :

$$\tilde{s} = 0\,|\,0.18 + 1\,|\,0.30 + 0\,|\,0.42\,,\quad \tilde{t} = 0\,|\,0.16 + 0.85\,|\,[0.38, 0.62] + 0\,|\,0.84\,. \tag{21}$$

The graphs of the updated $\mu$ - functions are plotted on the same Fig. 4 by dashed lines. The top line segment of $\mu_T$ has been slightly prolonged around the same central value 0.5 while its height has been correspondingly reduced in order to keep the area under the trapezium nearly unchanged.

## CONCLUDING REMARKS

Several existing fuzzy approaches to the seismic damage risk assessment have been presented, and some of them restated in order to make possible further developments of models of this type by combining different yet related methods. Four types of models were considered, from those based on probabilistic distributions only up to the purely fuzzy models. It would be interesting to develop mixed approaches, that is [ P | F | and [ F | P ].The problem of updating fuzzy models for seismic damage assessment has been discussed in a rather theoretical way. Example 2 shows a possible way to update fuzzy membership functions according to the conditions in Eq. (16). Obviously, further research is necessary. A direction to be followed could consist in using interval probabilities as being closer to the specific of fuzzy models.

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