BURIED PIPES UNDER EARTHQUAKE EXCITATION

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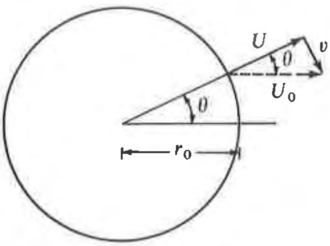
Introduction

The integrity of buried piping systems after a postulated earthquake attack has received greater attention recently. — This paper considers exclusively the "Near Field"—where soil-structure interaction is predominant. — It is assumed that by knowing the displacement field of the surrounding soil the deformation of the pipe can be found directly.

Fundamental Relationships

It is assumed that a circular footing of radius "r_0" is completely embedded in a homogeneous soil medium with material properties λ and μ, where λ is Lame's parameter and μ is the shear modulus of the soil. With the notation shown in the sketch below, the equations of motion in circular coordinates are:



$$(\lambda+\mu)\left(\frac{\partial\Delta}{\partial x}, \frac{\partial\Delta}{\partial y}, \frac{\partial\Delta}{\partial z}\right) + (\mu\nabla^2+\rho\omega^2)(u, v, w) = \varnothing$$

$$\Delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}; \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$h = \frac{\omega}{a}; \qquad a = \sqrt{\frac{\lambda+2\mu}{\rho}} \qquad k = \frac{\omega}{b}; \qquad b = \sqrt{\frac{\mu}{\rho}}.$$

In this summary only the case of horizontal vibration is considered, where according to Baranov the boundary conditions $w(r, \theta, t) = \varnothing$ simplify the equations substantially. Introducing the two potential functions $\varnothing$ and Ψ and substituting them into the equations of motion, one obtains after some reasoning the following expressions.

Around the circumference of the footing at distance "r" from the center we must have, since $U = U_0 \cos\theta$

$$U = A\cos\theta H_1^2(hr); \qquad v = D\sin\theta H_1^2(kr).$$

Elimination of A and D and rearranging for "U" and "v" and keeping in mind that $U = \bar{U}^* \exp(iwt)$ one gets

$$\bar{U} = U_0 \frac{H_2^2(hr)\,H_2^2(kr_0) - \frac{1}{hr}H_1^2(hr)\,H_2^2(kr_0) - \frac{1}{kr}H_2^2(hr_0)\,H_1^2(kr)}{H_2^2(hr_0)\,H_2^2(kr_0) - \frac{1}{hr_0}H_1^2(hr_0)\,H_2^2(kr_0) - \frac{1}{kr_0}H_2^2(hr_0)\,H_1^2(kr_0)}$$

$$\bar{v} = -U_0 \frac{H_2^2(hr_0)\,H_2^2(kr) - \frac{1}{hr}H_1^2(hr)\,H_2^2(kr_0) - \frac{1}{kr}H_1^2(kr)\,H_2^2(hr_0)}{H_2^2(hr_0)\,H_2^2(kr_0) - \frac{1}{hr_0}H_1^2(hr_0)\,H_2^2(kr_0) - \frac{1}{kr_0}H_1^2(kr_0)\,H_2^2(hr_0)}$$

These expressions are of course complex, where Hn^2 are the Hankel functions of the second kind of order n. Separating real and imaginary parts is achieved by expressing the Hankel functions in terms of Bessel functions and collecting the corresponding terms.

Conclusion

The above expressions give the displacement distribution around a circular disk with radius "r_0" which is assumed to vibrate horizontally with amplitude "U_0". — More complex cases can be treated by superposition methods.