SOIL-STRUCTURE INTERACTION PARAMETERS FOR A NUCLEAR CONTAINMENT VESSEL

J. R. HALL, Jr.

E. D'Appolonia Consulting Engineers, Inc., Pittsburgh, Pennsylvania 15235, U.S.A.

A. MORRONE

Westinghouse Electric Corporation, Advanced Reactors Division, Madison, Pennsylvania 1566 3, U.S.A.

SUMMARY

Considerable effort is spent on the analysis of a nuclear containment vessel to account for soil-structure interaction. In instances where the soil beneath foundation is relatively uniform, soil-structure interaction may be accounted for using theoretical solutions for a rigid foundation on an elastic half-space. These solutions indicate that the wave propogation and inertia effects of the soil have a pronounced that influence on damping of the soil-structure system.

In the present study, elastic half-space solutions for horizontal and rocking motions of a foundation in the form of frequency dependent displacement functions were utilized to study the response of a multiple degree of freedom lumped mass model of a typical containment vessel to ground motion input. The non-proportionally damped system response was solved assuming a unit harmonic ground motion input. Shear wave velocities were varied to represent soils ranging from that of a soft clay to a hard rock. Structural damping was assigned a value of two percent for all cases. The results of the analysis were used to derive an equivalent proportionally damped model that could be analyzed using more conventional techniques.

The results and conclusions derived from the study are as follows:

1. The damping contributed by horizontal soil-structure interaction is independent of the shear wave velocity of the supporting soil.
2. The percent damping contributed by rocking soil-structure interaction decreases as the shear wave velocity of the supporting soil increases.
3. The predominant response of the structure occurs at the first mode. For shear wave velocities less than 2,000 feet per second, the second mode is very highly damped.
4. The use of a proportionally damped model adjusted to account for soil-structure interaction will provide sufficient accuracy for most purposes. However, an overestimate of the response of the foundation occurs since, in the real system, a large amount of damping is provided at the foundation soil interface.

1.0 INTRODUCTION

In the analysis of soil-structure interaction for nuclear containment vessels where the soil beneath the foundation is relatively uniform, theoretical solutions may be used that assume the soil behaves as an elastic half-space. The purpose of this study is to evaluate the damping contributed by the soil due to the propogation of wave energy away from the foundation and to compute an equivalent proportionally damped system to give the same response.

2.0 SOIL-STRUCTURE INTERACTION

Theoretical solutions by Richart, Hall and Woods[1], for the response of a foundation supported on an elastic half-space indicate that damping at the foundation-soil interaction is produced by the propogation of wave energy away from the foundation. This form of damping is commonly referred to as radiation damping or geometric damping. It is also recognized that structural components have relatively low damping (less than ten percent for most cases) and that most computer programs utilized for dynamic analysis cannot specifically include different percentages of damping for the structure and for the foundation-soil interaction. In many cases, however, a sufficiently accurate model may be constructed by using an equivalent proportionally damped system.

In this study the response of a typical reinforced concrete nuclear containment vessel for a pressurized water reactor was analyzed on the basis of a unit free field horizontal harmonic ground motion. The foundation-soil interaction was computed on the basis of Bycroft's[2] elastic half-space solution for horizontal and rocking motions of a rigid circular foundation on an elastic half-space. Soil properties used in the analysis were varied from a soft clay to a hard rock (shear wave velocities from 500 feet per second to 6,000 feet per second) and structural damping was assigned a value of two percent in all cases. The frequency dependent stiffness and damping coefficients for the soil were computed on the basis of a series solution developed by Hall in 1967[3]. The results of the analysis were then used to derive an equivalent proportional damping for a mass spring model using the criteria that both models would have similar response characteristics.

The details of the analysis and the results are presented in the following sections of this paper.

3.0 STRUCTURAL MODEL

The structural model is shown schematically on Fig. 1. The foundation is modeled as a rigid mat having both a translational inertia as well as a rotational inertia. All of the other node points of the structure are represented as point masses. The stiffness elements connecting each node point were modeled as shear beam elements so that both bending and shear deflections were included. The properties of the masses and elements shown on Fig. 1 are given in Table I.

4.0 SOIL PROPERTIES

The soil properties required in the dynamic analysis of the foundation are the shear wave velocity, Poisson's ratio and mass density. It is recognized that the determination of the shear wave velocity and Poisson's ratio for a particular soil requires an understanding of the effects of confining stress, density, type of soil and average strain level associated with the earthquake considered in the analysis. However, in this study it is assumed that these parameters have properly been taken into account and the results are presented in terms of a parametric study considered typical of soil characteristics found at nuclear power plant sites. The soil properties used for the analysis of each model are tabulated in Table II.

5.0 ELASTIC HALF-SPACE SOLUTION

Bycroft's[2] solution was converted by Hall[3] into expressions for the equivalent stiffness and damping coefficients of the soil as a function of frequency. For horizontal motion the expressions are:

$$k_x = k_{1x} k_{xs} \tag{1}$$

$$c_x = c_{1x} k_{xs} r_o \sqrt{\frac{\rho}{G}} \tag{2}$$

where:

k_x = Frequency dependent horizontal spring constant,

c_x = Frequency dependent horizontal dashpot coefficient,

k_{xs} = Static horizontal spring constant,

ρ = Mass density,

G = Shear modulus,

r_o = Radius of the foundation, and

c_{1x}, k_{1x} = Frequency dependent coefficients.

$$k_{xs} = \frac{32\ (1 - \nu)\ Gr_o}{7 - 8\nu} \tag{3}$$

where: ν = Poisson's ratio.

$$k_{1x} = 1 + (-0.0464a_o - 0.00439)\ a_o \tag{4}$$

and

$$c_{1x} = 0.5709 + (0.0224a_o - 0.00276)\ a_o \tag{5}$$

where:

$$a_o = \frac{\omega r_o}{V_s} = \text{Dimensionless frequency coefficient.} \tag{6}$$

For rocking motion the analagous equations are as follows.

$$k_\theta = k_{1\theta} k_{\theta s} \tag{7}$$

$$c_\theta = c_{1\theta} \frac{k_{\theta s} r_o}{V_s} \tag{8}$$

$$k_{\theta s} = \frac{8\ Gr_o^{\ 3}}{3\ (1-\nu)} \tag{9}$$

$$k_{1\theta} = 1 + ((0.0735a_o - 0.1849)a_o - 0.0949)\ a_o \tag{10}$$

$$c_{1\theta} = -0.00131 + ((-0.0915a_o + 0.2089)\ a_o + 0.0483)\ a_o \tag{11}$$

The coefficients k_{1x}, c_{1x}, $k_{1\theta}$, and $c_{1\theta}$ are plotted on Figs. 2 and 3. Figure 2 shows that the stiffness coefficients decrease with increasing frequency for both the horizontal and rocking modes of vibration. Figure 3 shows that the damping associated with the horizontal mode of vibration is almost independent of frequency whereas the damping associated with rocking starts at zero and increases with increasing frequency.

It is important to note that the mass density of the half-space appears in the terms of the equation of motion as is sometimes assumed on an intuitive basis. Consequently, the concept of an in-phase mass has no theoretical justification and the mass term includes only the mass of the foundation. Hence, the elastic medium may be completely defined by frequency dependent stiffness and damping coefficients. Although the present analysis is for a uniform elastic half-space, the same concept may be applied to a non-homogeneous or layered medium.

6.0 MATRIX FORMULATIONS

The equation of motion for the structural model shown on Fig. 1 subjected to a ground motion input may be represented by the following:

$$\left[M_\searrow\right]\left\{\ddot{X}\right\}_R + \left[C\right]\left\{\dot{X}\right\}_R + \left[K\right]\left\{X\right\}_R = \left[M_\searrow\right]\left\{\ddot{X}\right\}_G \qquad (12)$$

where $\left[M_\searrow\right]$, $\left[C\right]$, and $\left[K\right]$ are the mass, damping and stiffness matrices respectively.

and $\left\{X\right\}_R$ = Displacement vector relative to the ground, and

$\left\{X\right\}_G$ = Displacement vector of the ground.

The solution to the above equation for steady-state ground motion may be computed by assuming displacements of the form:

$$X = (X_1 + iX_2)\, e^{i\omega t}$$

where X_1 = Real component of displacement amplitude, and

X_2 = Imaginary component of displacement amplitude.

The absolute motion of the structure is computed by adding the relative displacement to the ground displacement.

The dynamic analysis described above included two models. The first model utilized the elastic half-space solutions for foundation-soil interaction as defined by Eq's. (1), (2), (7), and (8). The structural damping was two percent. This results in a non-proportionally damped system, the response of which was solved for each frequency of input motion using an in-house EDCE computer program. The second model was a proportionally damped model having the same mass and stiffness matrices as the first model but with a damping matrix computed from a linear combination of the mass and stiffness matrices. This damping matrix was scaled until the response of the model was equal to that of the elastic half-space model at the top of the containment vessel.

7.0 RESULTS OF THE ANALYSIS

The response between 0 and 20 cycles per second was computed for five different soil conditions with shear wave velocities ranging from 500 feet per second to 6,000 feet per second. A response curve for the case with a shear wave velocity of 2,000 feet per second is presented in Fig. 4. The figure consists of three curves. The dotted line represents the response at the top mass of the containment structure, the solid line represents the rocking motion of the foundation multiplied by the distance to the top mass (provides a means of assessing the contribution of top mass response represented by shear and bending deflections of the structure) and the dashed line represents the horizontal response of the foundation. The response curves indicate that the first mode is predominant over the range of 0 to 20 cycles per second and that the second mode at 6.3 cycles per second is very highly damped. In fact, the frequency of the second mode could only be determined by considering the phase relationship of the various nodes of the structure. Other curves similar to that shown on Fig. 4 were generated for each value of shear wave velocity. The results indicated that as the ground becomes more rigid, the first mode natural frequency increases from about 1 cycle per second to a little over 3 cycles per second. For a shear wave velocity of 6,000 feet per second the structure behaves as if the foundation were fixed.

The results from the elastic half-space model were used as a basis for computing an equivalent proportional damping so that the response at the top of the containment structure for a unit harmonic ground motion input would be equal. The results of the analysis for both models are given in Tables III and IV and Figs. 5 and 6. The percentage of critical damping given in Tables III and IV is based on using the total mass and total mass moment of inertia of the structure in the computations for the critical damping coefficients. The results indicate that the percent damping for horizontal motion is independent of shear wave velocity whereas the percent damping for rocking motion decreases with increasing shear wave velocity and is less than two percent at a shear wave velocity of 6,000 feet per second. The percentages of damping for the equivalent proportionally damped model are given in the right hand column of Tables III and IV. The results show that as the shear wave velocity increases, the equivalent proportional damping decreases and in the limit approaches the damping of the structure used in the elastic half-space model. It is also noted that the equivalent proportional damping of the second mode is two to three times the equivalent proportional damping of the first mode with the higher ratios being associated with lower shear wave velocities.

8.0 CONCLUSIONS

The conclusions herein are derived from the model illustrated in Fig. 1 but are considered applicable to other containment structures having similar properties.

1. The percent damping contributed by horizontal soil-structure interaction is approximately forty percent for the first and second modes and is independent of the shear wave velocity.
2. For rocking motions the damping ranges from approximately ten percent at a shear wave velocity of 500 feet per second to approximately two percent at a shear wave velocity of 6,000 feet per second.

3. The predominant response of the structure occurs at the first mode. For shear wave velocities less than 2,000 feet per second, the second mode is very highly damped. For all velocities the second mode damping is two to three times the damping for the first mode.
4. The use of a proportionally damped model adjusted to account for soil-structure interaction will provide sufficient accuracy for most purposes. However, an over estimate of the response at the foundation occurs since in the real system a large amount of damping exists at the foundation-soil interface.

REFERENCES

[1] Richart, F. E., Jr., Hall, J. R., Jr., and Woods, R. D., Vibrations of Soil and Foundations, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1970.

[2] Bycroft, G. N. (1956), "Forced Vibrations of a Rigid Circular Plate on a Semi-infinite Elastic Space and on an Elastic Stratum," Philosophical Trans., Royal Society, London, Ser. A, Vol. 248, pp. 327-368.

[3] Hall, J. R., Jr., (1967), "Coupled Rocking and Sliding Oscillations of Rigid Circular Footings," Proc. International Symposium on Wave Propogation and Dynamic Properties of Earth Materials, Albuquerque, New Mexico, August.

TABLE I

MODEL PROPERTIES

Weight At Each Node In Kips

Node	Weight
1	6.20E+04
2	8.71E+03
3	1.49E+04
4	5.04E+03
5	8.67E+03
6	1.23E+04
7	8.25E+03
8	4.23E+03
9	4.22E+03
10	1.89E+03

Weight of Foundation Times Radius of Gyration Squared

$9.86E+07\ KIP*FT^2$

Radius of Foundation = 75 Ft

Properties of Elements Connecting Masses

MASSES		E(KSI)	G(KSI)	$I(IN^4)$	$A(IN^2)$	L(FT)	K
1	4	5.00E+03	2.10E+03	8.45E+10	1.13E+05	20.7	1.00
4	5	5.00E+03	2.10E+03	8.45E+10	1.13E+05	20.7	1.00
5	6	5.00E+03	2.10E+03	8.45E+10	1.13E+05	50.5	1.00
6	7	5.00E+03	2.10E+03	8.45E+10	1.13E+05	70.5	1.00
7	8	5.00E+03	2.10E+03	5.72E+10	8.07E+04	24.2	1.00
8	9	5.00E+03	2.10E+03	4.37E+10	8.07E+04	24.2	1.00
9	10	5.00E+03	2.10E+03	1.65E+10	8.07E+04	24.2	1.00
1	2	5.00E+03	2.10E+03	1.34E+09	7.45E+04	25.4	1.00
2	3	5.00E+03	2.10E+03	5.33E+08	1.12E+05	26.0	1.00
1	3	5.00E+03	2.10E+03	1.55E+09	1.08E+05	51.4	1.00

TABLE II

SOIL PROPERTIES USED FOR ANALYSIS

V_s ft/sec	G ksi	ν -	γ lb/ft^3
500	5.39	0.5	100
1000	26.9	0.45	125
2000	121.	0.35	140
4000	551.	0.3	160
6000	1320.	0.3	170

TABLE III

1st Mode Soil-Structure Interaction Parameters

ELASTIC HALF-SPACE MODEL

		SPRING CONSTANTS		DAMPING CONSTANTS				EQUIVALENT PROPORTIONAL DAMPING (COUPLED)
V_s FT/SEC	f_n CPS	HORIZONTAL K/FT	ROCKING K.FT/RAD	HORIZONTAL K.SEC/FT	%	ROCKING K.FT.SEC/RAD	%	
500	0.97	2.97×10^5	1.42×10^9	2.73×10^4	39	3.87×10^7	10	13
1000	1.86	1.44×10^6	6.53×10^9	6.61×10^4	43	8.33×10^7	10	9.7
2000	2.70	6.33×10^6	2.67×10^{10}	1.40×10^5	44	1.02×10^8	6.3	3.4
4000	3.09	2.88×10^7	1.20×10^{11}	3.12×10^5	46	9.43×10^7	2.8	2.1
6000	3.16	6.91×10^7	2.95×10^{11}	4.96×10^5	47	8.50×10^7	1.6	2.0

TABLE IV

2nd Mode Soil-Structure Interaction Parameters

ELASTIC HALF-SPACE MODEL

		SPRING CONSTANTS		DAMPING CONSTANTS				EQUIVALENT PROPORTIONAL DAMPING (COUPLED)
V_s FT/SEC	f_n CPS	HORIZONTAL K/FT	ROCKING K.FT/RAD	HORIZONTAL K.SEC/FT	%	ROCKING K.FT.SEC/RAD	%	
500	1.91	2.76×10^5	1.20×10^9	2.87×10^4	43	6.09×10^7	18	30
1000	3.8	1.34×10^6	5.46×10^9	6.97×10^4	47	1.38×10^8	19	26
2000	6.3	5.77×10^6	2.09×10^{10}	1.49×10^5	49	2.61×10^8	18	20
4000	8.5	2.75×10^7	1.01×10^{11}	3.21×10^5	48	3.95×10^8	12	8
6000	8.9	6.76×10^7	2.65×10^{11}	5.03×10^5	48	3.94×10^8	7.8	4

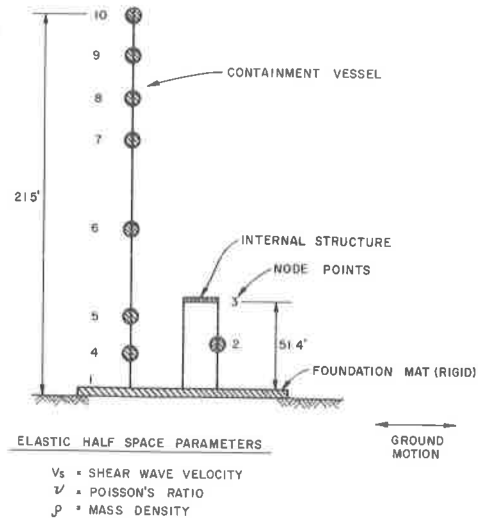

Figure 1 Containment Vessel Model

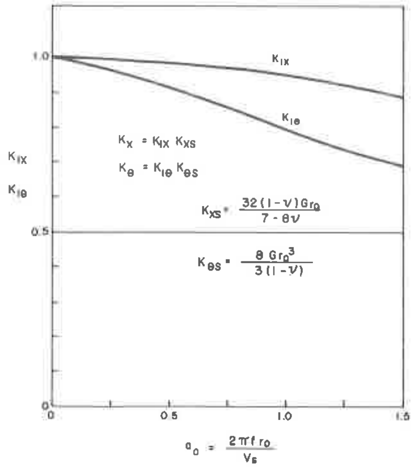

Figure 2 Variation of Half-Space Springs with Frequency

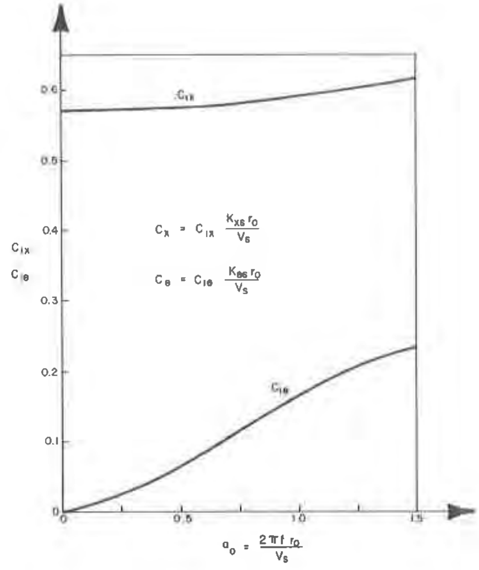

Figure 3   Variation of Half-Space Damping with Frequency

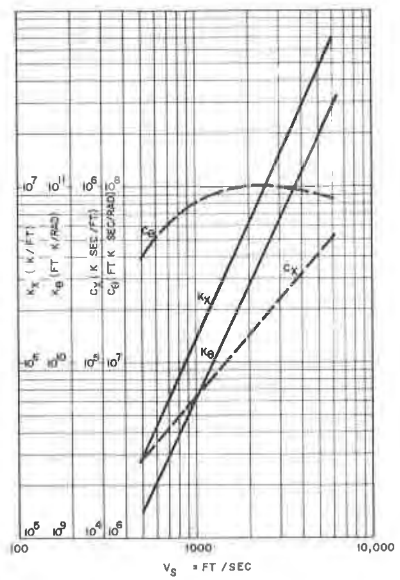

Figure 4   Containment Vessel and Foundation Response for V_s = 2000 Ft/Sec

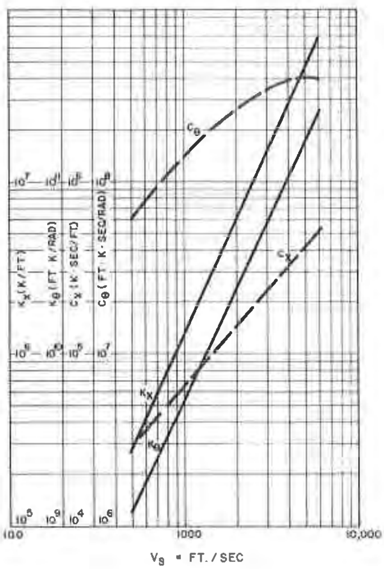

Figure 5   First Mode: Soil-Structure Interaction Parameters

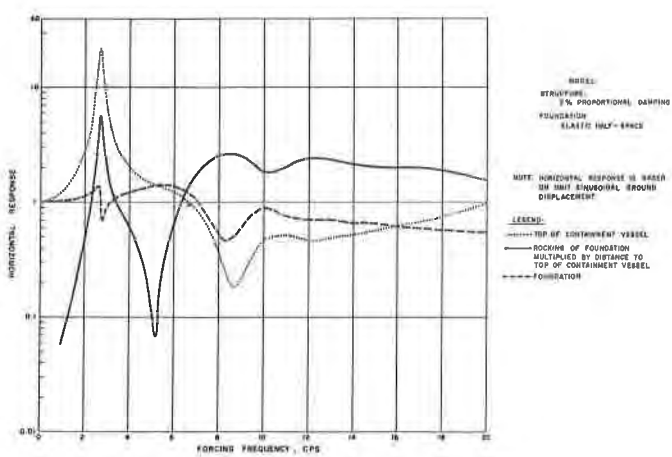

Figure 6   Second Mode: Soil-Structure Interaction Parameters