Impulsive Loading on Concrete Structures

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1. SUMMARY AND INTRODUCTION

Impulsive loadings and thereby response in the elastic regime can be analysed by means of the modal approach, but direct time integration is sometimes to be preferred since the local character of the load exites higher frequency modes. For non-linear response, - both material and geometrical non-linearities -, the modal approach is not of interest anymore since the eigenvalue problem has to be updated frequently. For such problems, the direct time integration method - implicit or explicit -, becomes a must. The implicit time integration needs at nearly every time step, - in particular for geometrical non-linearities -, the assembling and inversion of a matrix, and in addition, for material non-linearities, a certain number of iterations. For that reason, the explicit time integration method for impulsive loads became in vogue, in particular after the convincing work of Belytchko (see e.g. /1/), using the concept of convective coordinates to account for large rotations and, on purpose, simple elements. These techniques of explicit time integration and convective coordinates have been employed in previous work of one of the present authors /2,3,4/, for the more complex element SEMILOOF developed by Irons (see e.g. /5/), and comparisons with the results obtained by means of the simple elements have shown that the explicit time integration scheme can be applied for higher order elements competitively. Attention has to be given to the problem of diagonalizing (lumping) of the mass matrix.

The advantages of the explicit time integration scheme are manifold:

- ease of programming: no assemblage of matrices, less computer memory, no inversion of matrices, no problems of bandwidth and node and/or element numbering;
- straightforward implementation of all kind of constitutive equations;
- straightforward implementation of geometrical non-linearities due to large rotations, using convective coordinates.

The drawback of the explicit scheme is the limitation of the time step dictated by the travel time of acoustic waves through the smallest dimension of any element and for thin structures the shell formulation is to be preferred rather than a degenerated solid-element formulation. Anyhow, also in the implicit scheme, the non-linearities limit the time step, not from the point of view of stability but rather from the point of view of accuracy, or

alternatively iterations have to be applied.

In this paper, the more recent developments of the computer program SLOOFDYN are addressed, in particular the application of the element SEMILOOF for reinforced concrete, including dead weight, prestressing and cracking. The static stress distribution due to dead weight and prestressing are carried out using SLOOFDYN in a type-of-dynamic-relaxation-mode, and more recently by a simple numerical trick of zeroing the velocity of a node when the acceleration changes sign.

Applications are presented in the field of anti-missile design, simulating the hypothetical crash of an airplane onto the outer containment of a nuclear reactor.

2. EQUATIONS OF MOTION

An efficient way of formulating the equilibrium is the theorem of virtual work:

$$- \int \{\sigma\}^T \{\delta\epsilon\} dVol + \int \{\rho\ddot{u}\}^T \{\delta u\} dVol = \int \{f\}^T \{\delta u\} dVol \tag{1}$$

σ	stress	u	displacement	T	transposed
ϵ	strain	$\ddot{u}$	acceleration	$\{\ \}$	column vector
f	external load	ρ	density	$\{\ \}^T$	row vector

δu , $\delta\epsilon$ kinematically admissible variations of displacement and strain.

Using the concept of FEM and the notation of Zienkiewicz /6/, the discretized equations of motion result:

$$- [M]\{\ddot{u}\} + [K]\{u\} = \{f\} + \{f^0\} \tag{2}$$

$[M]$ mass matrix

$$[K] = \int [B]^T [D] [B] dVol \quad \text{stiffness matrix} \tag{3}$$

$$\{f^0\} = \int [B]^T [D] \{\epsilon^0\} dVol \quad \text{equivalent load due to initial strains } \epsilon^0 \tag{4}$$

$$\{\epsilon\} = [B]\{u\} \quad \text{total strain-displacement relation} \tag{5}$$

$$\{\sigma\} = [D]\{\epsilon^{el}\} \quad \text{Hooke's law} \tag{6}$$

$$\{\epsilon^{el}\} = \{\epsilon\} - \{\epsilon^0\} \quad \text{elastic strain} \tag{7}$$

For the explicit scheme, the equations are written in an alternative way:

$$- [M]\{\ddot{u}\} = \{f\} - \{f^{int}\} \tag{8}$$

where the internal load is given by:

$$- \{f^{int}\} = -\{f^0\} + [K]\{u\} = \int [B]^T \{\sigma\} dVol \tag{9}$$

The explicit scheme becomes attractive when the mass matrix [M] can be diagonalized (lumped), because then the equations of motion (8) are uncoupled. For simple elements, the

procedure is straightforward, for the SEMILOOF element special procedures have been tested numerically /2/. The decoupled equations (8) can be integrated using the central difference scheme. The combination of the central difference scheme and mass lumping has a compensating effect on the frequency spectrum as shown by the work of Krieg and Key /7/. Equations (2), - the implicit scheme -, allow for a static solution by inverting matrix [K], the assemblage of which is foreseen anyhow in implicit schemes. For the explicit formulation (8), the static solution ($\{f\} = \{f^{int}\}$) is more difficult. In SLOOFDYN two approaches are implemented by simply solving (8) for the static load as a step load at time = 0 and

(i) adding a damping $\{f^{damp}\} = [C]\ \{\dot{u}\}$, where [C] is a matrix proportional to the mass matrix, or

(ii) whenever the acceleration of a node changes sign ($f_i = f_i^{int}$), the velocity of that node is put to zero ($\dot{u}_i = 0$).

3. CONSTITUTIVE EQUATIONS

The explicit scheme facilitates the implementation of non-linear constitutive equations. The time steps and thereby apparent load steps are small and immediately an action can be taken when the stress pattern violates a yielding criterion or another failure criterion. The stress redistribution is accounted for when calculating the internal forces (9), to be used in the equations of motion (8) for the next time step. In SLOOFDYN, two types of non-linear material behaviour are implemented: (i) plasticity; (ii) cracking of concrete.

3.1 Plasticity

The plasticity model of SLOOFDYN is based on:

(i) experimentally observed monoaxial yield stress vs plastic strain;

(ii) the concepts of von Mises' stress, Drucker's normality rule and plastic energy dissipation to compare stress and plastification in a multiaxial state of stress with the experimentally observed monoaxial stress vs plastic strain;

(iii) isotropic hardening; for impulsive type of loadings, reversals into the plastic regime do not occur often.

The monoaxial yield stress vs equivalent plastic strain is represented by

$$\sigma_y = f(e^p), \quad c = \frac{df}{de^p} \tag{10}$$

The yield criterion is given by the comparison of the von Mises' stress (s) with yield stress (σ_y):

$$s - \sigma_y \leq 0 \tag{11}$$

The equivalent plastic strain increment de^p and the component plastic strain increments $\{de^p\}$ are related through Drucker's normality rule, and the equality for plastic energy dissipation:

$$\{ d\epsilon^p \} = \lambda \left\{ \frac{\partial s^2}{\partial \sigma} \right\} = \lambda \{r\}, \quad s d\epsilon^p = \{\sigma\}^T \{ d\epsilon^p \} \qquad (12) \quad (13)$$

where λ is a positive unknown parameter to be defined from elastic constraints and $\{r\}$ is the deviatoric stress.

The parameter λ is defined as follows. The updated displacements, together with updated accumulated plastic strains, give a stress distribution such that the yield criterion (11) is violated. Additional plasticity will occur, which (i) relaxes the stresses (s decreases) and (ii) the yield stress σ_y increases (eventually) due to strain hardening. The relaxation of the stress is governed by Hooke's law:

$$- \qquad \{d\sigma\} = -[D]\{ d\epsilon^p \} = -\lambda [D] \{r\} \qquad (14)$$

The equivalent stress will change by an amount:

$$- \qquad ds = \left\{ \frac{\partial s}{\partial \sigma} \right\}^T \{d\sigma\} = \frac{1}{2s} \left\{ \frac{\partial s^2}{\partial \sigma} \right\}^T \{d\sigma\} = -\frac{\lambda}{2s} \{r\}^T [D] \{r\} \qquad (15)$$

The variation of the yield stress is evaluated as:

$$- \qquad d\sigma_y = c d\epsilon^p = \frac{c}{s} \{\sigma\}^T \{ d\epsilon^p \} = \frac{c\lambda}{s} \{\sigma\}^T \{r\} = 2\lambda cs \qquad (16)$$

At the end of the relaxation, the yield criterion should be satisfied:

$$- \qquad s + ds - \sigma_y - d\sigma_y = 0 \qquad (17)$$

From (15), (16) and (17) λ can be defined:

$$- \qquad \lambda = \frac{2s\,(s - \sigma_y)}{\{r\}^T [D] \{r\} + 4cs^2} \qquad (18)$$

The implementation is straight forward as shown in Table 1. It includes an option of concrete cracking described in the next paragraph. It is noted that iterations can be carried out, however, numerical experiments have shown that it is not necessary. The advantage of the present plasticity model is that it is not needed to memorize the stresses: the deformation history is memorized in $\{\epsilon^p\}$ and e^p.

3.2 Cracking of concrete

The model for concrete cracking in SLOOFDYN is based on the concept of zero tensile strength of concrete. For reinforced concrete, the model is realistic, the tensile strength is taken up by the reinforcement. For non-reinforced concrete, - where anyhow the design should be such that the load does not give rise to tensile stresses -, the model could give numerical difficulties, since a crack initiated by a small tensile stress due to rounding off, will propagate through the structure. Anyhow, for concrete structures where severe impulsive loads are expected, reinforcement or even prestressing is applied. The great advantage of the method is the ease of implementation, in particular when the explicit time integration scheme is used. Moreover, nearly no additional calculations and no additional

storage is required. As seen from the conceptual flowchart for the constitutive equations (Table 1), the stresses are calculated in the Gauss points from the total strain and accumulated plastic strain using Hooke's law. The stresses are expressed in a local coordinate system related to each integration station:

- $$\{\sigma\} = [\sigma_x, \sigma_y, \tau_{xy}]^T \tag{19}$$

For the concept of zero tensile strength, they have to be transformed into principal stresses according to Mohr's circle (Fig. 1)

- $\{p\} = [p1, p2]^T, \quad p1 \geq p2$

- $$SAV = (\sigma_x + \sigma_y)/2, \quad RAD = SQRT\left(((\sigma_x - \sigma_y)/2)^2 + \tau_{xy}^2\right) \tag{20}$$

- $p1 = SAV + RAD, \quad p2 = SAV - RAD$

Three cases have to be analyzed:

(i) $p_2 \leq p_1 \leq 0$ compression only, no cracking

(ii) $p_2 \leq 0, \quad p_1 > 0$ cracking perpendicular to direction 1. A smeared crack strain ϵ_1^{cr} is introduced such that the principal stress in direction 1 becomes zero:

- $$p_1^{new} = p_1 - D_{11}\epsilon_1^{cr} = 0 \rightarrow \epsilon_1^{cr} = p_1/D_{11} \tag{21}$$

- $$p_2^{new} = p_2 - D_{21}\epsilon_1^{cr} = p_2 - \nu p_1$$

D_{11} and D_{12} are components of Hooke's matrix (6), $D_{12}/D_{11} = \nu$ (Poisson's ratio).

(iii) $0 < p_2 \leq p_1$ In this case cracking may occur in both directions, but not necessarily:

a) cracking in both directions if the expression of p_2^{new} in (21) becomes positive

$$p_2 - \nu p_1 > 0 \tag{22}$$

In this case all stresses vanish, also the shear stresses.

b) if condition (22) is not satisfied, cracking occurs only perpendicular to direction 1 and p_2^{new} is calculated from (21).

In terms of the stresses in local directions to be used to calculate the internal forces (9), the actions to be taken are:

(i) no action, stresses remain unchanged;

(ii) and $(iii)_b$ Mohr's circle translates and shrinks by an amount $\Delta\sigma$ and a factor β, respectively (see Fig. 1):

- $$\Delta\sigma = \frac{1}{2}(p_1 + p_2)^{new} - \frac{1}{2}(p_1 + p_2) = -\frac{1}{2}(1 + \nu)p_1 \tag{23}$$

- $\beta = (p_1 - p_2)^{new}/(p_1 - p_2) = -(p_2 - \nu p_1)/(p_1 - p_2)$ (23)

(Note $\Delta\sigma \leq 0, \quad 0 \leq \beta \leq 1$)

The new local stresses become:

- $\sigma_x^{new} = SAV + \Delta\sigma + \beta(SAV - \sigma_x)$

- $\sigma_y^{new} = SAV + \Delta\sigma + \beta(SAV - \sigma_y)$ (24)

- $\tau_{xy}^{new} = \beta\tau_{xy}$

$(iii)_a$ all stresses, including τ_{xy} vanish. The concrete has lost its strength completely ($\Delta\sigma = -SAV$, $\beta = 0$ in (24))

The conceptual flowchart of the routine CRACK is given in Table 2. It should be noted that the smeared crack strain is not going to be stored. In a subsequent time step, the calculated principal stresses might become negative and the crack closes immediately. The closed crack has, apart from its full compressive strength, also its full shear strength. The routine is used within the plasticity iteration: (i) plasticity in compression might occur in direction 2 and cracking perpendicular to direction 1; (ii) accumulated plastic strains in the compression regime cannot be recuperated and in case of load reversals, cracking will occur earlier (in terms of total strain) than for virgin material.

4. REINFORCEMENT AND PRESTRESSING OF CONCRETE

In the previous chapters, the advantages of explicit time integration for the implementation of non-linear constitutive relations have been addressed. In this chapter, the ease of implementing reinforcement and prestressing in explicit programs is discussed.

4.1 Reinforcement

As shown in Table 1, the constitutive box starts from known strains throughout an element and in particular at integration stations. From the strains, stresses are defined, taking into account plasticity and/or cracking, in order to define the contribution of the Gauss point to the internal forces (9). The modelling of reinforcements consists of adding into the internal forces the stresses due to steel membranes (smeared bars) with Poisson's ratio $\nu = 0$ (Fig. 2). The membranes undergo the same total strains as the concrete (no slippage!). The thickness of the membranes follows from the design specifications of percentage of reinforcement.

4.2 Prestressing

When the reinforcement membrane described in the previous paragraph is given a (negative) prestrain $\{\epsilon^0\}$, the contribution ot the internal force (9) of the reinforcement becomes tensile, and the concrete will be precompressed. In SLOOFDYN, the amount of precompressing of the concrete is an input parameter. Running the program in the static

mode, by one of the two methods discussed in chapter 2, the stress distribution in the concrete and reinforcement is established automatically, including eventually the effect of dead weight, being a natural way of precompressing. The RESTART option allows to introduce the impulsive load in a subsequent run.

5. APPLICATIONS

Examples are given to compare the SLOOFDYN concepts (SEMILOOF, explicit time integration) and the simplified cracking model, with published results (Rebora et al. /8/) using a comprehensive plasticity formulation for concrete in a frame of an implicit computer program with a degenerated solid-element for the shell.

5.1 Circular plate

The reinforced concrete plate has radius R = 10 m and thickness t = 1 m, and is loaded dynamically by a pressure p = 1.4 kg/cm^2 with a rise time T/2 (half of fundamental period). The reinforcement is 2% in radial and circumferential direction. One quarter of the plate is modelled by means of 6 SEMILOOF elements. In Fig. 3 a comparison between the present linear and non-linear results is made with those given by Rebora /8/. Although the linear response corresponds quite well (curves 1 and 2), the non-linear response of SLOOFDYN with no prestressing (curve 5) gives a much larger displacement as compared to Rebora (curve 4) who used a concrete plasticity formulation with ultimate tensile strength of 56 kg/cm^2 and ultimate compressive strength of -350 kg/cm^2. The SLOOFDYN results with a prestress of -56 kg/cm^2, which can also be considered as an artificial way of delaying cracks, show a response (curve 3) which is less flexible than those given by Rebora. It seems therefore that the energy dissipation in plasticity models can be modelled by the simple crack model using a prestress to delay cracks, or alternatively delaying the first appearance of a crack by a small tensile strength. The crack pattern as defined by SLOOFDYN is consistent (Fig. 4), tangential cracking at upper face near the clamped edge, radial and circumferential cracking at lower face in the central region. For the case of prestressing, much less cracking is observed.

5.2 Reactor containment loaded by an aircraft impact

This problem is also studied by Rebora et al. /8/ using the same constitutive relations as for the plate. The SLOOFDYN results, using 26 SEMILOOF elements, show a good comparison (see Fig. 5). Again SLOOFDYN is a bit more flexible, probably due to the SEMILOOF. The non-linear response showed the same phenomena as for the plate, i.e. a much larger displacement. The crack pattern, however, was again consistent. These results are not presented yet, on the one hand because of lack of space, on the other hand because of the preliminary character of the results. They will be given during the oral presentation.

6. CONCLUSIONS

A simple crack model for reinforced concrete has been implemented. As compared to

a comprehensive plasticity model with ultimate tensile strength of 56 kg/cm^2, it seems that the model dissipates less energy when the first appearance of a crack is assumed for zero tensile strength. Experimental research into the constitutive relations for concrete under dynamic loading to define tensile strength seems to be a must in order to use the results of computer programs with confidence.

7. REFERENCES

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/5/ B. Irons, S. Ahmad, "Techniques of finite elements", Wiley and Sons, 1980.

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/7/ R. D. Krieg, S. W. Key, "Transient shell response by numerical time integration", Int. J. Num. Meth. Eng., 17, 1973.

/8/ B. Rebora, Th. Zimmermann, J. P. Wolf, "Dynamic rupture analysis of reinforced concrete shells", N. E. and D. , Vol. 37, 1976.

Table 1 - Conceptual flowchart of constitutive equations

DO 1, i = 1, N GAUS (loop over integration stations)
Iter = 0
Define strains $\{\epsilon\} = [B]\{u\}$
3 Define stresses $\{\sigma\} = [D](\{\epsilon\} - \{\epsilon p\})$
If (elastic calculation) GO TO 2
If (concrete) CALL CRACK ($\{\sigma\}$)
If (iter .GT. itermax) GO TO 2
Define von Mises' stress s
If (s .LE. $\sigma_y(e^p)$) GO TO 2
Define λ from (18)
Define $\{d\epsilon^p\}$ from (12), $\{\epsilon^p\} = \{\epsilon^p\} + \{d\epsilon^p\}$
Define de^p from (16), $e^p = e^p + de^p$
Iter = iter + 1
GO TO 3
2 Define contribution to $\{f^{int}\}$ from (9)
1 CONTINUE

Table 2 - Conceptual flowchart of cracking model

SUBROUTINE CRACK (σ_x, σ_y, τ_{xy})
Define principle stresses from (20)
If (p1 .LT. 0) RETURN (No cracking)
If (p2 – νp1) .GT. 0.) GO TO 3
Define new stresses from (24) (cracking perpendicular to direction 1)
RETURN
3 Zero all stresses (cracking in both directions)
RETURN
END

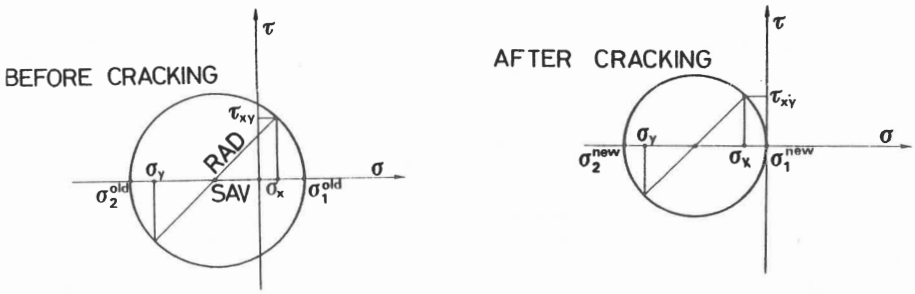


Fig.1 Stress redistribution due to cracking

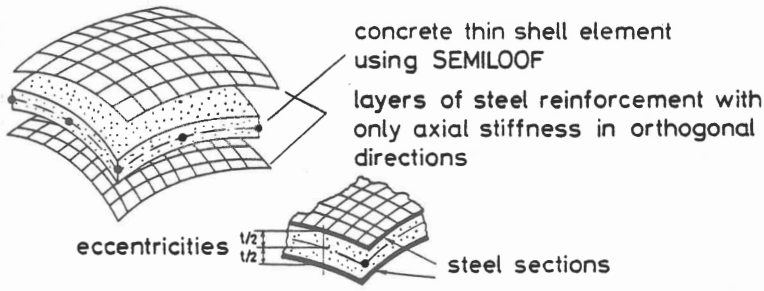


Fig.2 Model in for reinforcement

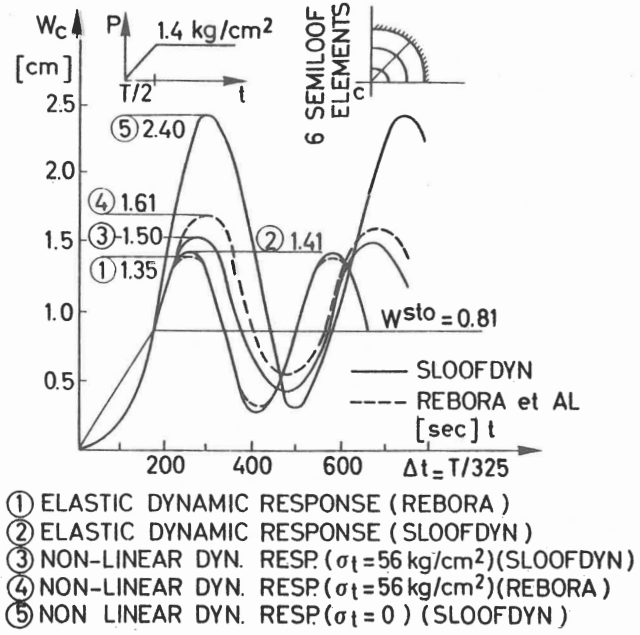
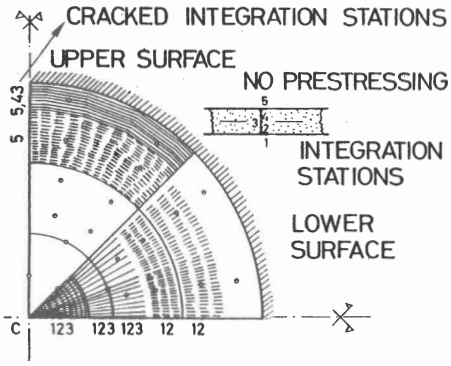
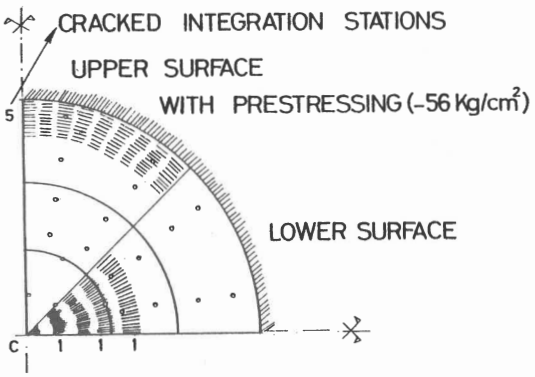


Fig.3 Response for dynamic load on circular plate

Fig.4 Crack patterns



a) without prestress



b) prestress of -56 kg/cm^2

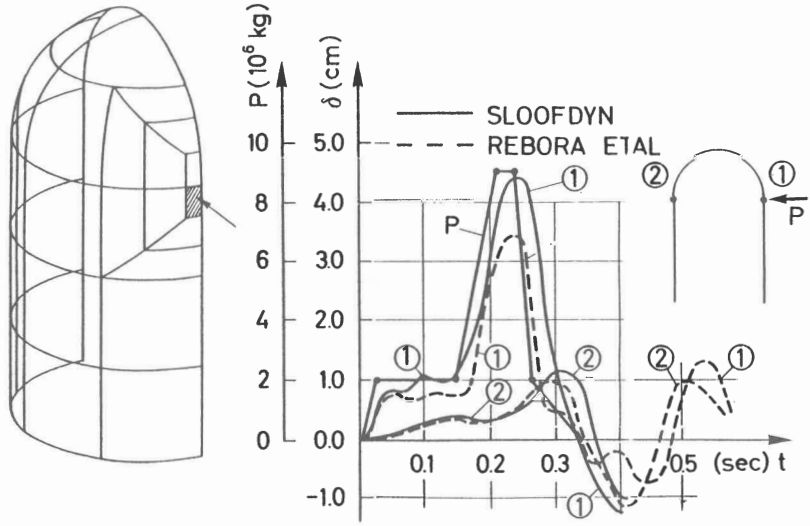


Fig.5 Mesh and dynamic (linear) response of reactor containment due to aircraft crash