TIME-DEPENDENT CREEP IN METALLIC COMPONENTS : NOZZLE ENTERING A SPHERICAL PRESSURE VESSEL

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ABSTRACT

A method of calculating creep deformations based on the corresponding elastic problem is described. The starting point in the analysis is the elastic solution so that a single computer program can be used to calculate stress redistribution caused by creep to a state of stationarity.

This procedure has been applied to the nozzle problem in order to arrive at conclusions that will be useful in design.

1. INTRODUCTION

The importance of creep as a major design criterion is increasing in practically all branches of high temperature technology. In pressure vessel design there is every incentive to raise operating temperatures and pressures to achieve higher plant efficiencies while capital costs can be reduced significantly by reducing thicknesses. Creep will occur to different degrees in all these circumstances and must therefore be designed against. The problem is to guard against excessive deformation which could lead to mismatch of parts or to rupture.

Calculation procedures for creep are rare in their availability. The reason for this invariably stems from the lack of information regarding material behaviour on the one hand and a lack of understanding of how creep affects structural behaviour on the other. Most investigations on material behaviour are aimed at collecting data from the uniaxial, isothermal creep test followed by, mainly, empirical curve fitting. A qualitative understanding of underlying mechanisms has enabled metallurgists to go a long way in helping to design creep resistant alloys. However it is true to say that the quantitative laws needed by engineers so that they can reliably account for the practical conditions to be catered for in component design have not been forthcoming. To a large extent engineers have been sidetracked into trying to understand the creep test rather than towards generating methods for utilising the data from such tests. A notable exception to this is in the efforts of Hoff [1] and others in the study of structures subjected to creep in which elastic and primary creep strains are supposed negligible compared with secondary creep components. The advantage of such an approach is that structural actions and material effects can usually be isolated with the result that materials can be properly chosen and disposed in a design. At the same time experimental effort aimed at the collection of useful data can be properly planned.

Steady state calculations are, in fact, no easier to perform than those in which elastic and primary creep strains are also included. If these strains are included, additional information regarding the effects of stress redistribution can also be estimated and it is upon this theme that the present paper is concerned (Figure 1). Within the paper well tried procedures are applied to the nozzle problem under conditions of constant applied loading and temperature with a view to discovering, in broad terms, the response of such a component to the creep process. It is fully realised that constant operating conditions are not always a feature of a practical design and all that can be offered to cater for these at the present time are some conjectural methods outlined elsewhere (Penny and Marriott [2]).

2. ANALYSIS

2.1 Assumptions

In order to isolate the most important features of the problem a number of simplifying assumptions is made:

a) all the usual thin shell assumptions which are reasonable in elasticity apply during creep. Thus, originally plane sections are assumed to remain plane after bending, and through-thickness stresses are ignored. Furthermore, displacements are assumed to be small ($< \frac{1}{2}$ x shell thickness).

b) the nozzle is taken to intersect radially with the spherical pressure shell and it is assumed that the intersection occurs at the mid-thicknesses of the nozzle and sphere. While it is realised that the latter assumption is unrealistic it is known from elastic theory to yield reasonable results for stresses in the sphere and overestimates of stresses in the nozzle in the region of the intersection. The nozzle is taken to be of infinite extent away from the intersection.

c) the loading is constant internal pressure and the vessel is maintained at constant temperature. The loading and temperature are assumed to be applied instantaneously. On first loading it is assumed that the response is elastic and that thereafter creep is allowed to occur.

d) the material is homogeneous, isotropic and follows a time hardening creep law of the form

$$\frac{d\varepsilon'}{dt} = \sigma^m f(t) \qquad (1)$$

in which ε' is the creep strain, t time, and σ stress while m and $f(t)$ are determined from a creep test. Although this law is used in the interest of an expedient solution others could be inserted. It is recognised that the time hardening description has little physical appeal but experience has shown that component response is hardly affected by the type of law in situations for which stresses are slowly varying.

As a further convenience equation (1) is simplified in the manner of other publications (Penny [3]) to remove dependence on the time function $f(t)$ during the analysis.

This is accomplished by non-dimensionalising stresses and normalising strains as follows:

$$\Sigma = \frac{\sigma}{\sigma_o} , \quad \lambda = \frac{\varepsilon}{\varepsilon_o} , \quad (\varepsilon_o = \frac{\sigma_o}{E})$$

This device is used throughout and leads to a modified creep law,

$$\dot{\lambda}' = \Sigma^m \tag{2}$$

where the dot in (2) refers to differention with respect to a non-dimensional time parameter (Penny [3])

$$\tau = E\sigma_o^{m-1} \int_0^t f(p)dp \tag{3}$$

In these equations the reference quantities σ_o, ε_o may be chosen at will; in this paper the equilibrium stress $\sigma_o = \frac{pR}{2T}$ is chosen so that all stresses that are calculated refer to proportions of the membrane stresses in the unpierced spherical shell.

e) the multidimensional version of equation (2) is derived through the use of the Von-Mises yield criterion and the Prandtl-Reuss relations usually regarded as being valid for time-independent plasticity. In this way the meridional (ϕ) and circumferential (θ) creep strain increments are

$$\Delta\varepsilon_\phi' = \frac{\Delta\varepsilon^*}{\sigma^*} (\sigma_\phi - \frac{\sigma_\theta}{2}); \quad \Delta\varepsilon_\theta' = \frac{\Delta\varepsilon^*}{\sigma^*} (\sigma_\theta - \frac{\sigma_\phi}{2}) \tag{4}$$

where $\Delta\varepsilon^* = \frac{1}{\sqrt{3}} (\Delta\varepsilon_\phi'^2 + \Delta\varepsilon_\theta'^2 + \Delta\varepsilon_\theta'\Delta\varepsilon_\phi')^{\frac{1}{2}}$

$$\text{and} \quad \sigma^* = (\sigma_\phi^2 + \sigma_\theta^2 - \sigma_\phi\sigma_\theta)^{\frac{1}{2}} \tag{5}$$

are the effective strain increment and stress respectively. These are taken to be fitted by data from a uniaxial test which can be expressed by

$$\Delta\varepsilon^* = \sigma^{*m} f(t)dt$$

Thus, by eliminating $\frac{\Delta\varepsilon^*}{\sigma^*}$ from (4) by use of (5) and at the same time utilising the non-dimensional stresses as in d), together with the non-dimensional time τ, the multidimensional versions of (2) become

$$\Delta\lambda'_\phi = \Sigma_*^{m-1}(\Sigma_\phi - \frac{\Sigma_\theta}{2})d\tau \;;\; \Delta\lambda'_\theta = \Sigma_*^{m-1}(\Sigma_\theta - \frac{\Sigma_\phi}{2})d\tau \qquad (6)$$

The advantage of this scheme is that the material is specified by one parameter only - the stress exponent m.

2.2 Details of Analysis

It has been shown elsewhere (Penny [4]) that the governing equations for shells of revolution (or indeed any structure) can be expressed in the form

$$D(u,v) = D_1(P) \;, \quad (\tau = o)$$

$$D(\dot{u},\dot{v}) = D_1(\dot{P}) + D_2(\dot{\lambda}'), \quad (\tau \geq o) \qquad (7)$$

In these D, D_1, D_2 are differential operators, P is the generalised loading and $\dot{\lambda}'$ terms represent creep strain rates following initial loading. In addition, u and v are independent variables chosen from convenience for a particular problem. For example in the case of spherical shells (Penny [3])

$$D(u,v) \equiv \begin{cases} (L^2 + \nu)S - 2\kappa^2\Theta \\ (L^2 - \nu)\Theta + 2\kappa^2 S \end{cases}$$

where $L^2(..) = \frac{\partial^2(..)}{\partial\phi^2} + \cot\phi\,\frac{\partial(..)}{\partial\phi} - \cot^2\phi(..)$, S being the lateral shear force and Θ the rotation of the tangent to the meridian. Also, for constant pressure loading $D_1(P) = D_1(\dot{P}) = 0$. The corresponding meaning of these terms for cylindrical shells can be similarly derived. (Penny [5])

a) Solution procedure for a given shell.
Equations of the form (7) can be re-written in finite difference form, for a given shell shape, to yield matrix equations of the form

$$\underline{A}X = \underline{B} \;, \quad (\tau = o) \qquad (8)$$

$$\underline{A}\dot{X} = \underline{C} \;, \quad (\tau \geq o) \qquad (9)$$

where, it should be noted, the coefficient matrix $\underline{A}$ occurs in both the instantaneous equations ($\tau = o$) and their rate version ($\tau \geq o$). The vector $\underline{B}$ depends only on the loading while $\underline{C}$ depends only on the current stresses if the loading is constant, equation (7). X and $\dot{X}$ are solutions to (8) and (9) which enable stresses and stress rates to be determined. Steps in the solution procedure are then:

i) solve the initial problem (equation (8)).
ii) evaluate stresses (and any other quantity required) from a knowledge of X.
iii) insert these stresses into expressions of the form $D_2(\dot{\lambda}')$ (equation (7)) to evaluate $\underline{C}$ and hence solve the rate problem (equation (9)) to obtain $\dot{X}$.
iv) evaluate stress rates, etc., from a knowledge of $\dot{X}$.
v) over a given time interval $\Delta\tau$, evaluate new stresses from $\Sigma_{\tau+\Delta\tau} = \Sigma_\tau + \dot{\Sigma}_\tau \times \Delta\tau$.
vi) repeat steps iii) to vi), having started from $\tau = o$, to any stage required.

This solution procedure amounts to solving a number of elastic problems and is directly analogous to Ilyushin's [6] method for instantaneous plasticity. Such an approach has only become practicable relatively recently, following the advent of the automatic computer. Now that the computer is a commonplace tool it can be confidently said that if a procedure is available for solving an elastic problem then the corresponding creep solution can also be found. This device does not seem to be widely known - something of a pity, since a wide range of elastic programs is generally available.

b) Solutions for multiple shells.

In practice most pressure carrying shell structures are composed of elements of different shapes; boilers with torispherical or ellipsoidal ends, spherical nuclear vessels containing a graphite core with the whole supported on cylindrical skirts, cylindrical cooling ducts or charge chutes entering a spherical pressure vessel and so on. The procedure for dealing with these shell assemblies is well known for elastic materials; the work of Kalnins [7] is typical. The method is to force the unconstrained displacements of adjoining shells acted upon by their separate loadings, to be compatible, by imposing self-equilibrating forces at their junctions. The magnitudes of these forces determine the level of stresses at shell junctions and, to a large extent, the need for reinforcement there.

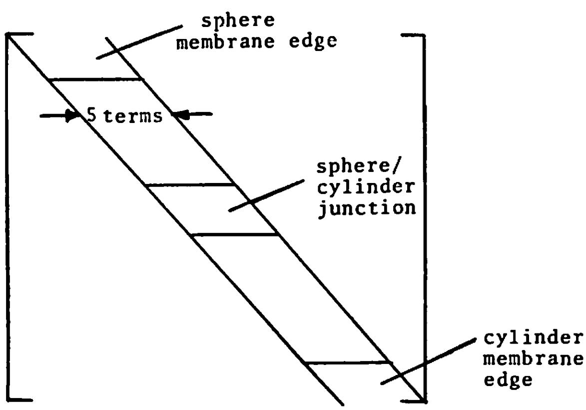
In the case of the creep of discontinuous shells the same procedure can be followed but the boundary conditions at the junctions of the shell elements must be expressed in terms of their current states and also their (time) rates at that state. So far as the particular problem of interest in this paper is concerned - the cylindrical nozzle entering a spherical vessel - the appropriate compatibility requirements are

$$\left.\begin{array}{l}\Theta_c = \Theta_s \\ \delta_c = \delta_s\end{array}\right\} \tau = o \qquad \left.\begin{array}{l}\dot{\Theta}_c = \dot{\Theta}_s \\ \dot{\delta}_c = \dot{\delta}_s\end{array}\right\} \text{all } \tau$$

where Θ and δ are the element rotation and displacement perpendicular to the axis of symmetry respectively.

With these boundary conditions the governing equations for the complete assembly can be written in finite difference form to yield matrix equations of the form (8) and (9). However in the case of discontinuous shells the coefficient matrix $\underline{A}$ takes the schematic form

$\underline{A}$ =



By careful ordering of the equations in this coefficient matrix the qui-diagonal form, characteristic of single shells, and for which efficient inversion schemes are available, can be preserved. The number of discontinuous shells in any assembly can be extended indefinitely subject only to the limitations of the computer available. The vectors $\underline{B}$ and $\underline{C}$ contain loading and other terms appropriate to both the sphere and the cylinder.

The solution procedure hereafter is precisely similar to that for continuous shells (Section 2.2(a))

c) Computational details.

Although the scheme for creep which has been described is similar in form to the elastic procedure it contains some essential differences which are worth noting.

Terms in the vector $\underline{C}$ are found to require integration of various combinations of stresses through the shell thickness and because of the non-linear distribution of stresses this integration must be performed numerically. The least number of sections through the half-thickness which has been found to be satisfactory is (m+2), (Penny [3]), where m is the stress exponent. This integration has to be performed at every station in the finite difference mesh around the shell for which angular steps of $\frac{1}{2}^{o}$ intervals have been found to be adequate.

The question of choosing a suitable time interval requires careful thought because after initial loading the stresses change rapidly and small time steps are needed whereas later in the calculation, as stationary conditions are approached, a small interval is unnecessary. A satisfactory way of incorporating these requirements is to choose a time interval which is a fraction of a current variable divided by its rate. In general this can be expressed as

$$\Delta\tau = \frac{1}{f} \min. \left| \frac{M_\theta}{\dot{M}_\theta} , \frac{M_\phi}{\dot{M}_\phi} , \frac{\Sigma_\theta}{\dot{\Sigma}_\theta} \; .... \right| \qquad (13)$$

the symbols having their usual meanings.

Generally, M_θ has been found to be a suitable choice, together with f = 10.

Another problem to be faced with the time integration is in knowing when to stop the calculation. This point can be chosen at will but in the present context, where the state of stationarity is of most interest, the calculations have been allowed to run that far. For this purpose a limit is placed on the stress rates; in the present work this has been $|\dot{\Sigma}| < 10^{-2}$.

The total computational times for work performed on the Cambridge University TITAN computer were typically one minute for single shell problems and two minutes for shell/nozzle intersections.

3. DISCUSSION OF RESULTS

Extensive results for spherical and cylindrical shells have already been published elsewhere (Penny [3], [5]) from which a number of conclusions relating to the character of the creep process have been derived.

A full parametric survey for the nozzle problem has not yet been completed but some results for nozzles with geometries typical of nuclear pressure vessels are included in figures 3 - 6.

These figures give results for two different nozzle sizes ($\frac{r}{R}$ = 0.083, 0.166) while the main vessel radius to thickness ratio and the nozzle to main vessel wall thicknesses are maintained constant at 120 and 1 respectively. In each case the material creep property has been changed by performing calculations for values of the stress exponent m at 3, 5 and 7, these values covering the range thought to be valid in practice.

Figures 3a and 3b show how the maximum surface circumferential stresses vary around the spherical shell at the stationary state; in these figures the stresses are plotted non-dimensionally as proportions of the membrane stress $\frac{pR}{2T}$. In each case the character of the stress distributions is very similar in that beyond an angular distance of about 5^{o} from the nozzle intersection the shell is elastic. Within this region the creep process acts quite powerfully in reducing the peak stress levels from their initial elastic values although the reductions become smaller as the stress exponent increases. Most significantly, the location of the maximum stress moves away from the nozzle into the main shell - a feature in accordance with experimental observations of crack locations during creep.

Figure 4 shows how the maximum stresses for the larger nozzle redistribute in time depending on the value of the stress index m. Not surprisingly the redistribution time is strongly influenced by m - to the extent of about an order of magnitude in τ for each change in m. This redistribution process is much slower as the nozzle size reduces as can be seen from Figure 5. Finally, Figure 6 gives some idea of the effectiveness of nozzle reinforcement during creep. This figure gives two points of information of some importance. First, the frequently used plate analysis as an approximation to the behaviour of unreinforced openings is clearly inadvisable both for elastic and creep responses; it is grossly unconservative so far as stress levels are concerned. Second. the character of the reinforced nozzle stress redistribution is closely similar to that for the unreinforced nozzle. The reason for this is that the nozzle reinforcement acts as intended in reducing stresses, and for the case under consideration, quite closely to the stresses appropriate to the flat plate. This similarity will be even closer as the nozzle size is reduced although the stress levels will always be lower in the case of the plate.

4. CONCLUSIONS

The method of analysis which has been outlined is a simple extension of the elastic shell solution procedure. Thus it is possible to write a single computer program to predict the initial elastic stresses and the redistribution of these stresses in reaching a stationary state of stress; the stationary state solution represents a simple plasticity solution (Hoff /1/). The intelligent use of elasticity theory can therefore give powerful methods to pressure vessel designers who have to make decisions regarding the effects of creep, plasticity, shakedown and fatigue in sizing parts and assessing lifetimes. This apparently paradoxical conclusion has been derived through the manipulation of the governing equations which are well known in elasticity and is only possible through the use of high speed computers. Clearly such methods will assume greater importance as the use of computers becomes more widespread but careful attention must be paid to the relatively crude state of knowledge regarding material behaviour before carrying these methods to a high degree of sophistication. It is essential, for example, that more attention be given to the effects of multiaxiality of loading on deformation accumulation and rupture before the implications of computed results for components can be judged in economic design. Furthermore the effects of variable loading, as will be important in practice, must be studied through computer calculations and by laboratory experimentation.

For the particular problem of the nozzle in a spherical pressure vessel, which has been studied in this paper, a number of useful conclusions can be stated:

a) maximum stresses which occur at the nozzle intersection diminish rapidly with time. The higher the initial stress - and this is determined by the size of the nozzle - the quicker is the redistribution process.

b) maximum stresses are governed by the same geometrical features as for the elasticity problem. In the case of creep the position of the maximum stress moves away from the intersection into the main vessel. The magnitudes and positions of maximum stresses during creep are relatively insensitive to the stress exponent in the creep law.

c) the time taken for stress redistribution depends on the nozzle geometry and strongly on the stress exponent.

5. LIST OF REFERENCES

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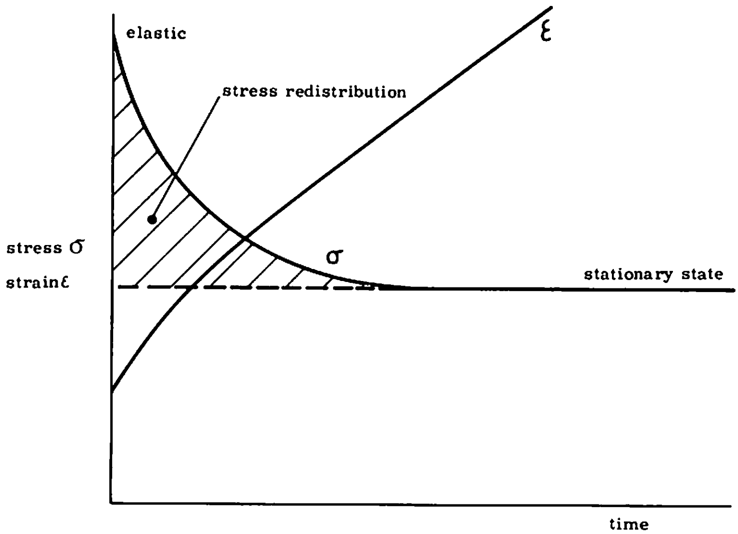
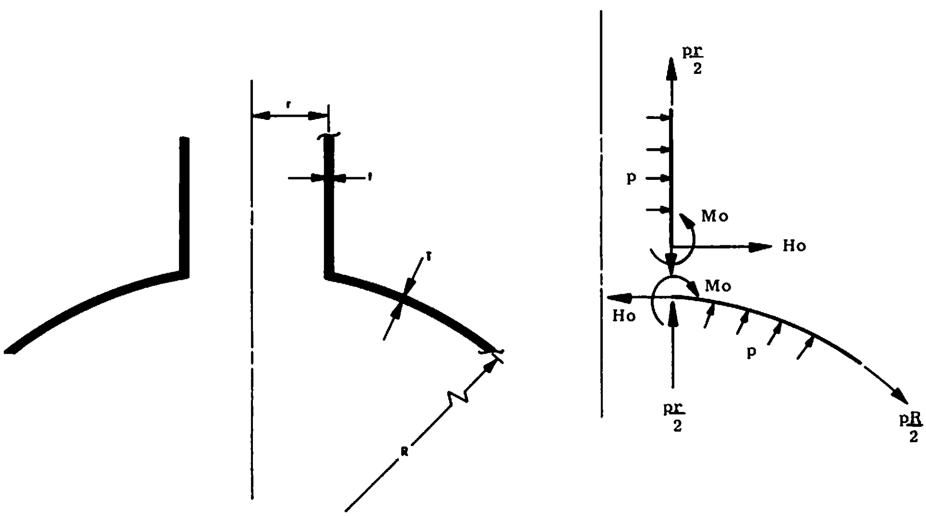


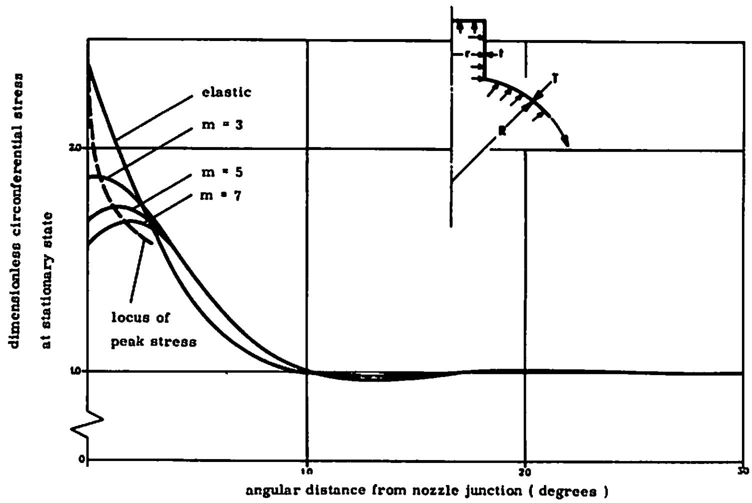
1. The character of stress redistribution during creep.



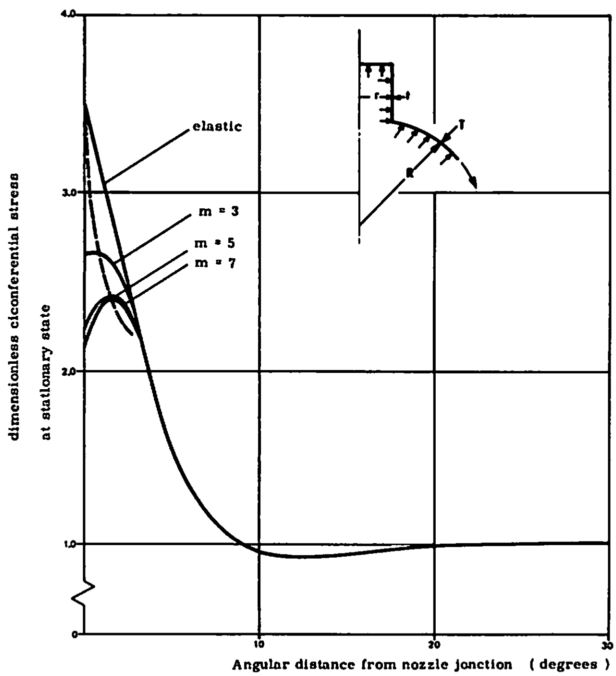
a) Nozzle geometry

b) Jonction discontinuity forces

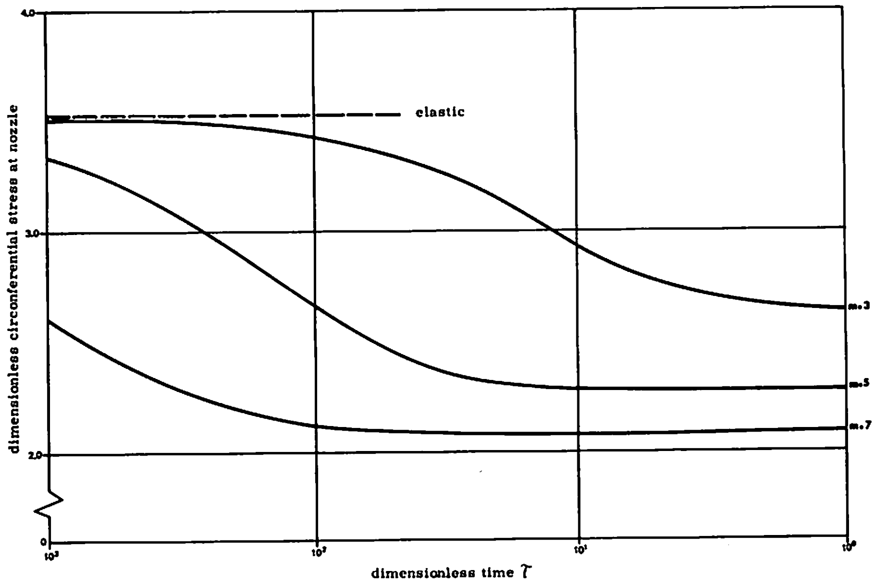
2. Nozzle geometry and junction discontinuity forces.
(a) Nozzle geometry. (b) Junction discontinuity forces.



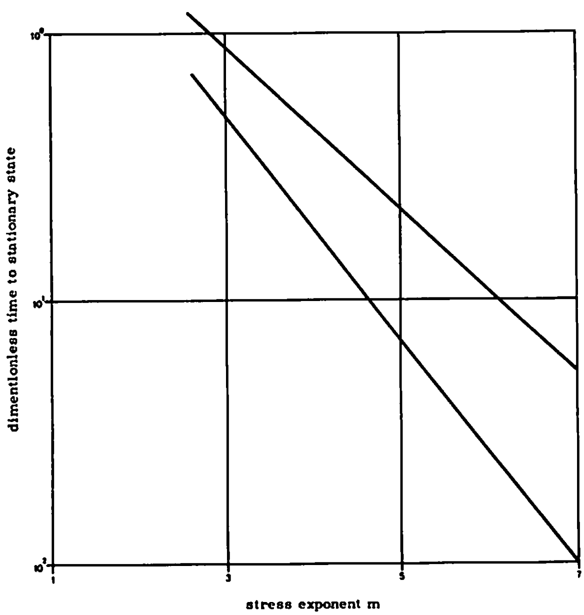
3a. Variation of circumferential stress in a spherical shell ($\frac{r}{R}$ = 0.166, $\frac{R}{T}$ = 120, $\frac{t}{T}$ = 1.0).



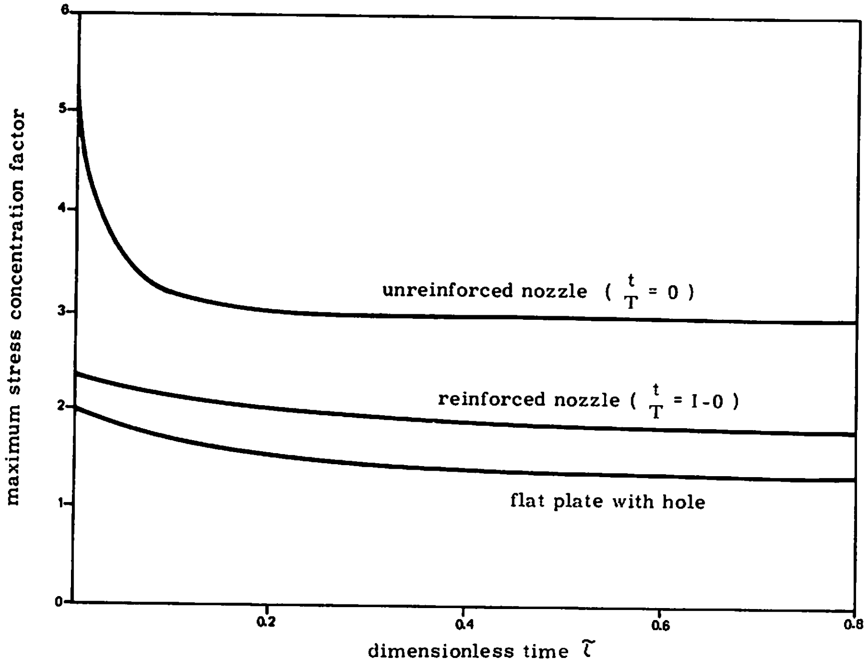
3b. Variation of circumferential stress in a spherical shell ($\frac{r}{R}$ = 0.166, $\frac{R}{T}$ = 120, $\frac{t}{T}$ = 1.0).



4. Redistribution of stress in spherical shell at nozzle junction ($\frac{r}{R}$ = 0.166, $\frac{R}{T}$ = 120, $\frac{t}{T}$ = 1.0).



5. Variation of time to stationary state with stress exponent.



6. Stress redistribution at nozzle; effect of reinforcement ($\frac{r}{R}$ = 0.083, $\frac{R}{T}$ = 120, $\frac{t}{T}$ = 1.0).

DISCUSSION

Q Y. R. RASHID, U. S. A.

In order to generalize the results of tests done on a specific component to other situations some sort of theory has to be used. I think then that more fundamental work is needed as well as component testing. I agree, however, with Prof. Penny's thesis.

A R. K. PENNY, U. K.

I agree that some sort of theory is needed by which one brings together results of component tests. My plea is that the first theories looked at and the first tests performed should be simple ones. As I see the situation at present, many theories are overly complex, analytical techniques are applied in an ad-hoc fashion, while component tests are almost non-existent. I also agree that fundamental work is needed. However "fundamental" should not be equated to "mathematical" as is often done. I regard as fundamental the process of observation, speculation, analysis and test, applied in an iterative fashion proceeding from the particular to the general.

Q E. Y. W. TSUI, U. S. A.

1. Would you like to comment on the strain variations vs time ?
2. If stationary stress is used as a basis could one obtain approximate strains from one isochronous stress-strain curves ?

A R. K. PENNY, U. K.

1. The strain variation is calculated by the procedure outlined in the paper. It takes the schematic form of Fig. 1. I do not have values with me but it is true to say that there is little error in superposing the stationary state values on the initial elastic ones. This will be unconservative, however.
2. I suppose so but I would use the calculation procedure as described, for consistency.