EFFECTS OF CRACKS ON THE RESPONSE OF SHELL STRUCTURES

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Summary

A simple model that predicts the effects of axial cracks on the elastic deformation of thin cylindrical shells has been developed. This model provides an efficient tool for performing parametric studies and for interpolating, extrapolating, and generalizing finite element analyses.

Since the symmetry of a cylindrical shell is destroyed by the presence of axial cracks, significant bending deformation can be induced by the cracks even when the loading itself is axisymmetric. This additional deformation depends upon the number, size, and location of the cracks, and it can dominate the response for moderately deep cracks. The dynamic response of a shell is especially sensitive to the presence of cracks, and the amplitude of vibration of a cracked shell can be an order of magnitude greater than the static deflection response of the same shell without a crack.

The model employs Fourier series representations for the deformation components of the shell and represents a crack by pairs of concentrated couples at the crack location. These couples induce bending modes of deformation, with the lower order bending modes dominating the response. Hence excellent results can be obtained with only about twenty-five bending modes superimposed on the axisymmetric breathing mode, which alone characterizes the response of an uncracked shell to an axisymmetric pressure load. The model can be calibrated from a single static finite element analysis of the shell of interest. The calibrated model then accurately predicts the effects of different numbers, sizes, and locations of cracks under both static and dynamic loading of the shell.

If plastic yielding or crack propagation do not occur in the shell, the principal concern for design and safety analyses would appear to be the amplitudes of the deformations induced by cracks, especially under dynamic loading conditions. Since these amplitudes can be considerably affected by the cracked geometry, an analysis that ignores the effects of cracks will not be conservative. In cases where clearance and interference of adjacent components are of concern it is imperative that the presence of cracks and crack-like imperfections be taken into account.

The simple model described in this paper provides a useful tool for understanding the phenomena involved in the response of cracked shells and for identifying situations where unacceptably large deformations may result in the presence of cracks. Cases so identified may then be subject to more accurate, and more expensive, finite element analyses.

1. Introduction

Many of the components of fast reactor plants may be treated as circular cylindrical shells for the purposes of structural dynamics analyses. Standard analyses consider these shells to be flawless, i.e., the geometry is considered to be perfectly circular, and imperfections, such as cracks, are ignored. In actuality real shells may be built up of sections with irregular welds, be out of round, have variations in thickness, and possess nicks, scratches, gouges, and other crack-like discontinuities in geometry. Although all these deviations from uniformity destroy the axisymmetry of the shell, their effects on the fundamental structural response are generally of higher-order and ignorable. This is especially true when more substantial discontinutites such as nozzels and structural supports are present.

Nevertheless, the safety analyst should have some understanding of the effects of irregularities such as cracks on the static and dynamic response of shells not only to understand the basic phenomena involved but also to know when the effects are or are not indeed ignorable. While one short and shallow nick in a reactor vessel with massive nozzles may be insignificant to the overall dynamic response of the vessel, a long, deep crack in a large diameter pipe may have significant effects on the response of that component to a pressure surge.

In order to be able to classify and quantify the effects of cracks on the response of cylindrical shells, a simple model has been developed. This model may be calibrated from a few quasistatic finite element solutions for the shell of interest, and the calibrated model may be used to conduct parametric studies of the cracked shell under a variety of static and dynamic loading conditions and crack geometries. Cases identified as potentially troublesome may be studied in more detail by finite element techniques.

2. Description of the Plane Strain Model

The cracked shell is modelled as a geometrically uniform thin elastic shell with concentrated moments applied at the crack locations, as illustrated in fig. 1 for the case of a uniformly pressurized cylinder with a single internal crack. The model generalizes ideas introduced by Hetényi [1] and used by Thompson [2] and represents a crack as a notch of arbitrarily small width c. The couples M´ are to be determined as functions of the load, geometry and stiffness. Fourier representations for the dimensionless radial and tangential displacements are employed:

$$\zeta = \bar{\zeta}_o + \sum_{n=2}^{\infty} \bar{\zeta}_n \cos n\theta \qquad (1)$$

$$\psi = \sum_{n=2}^{\infty} \bar{\psi}_n \sin n\theta \qquad (2)$$

where the loading is assumed to be symmetric about the crack plane $\theta = 0$. Since the major flexural wavelengths are expected to be long compared to the wall thickness, it is reasonable to assume the inextensionality condition $\zeta + \psi' = \bar{\zeta}_o$, which implies that $\bar{\psi}_n = -\bar{\zeta}_n/n$. Energy methods may be employed to determine the coefficients $\bar{\zeta}_n$.

3. Calibration of the Model

The Fourier coefficients $\bar{\zeta}_n$ depend upon the crack depth e and width c and the couples M´ in addition to the shell geometry and stiffness H. For a uniform quasistatic pressure p_o applied to the shell in fig. 1, the principle of stationary potential energy gives

$$\bar{\zeta}_o = \frac{ap_o}{Hh} \qquad (3)$$

$$\bar{\zeta}_n = -\frac{2M'}{\pi Hha\alpha^2}\frac{\sin n\gamma}{n(n^2-1)} \qquad (4)$$

where $\alpha^2 = h^2/12a^2$ and $\gamma = c/2a$. These coefficients decrease rapidly with increasing n, and the deformation of a cracked cylinder may be represented by relatively few terms of the Fourier series.

For the present case of a single crack the infinite series (1) may be summed in closed form to give expressions for the radial displacements $\bar{\zeta}_C$ at the crack plane $\theta = 0$ and $\bar{\zeta}_A$ at $\theta = \pi$ diametrically opposite the crack plane. Then it follows for small γ that

$$M'\gamma = \pi Hha\alpha^2 \left[\bar{\zeta}_o - \frac{1}{2}(\bar{\zeta}_A + \bar{\zeta}_C)\right] \qquad (5)$$

and this formula may be used to calibrate the model from finite element or other independent results. The calibration employed in the present work was based on three quasistatic finite element analyses of a 2.54 cm (1 in.) thick cylindrical shell with a/h = 25 and H = 208 GPa (30×10^6 psi). The cylinder was uniformly pressurized and had one internal crack. All subsequent predictions of the simple model, both static and dynamic, follow from this single calibration [3].

Nonuniform loading and different numbers of cracks may also be treated with a generalization of the simple model once it has been calibrated as described above. The simple model may also be employed without further calibration to illustrate the effects of having the crack located externally and of applying an external pressure.

4. Deformation of Cracked Cylindrical Shells

Figure 2, which represents an externally cracked cylinder subject to uniform internal pressure, illustrates the basic phenomena involved in the deformation of cracked cylindrical shells. The deformed shapes, magnified in this figure for clarity, were calculated using 100 terms of the Fourier series, but calculations using only 25 terms give shapes indescernible from these except at the crack location, where fewer terms give a blunter shape.

If a flawless shell were subjected to uniform internal pressure p_o, then the deformed shape would be simply a larger circular cylinder of radius $a(1 + \bar{\zeta}_o)$. However, the presence of a crack reduces the local stiffness of the shell and induces bending modes of deformation which are superimposed on the axisymmetric mode. The simple model employed herein uses the couples M' to represent the effects of reduced stiffness, and these couples induce the bending modes whose amplitudes are given by eq. (4). Since these amplitudes decay rapidly with increasing n, the lower order bending modes dominate the response.

For shallow cracks ($e < h/4$), the amplitudes of the bending modes are so small that the axisymmetric mode of deformation itself dominates the response, and the effects of the crack are clearly negligible. However, for moderately deep cracks ($e \simeq h/2$), the bending deformations are of the same order of magnitude as the uniform extensional mode, and the bending causes the crack plane to move outward significantly relative to the undeformed shell. Another consequence of the presence of a moderately-deep crack is to cause sections of the shell diametrically opposite the crack to deflect outward more than the uncracked shell would, while other sections actually move inward relative to the deflected shape of an uncracked shell. Finally, for deep cracks ($e \simeq 3h/4$), the bending modes of deformation dominate the response and considerable out-of-roundness results under uniform internal pressure, and some sections actually move inward relative to the undeformed shape.

The four possible combinations of crack location and pressure direction are easily handled by the simple model by changing the signs of p_o and M'. The different responses for the case of a crack half through the wall are illustrated in fig. 3, and the basic phenomena described above can be seen to be involved in all cases.

5. Dynamic Response of Cracked Cylindrical Shells

The dynamic response of the cracked shell is also predicted by the quasi-statically calibrated model. Forming the Lagrangian of the cracked shell loaded by a uniform step pressure $p_oH(t)$, where $H(t)$ is the Heaviside step function, leads to the equations of motion which show the dynamic response to be made up of the free vibrations of the natural modes about their corresponding static values. The total motion is given by

$$\zeta(\tau) = \bar{\zeta}_o\,(1 - \cos\tau) + \sum_{n=2}^{\infty} \bar{\zeta}_n\,(1 - \cos\omega_n\tau) \tag{6}$$

where the amplitudes are given by eqs. (3) and (4). The dimensionless time τ is related to real time t and the mass density ρ by

$$\tau = \frac{t}{a}\sqrt{\frac{H}{\rho}} \tag{7}$$

and

$$\omega_n = \frac{\alpha n(n^2 - 1)}{\sqrt{n^2 + 1}} \tag{8}$$

are the familiar natural frequencies for the bending vibrations of uniform rings. Although each mode of vibration gives rise to a dynamic peak displacement which is exactly twice the corresponding static value, the total motion of the cracked cylinder can exhibit peak dynamic amplitudes much in excess of twice the corresponding static deflection. In fact, when compared to the uniform static deflection $\bar{w}_o = a\,\bar{\zeta}_o$ of an uncracked shell, the maximum displacements under dynamic loading can be an order of magnitude greater than $\bar{w}_o$.

The dynamic response to pulse loadings will not be so severe as this worst case condition, of course, but the step load serves as a convenient benchmark problem for comparison of effects. The simple model may be easily adapted to treat other dynamic loadings.

6. Effects of Cracks on the Dynamic Response of Cylindrical Shells

The simple model, calibrated quasistatically, has been employed to study the dynamic response of cracked cylinders, and results for the case of a shell with one external crack subjected to a uniform internal step pressure is illustrated in fig. 4. These response curves represent only the breathing and first nine bending modes of the model, but these ten alone have been found sufficient to give adequate results [3].

The outward radial displacement of the crack plane (solid curve) and of a plane $\pi/2$ radians from the crack plane (dashed curve) are plotted as functions of time. All displacements have been normalized with respect to the uniform static displacement $\bar{w}_o$ that would result if the cylinder were flawless, so that the displacements plotted will represent the dynamic amplifications. For a perfectly uniform and flawless cylinder, the dynamic response to a step pressure would, of course, be $w/\bar{w}_o = 1 - \cos\tau$, and all planes would respond in this same "breathing mode" with unit amplitude about the static value of unity.

When a crack is present, the bending modes of vibration are superimposed upon the fundamental breathing mode, and their amplitudes increase relative to the breathing mode with increasing crack depth. This is demonstrated in fig. 4, which illustrates the effects of the three crack depths studied. The left-hand part of this figure represents the response for the case of a crack one-quarter through the wall, and there is very little discernible difference between the motions of the two planes and what would be expected for the response of the uncracked cylinder. When the crack depth is increased to half the wall thickness, represented in the middle part of fig. 4, the effects of the crack may be readily seen. Here the two planes clearly have distinct motions, showing the out-of-roundness of the response, and the overall amplitude of vibration is of the order of twice that of an uncracked cylinder. Dramatic dynamic amplifications occur in the case of a crack three-quarters through the wall, as illustrated in the right-hand part of fig. 4. Here the overall amplitude of vibration is an order of magnitude greater than that of the uncracked cylinder.

The effects of different crack depths on the dynamic response of an internally-cracked cylinder are shown in fig. 5. Since the pressurized cylinder now bulges in at the crack plane, the bending mode responses are the negatives of those for the case of an external crack. The sign of the breathing mode is determined by that of the pressure, however, so the overall responses for external and internal cracks are not quite mirror images.

The case of external pressure may be treated also, but the dynamic responses to such a loading may be inferred from the above examples.

7. Conclusion

A very simple model has been seen to give the static and dynamic response of uniformly pressurized cylindrical shells with a minimum of computational effort. This model has the advantage not only of illustrating clearly the structural dynamic phenomena involved but also of providing an efficient analytical tool for performing parametric studies of cracked cylindrical components where interference with nearby components might be a concern. Situations found to be questionable structurally may be subjected to more careful analysis with finite element models. The simple model is very versatile and may easily be modified to study the effects of different numbers of cracks and different loading conditions.

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References

[1] HETENYI, M., "Deflections of Beams of Varying Cross Section," J. Appl. Mechanics 4, A49-A52 (1937).

[2] THOMPSON, W. T., "Vibration of Slender Bars with Discontinuities in Stiffness," J. Appl. Mechanics 16, 203-207 (1949).

[3] PETROSKI, H. J., GLAZIK, J. L., "The Response of Cracked Cylindrical Shells," to appear.

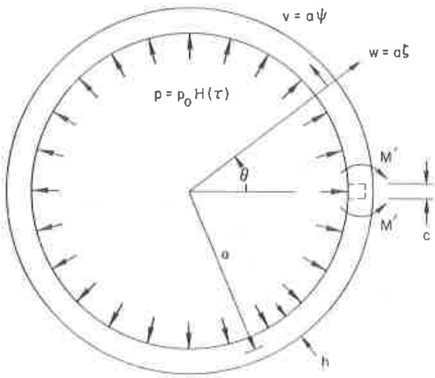

Fig. 1. Simple Model of a Uniformly Pressurized Cylinder with an Internal Crack.

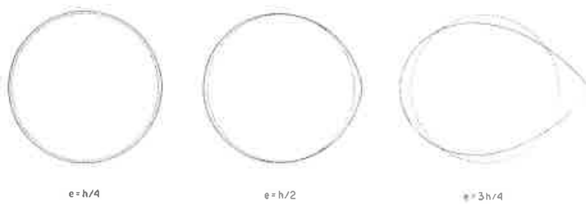

Fig. 2. Deformed Shapes of a Uniformly Pressurized Cylinder for Three Depths of External Crack.

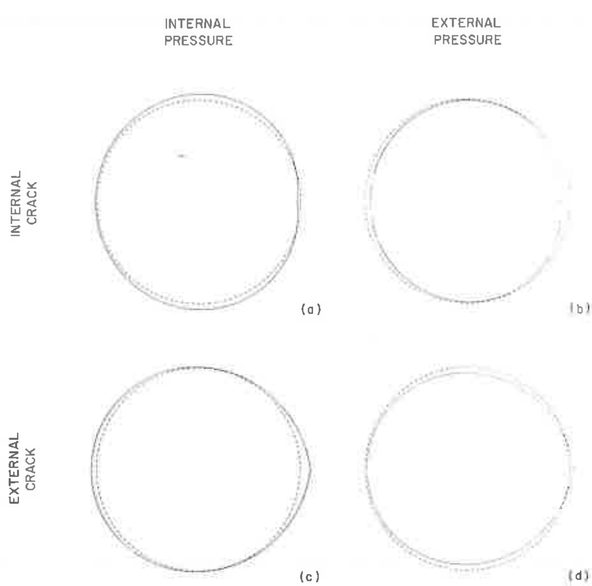

Fig. 3. Deformed Shapes of a Cylinder with an Internal or External Crack h/2 Deep Subject to Uniform Internal or External Pressure.

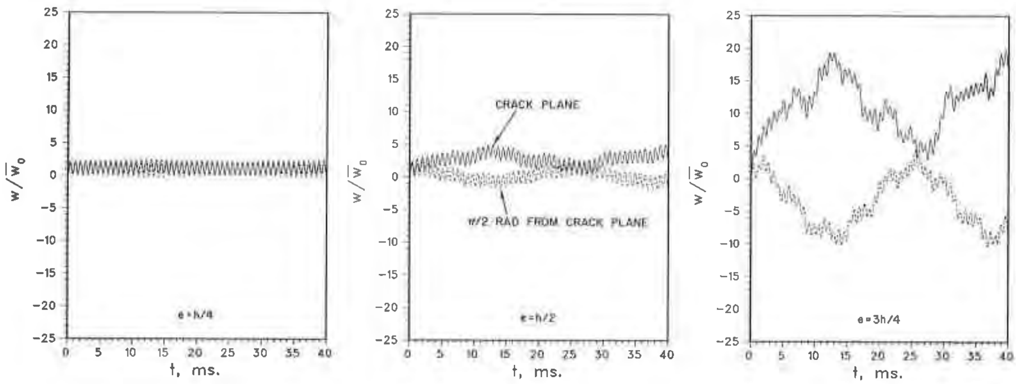

Fig. 4. Effect of Crack Depth on the Dynamic Response to Step Internal Pressure of a Cylinder with an External Crack.

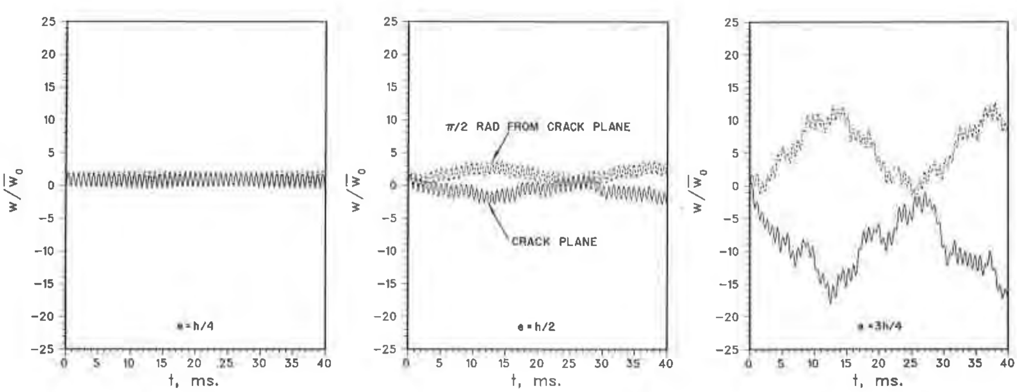

Fig. 5. Effect of Crack Depth on the Dynamic Response to Step Internal Pressure of a Cylinder with an Internal Crack.