FATIGUE CRACK EXTENSION IN NOZZLE JUNCTIONS; COMPARISON OF ANALYTICAL APPROXIMATIONS WITH EXPERIMENTAL DATA

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SUMMARY

Introduction

The fracture mechanics based stress intensity factor (K-factor) concept has obtained widespread acceptance as a tool for quantitative analysis of both fatigue crack growth and instable fracture.

The high local stresses associated with discontinuities such as perforations (pipe plates) and cylinder-to-cylinder or cylinder-to-sphere connections enhance crack initiation and extension. Such cracks generally develop in the shape of quarter ellipses.

Due to the complexity of such crack configurations a fully 3-D finite element treatment seems mandatory to obtain accurate solutions, but the associated high computation costs makes this procedure extremely expensive for standard design procedures. By contrast, analytical approximations, allowing for the application of available analytical solutions by strongly schematizing the complex geometry and stress distribution, are cheap, but generally unreliable, unless checked with accurate experimental and/or finite element data.

The present study discusses the applicability of various simple analytical approximations, some of which have been advocated in literature, and of a specially developed semi-analytical procedure, by comparing results with experimental data.

Analysis

Simple approximations are summarized in the order of increasing sophistication, reaching from the uniformly loaded infinite long surface crack model to various combinations of the Bowie solution for cracks emanating from a hole and solutions for penny shaped or elliptical cracks under uniform normal loading, with corrections for stress concentrations and free surfaces. However, none of these approximations can account for a nozzle corner radius and for the true complex stress distribution at nozzle transitions. Therefore, a semi-analytical procedure has been developed whose main characteristics are:

— the true stress distribution perpendicular to the crack plane (either mechanical or thermal) for the uncracked structure is used as input data;
— an extended version of the Shah and Kobayashi solution for elliptical cracks, loaded on their surfaces by tractions described by fourth order double symmetrical polynomials fit through the data of the previous step is used to calculate full K-factor variations along the crack fronts;
— several corrections, a.o. to correct for free surfaces and for a corner radius are incorporated.

Experiments

The experiments concern careful monitoring crack growth rates (da/dN) under uniaxial fatigue loading of precracked nozzle-on-plate models, a.o. using a closed T.V. circuit. Resulting da/dN versus crack length (a) curves are converted into K versus a curves using da/dN versus ΔK curves for the same material (ASTM A 508 C12) obtained by standard procedures.

Conclusions

Comparison of theoretical and experimental data yields the conclusions that

— simple analytical approximations as sometimes recommended in literature may largely overestimate or underestimate K-factors for nozzle corner cracks. Some guidelines for the range of applicability of specific approximations are provided.
— a computer program based on the semi-analytical procedure yields results within seconds of CPU-time once the input data have been generated. These results compare well with experimental and available finite element data for the range of crack depths of practical concern.
However, the theoretical K-factors vary considerably along the crackfronts, which is not observed in the experiments.

1. INTRODUCTION

Availability of sufficiently accurate procedures for quantitative determination of the extension behaviour of existing or postulated cracks, both under fatigue loading and at fracture, is a prerequisite in particular for those structures whose failure (either leakage or fracture) can imply extreme personal and/or economic dangers, such as aircraft and spacecraft structures and nuclear or large (petro-) chemical vessels. Linear elastic fracture mechanics (LEFM), though restricted in its applicability to situations of overall linear elastic material behaviour, has proven to be a highly valuable tool to describe crack extension under fatigue loading and final fracture conditions. This approach, adopted in App. G of the ASME Code (Section III) [1], requires the determination of stress intensity factors (K-factors) relevant for the configurations of concern. Procedures for application of these K-factors together with appropriate material data in order to establish the crack extension behaviour are well documented in literature.

Due to the associated high stresses cracks developing from discontinuities (perforations in plate or shell structures) are of special concern. Estimating K-factors for cracks in nozzle junctions (fig.1: the most dangerous crack is located in a plane through the axes of nozzle and cylinder) is hampered both by the complex geometry (3-D quarter crack with two perpendicular curved free front surfaces joined by a corner radius, and a complicated free back surface) and by the complex local stress distribution with high stress gradients. Geometry and, consequently, stress distribution vary considerably with the various nozzle-to-shell designs applied in practice. Availability of an analytical solution, requiring much less computation costs than a 3-D finite element analysis, is therefore especially desirable as, contrary to a finite element analysis, it allows for variation of the geometrical parameters governing the problem at little additional costs. The complexity of this crack configuration however, seems to rule out an exact analytical solution, so that any analytical treatment will be of an approximative nature, requiring verification of its results by carefully designed experiments and/or finite element results.

Because of the scarceness of LEFM-based studies on nozzle corner cracks this problem has been adopted as the main area of research within a Dutch cooperative research program [2]; some preliminary results obtained within this program have been presented at the second SMIRT conference [3]. Information obtained since regarding both experimental investigations and simplified and advanced analytical treatments are summarized in the next section. All results concern a uniaxially loaded nozzle-on-flat plate geometry (model of the main coolant inlet nozzle N5 of the Dutch Dodewaard 50 MWe BWR vessel). The choice of this geometry is explained in the next section. 3-D finite element results for this geometry are presented in a companion paper to this conference [4]. All K-factors shown are for a membrane stress σ_m of 1 $N.mm^{-2}$, perpendicular to the crack plane.

2. EXPERIMENTAL INVESTIGATIONS

The experimental investigations concern monitoring the growth of corner cracks in nozzle-on-flat-plate models under uniaxial fatigue loading. Based on the principle illustrated in fig.2 the crack growth rate data from these model tests can be translated to K-factors as a function of crack depth using da/dN vs. ΔK curves for the same material. This principle, also used by other investigators (a.o. [5]), has been chosen mainly because it permits the experimental determination of K-factors for a series of cracks of continuously increasing size in a single experiment. However, it is based on the simplifying assumption that a specific fatigue crack growth rate is uniquely related to a specific ΔK value at the same R (= K_{min} over K_{max}) ratio, irrespective of such factors as geometry and K-history.

Some practical considerations, i.e. good accessibility for continuous accurate crack growth monitoring, available loading equipment and production costs, were instrumental in selecting uniaxially loaded nozzle-on-flat-plate models (fig.3) for the experiments. The deviation from the nozzle-on-pressurized (and thus biaxially loaded) cylinder configuration does not affect the typical complex character of the problem as elucidated in [4].

Results of 3 model tests concerning the same nozzle (N5) are reported. One model, already available at the start of the present program from previous other investigations [6], is entirely manufactured from a common construction steel (St42), the other two consist of a nozzle of A508 Cl 2 material welded into a flat plate of St52 steel. Manufacturing of these latter models involved pre-machining of an oversized nozzle and plate, manual arc welding, stress relief annealing, non destructive testing and machining of all surfaces to final dimensions. Initial part-circular cracks were provided by spark erosion with a = 3.9, 3 and 2.9 mm respectively*. With r_n = 5 mm, this yields "real" initial crack depths at $\theta = 45^o$ of ca. 1.8, 0.9 and 0.8 mm. The models were load-cycled in a 1 MN Amsler fatigue machine at ca. 8 Hz (max. loads 0.692 MN (St42 model) and 0.942 MN (A508 models); R = 0.1). Crack growth rates were monitored at the inside nozzle surface and at the plate surface using the arrangement schematized in fig.4. Visual observation using a microscope made it possible to locate the crack tip generally within some hundredths of mm.

In order to eliminate the laborious and time consuming visual observations, two procedures for automatic crack growth monitoring were elaborated. Using a slightly modified version of the arrangement of fig.4, two robot photo cameras, triggered by a clock and provided with an alarm system for the case of malfunction, photographed the crack tips at $\theta = 0^o$ and 90^o simultaneously at fixed intervals. In the second procedure the photo cameras were replaced by television cameras, which monitor the crack tip during short periods at fixed intervals. This information is stored on a video tape which can be displayed afterwards.

Some typical results of visual, photographical and TV observations are shown in figs.5 and 6. The visual observations turned out to be the most accurate, followed by TV and photographical observations in this sequence. Extremely well polished surfaces and experience in adjusting the illumination (especially of the inside nozzle surface) turned out to be of paramount importance for obtaining good results at all three methods.

da/dN vs. ΔK data have been obtained by standard procedures for both nozzle materials applied, using 4 point bend specimens; the scatterbands in figs.7 and 8 contain over 95% of all data points.

From interpreting the model test results by the procedure of fig.2 it followed that the difference between the K-factors at $\theta = 0^o$ and 90^o respectively was within a few percent for crack depths up till about 25 mm. A similar minor variation of the "experimental K-factors" for $0 < \theta < 90^o$ then follows from the assumption that the crack fronts maintain an

*All crack depths are measured from the reference point x,y = 0, see fig.1.

approximately quarter elliptical shape during the total fatigue crack growth process. This assumption is supported by experimental evidence from other investigations (f.e.[7,8,9]) and by a marking detected at the crack surface of the St42 model (the other two models have been kept intact for subsequent fracture tests under preparation now). Therefore the experimental K-factors have been plotted as average values for $\theta = 0^{o}$ and 90^{o} versus average crack depths in fig.9, together with their scatterbands. (Due to insufficiently polished surfaces no growth data for $a < 9$ mm have been obtained for the St42 model).

3. SIMPLIFIED ANALYTICAL APPROXIMATIONS

Numerous approximate analytical solutions, several of which have been proposed in literature, can be obtained by simplifying the geometry and/or the stress distribution to a more or less extent, thus allowing for the application of existing analytical solutions for less complicated configurations as compiled in several handbooks (f.e. [10,11]), which in general do not require the use of a computer.

Some geometrical simplifications are (fig.10) an infinitely long surface crack (2-D edge crack), a part circular or elliptical surface crack, a quarter circular or elliptical corner crack, a through crack emanating from a hole in a flat plate and a semi-elliptical surface crack whose length at the plate surface equals the sum of hole diameter d plus a at $\theta = 0^{o}$. These simplifications allow for the use of basic analytical solutions for 2-D edge cracks, Sneddon's solution for a circular crack, Irwin's solution for elliptical cracks and Bowie's solution for a crack emanating from a hole, together with relevant correction factors to account for the influence of the free surface(s) provided in the literature (f.e.[12,13]).

Several procedures are open for approximating the stress distribution. If no information about the uncracked stress distribution is available a value for the peak stress can be derived from the similarity between a hole in an infinite flat plate (yielding 3 and 2.5 times the maximum membrane stress for 1:0 and 1:0.5 biaxial loading respectively) and the nozzle-on-plate or cylinder configuration. Using these approximated peak stresses as input for a K-factor computation by the solutions mentioned above yields a considerable over-estimation, except for very shallow cracks, as the true stresses reduce very quickly with distance from the nozzle corner (f.e.[4,6]). In the view of the afore-mentioned similarity the use of Bowie's solution implies a fair approximation of both the geometry of the structure and the associated stress distribution, be it that the real crack shape is badly approximated. Therefore its combination with a solution for circular or elliptical cracks [14,15] will lead to considerably better results. If the true uncracked stress distribution perpendicular to the plane of the crack (both at the surface and at interior points) is available this can be used to improve the accuracy of the procedures mentioned above. At first, the true peak stress can be used instead of an approximated value. Secondly the true stress distribution can be approximated by a combination of constant and linear varying stresses, allowing for the application of solutions given in tabulated or graphical form in literature for the geometrical approximation of shallow surface cracks [10,11,12]. Thirdly, for the deeper cracks, this true stress distribution can be used in combination with the more appropriate geometrical approximation of quarter circular or elliptical cracks, necessitating the use of advanced analytical solutions (f.e.[16]) which require the use of a computer facility. Developments along this line are presented in the next section.

The results of a limited selection (a.o. based on suggestions in literature) from the numerous solutions that can be obtained by combining the various geometrical and stress approximations are shown in fig.10. A comparison with the results of experiments and a semi-analytical procedure is discussed in section 5.

4. SEMI-ANALYTICAL PROCEDURE

This procedure is aimed at obtaining optimal accurate results at minimum costs by the use within a computer code of a combination of analytical solutions for complex loaded cracks and appropriate correction factors, that takes as good as possible account of both the real geometry and the true stress distribution. A short outline of this procedure, presented previously in a provisional form [3], is given here stepwise.

a. The true stress distribution perpendicular to the crack plane for the uncracked nozzle transition area, to be obtained from an economical finite element analysis (f.e.[6,17] , requiring less than USA $ 400 computer costs) is described by a least square fit 4th order polynomial expression (15 terms):

$$\sigma_z(x,y) = \sum_{i=0}^{4} \sum_{j=0}^{4} a_{ij} \, x^i y^j \qquad (1)$$

b. For an area slightly exceeding the crack area, the crack front being described by (fig.1)

$$\frac{x^2}{a_0^2} + \frac{y^2}{a_{90}^2} = 1, \quad x,y \geq 0 \qquad (2)$$

this stress distribution is fit by the least square method into a double symmetrical (in x and y) 4th order polynomial expression:

$$\sigma_z(x,y) = A_{00} + A_{20}x^2 + A_{02}y^2 + A_{22}x^2y^2 + A_{40}x^4 + A_{04}y^4 \qquad (3)$$

c. This stress distribution is used as input to calculate the K-factor distribution along the front of a full elliptical crack (eq.(2) without the restriction $x,y \geq 0$) embedded in an infinite solid by an extended version of a solution developed by Shah and Kobayashi [16] on the basis of Segedin's potential function [18] . (Derivation of the solution of [16] has been restricted to $i + j \leq 3$ in eq.(1) for practical reasons. The resulting 3 terms $i + j \leq 3$ in the double symmetrical polynomial turned out to be insufficient to appropriately match the true stress distribution. Therefore the solution has been extended to $i + j \leq 4$ through the laborious derivation of necessary additional derivatives of the potential function. The resulting 6 terms of eq.(3) allow for a sufficiently close approximation of the true σ_z distribution).

d. For the real crack configuration no material and no stresses exist within the region bounded by $x = 0$, $y = 0$ and $(x - r_n)^2 + (y - r_n)^2 = r_n^2$, r_n being the inside nozzle radius. A correction for the non-existence of stresses in this region, which are necessarily included in eq.(3) and thus in the results of step c, is provided by distracting from these results the K-factor distribution for a circular crack(as only available approximation) loaded with the stress distribution to be removed, using the solution for such a crack embedded in an infinite solid and loaded on its surfaces by symmetrical point loads [19,10] .

e. Subsequent introduction of two perpendicular free surfaces ($x=0$, $y=0$) requires the introduction of free surface correction factors. Approximate correction factors have been taken from literature; similar to [20] the θ-dependent free surface correction factor is established as the product of the correction factors for the semi-cracks $0 \leqslant \theta \leqslant \pi$ and $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$.

f. An appropriate correction for the free back surface (i.e. the outside nozzle surface) seems hard to derive; approximate correction factors are only available for much simpler geometries (surface cracks in plates). Though the presence of the back surface will increase the K-factors in particular for the deeper cracks, no correction has been applied as yet; applicable correction procedures are being studied.

g. Because the crack treated in the preceding steps is located at the edge of a quarter space, a final correction is required to account for the doubly connected character of the real perforation-in-a-shell or plate geometry. An approximate correction has been derived from a 2-D simulation model, i.e. by comparing the K-factor for a through crack emanating from a hole in a plate (Bowie's solution) with the K-factor for an edge crack in a half plane loaded on its surfaces by a stress system equal to the stress system present at the edge of that hole. For this latter case a Westergaard type solution (see [21] App. 5) can be used.

All computations associated with the previous steps are performed with a computer program, allowing for the computation of the full K-factor variation along the front of a nozzle corner crack of arbitrary quarter elliptical shape within seconds of CPU-time once the input stress data for the uncracked geometry are available. Input data may also concern thermal or residual stresses.
A typical plot of results for the uniaxially loaded ($\sigma_m = 1\ N.mm^{-2}$) nozzle-on-plate model of fig.3 is shown in fig.11 as K vs. θ. K-values averaged along the crack fronts are shown in figs. 9 and 10.

5. DISCUSSION OF RESULTS AND CONCLUSIONS

Taking into account the data scatter generally associated with fatigue crack growth experiments, the mutual agreement between the experimental results from the 3 model tests shown in fig.9 seems very good; especially because the results concern models of different materials, tested at fatigue load levels that differed by approximately 30%.
Though the results presented here concern only one geometry some preliminary conclusions regarding the agreement between analytical and experimental data (figs.9 and 10) seem justified.

. Simplified analytical K-factor approximations may considerably over- or underestimate the real crack extension behaviour. Care should be taken to apply these approximations only for specific ranges of crack depths. Such a specification however, requires considerably more data than provided here. For the specific configuration treated here a smooth curve joining the curves marked with 1 (or 2), 4 and 6 in fig.10 seems to be in fair agreement with the experimental data.
. For this configuration results of the semi-analytical procedure in terms of K-values averaged along the crack fronts are in close agreement with experimental results for crack depths up to about 10 mm; fair agreement appears up to about 16 mm (fig.9). For higher values

of a this procedure tends to underestimate the experimental results.

. The strong variation of K along the crack fronts predicted by the semi-analytical procedure (as well as by a detailed finite element treatment [4]) is not reflected by the experimental results. This discrepancy may be attributed to several factors, such as
  - simplifications in the interpretation model of fig.2, that does not make allowance for differences between nozzle model and test specimen regarding f.e. K-history, stress state and crack closure effects
  - simplification in the analytical treatment that does not appropriately account for free surface effects.

A further comment regarding the significance of the K-factor variation is provided in [4].

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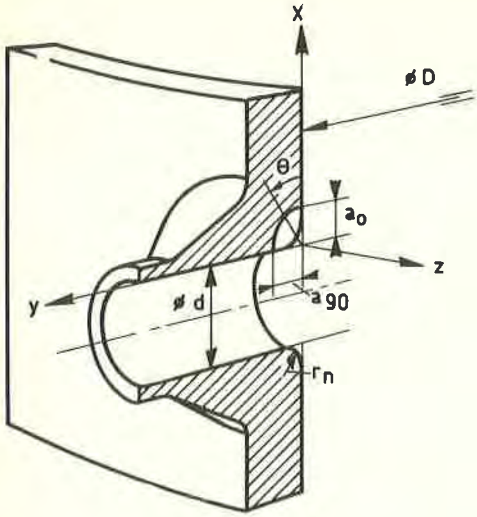


Fig. 1. Nozzle-to-cylinder junction with corner crack

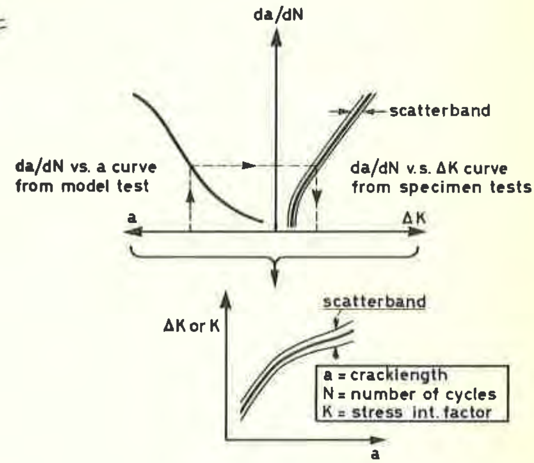


Fig. 2. Principle of experimental technique to determine K as function of a

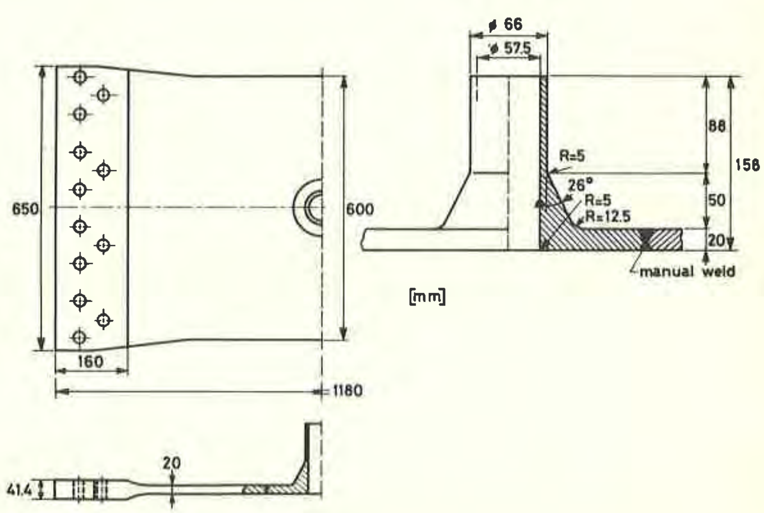


Fig. 3. Typical nozzle-on-flat-plate model (N5/A508)

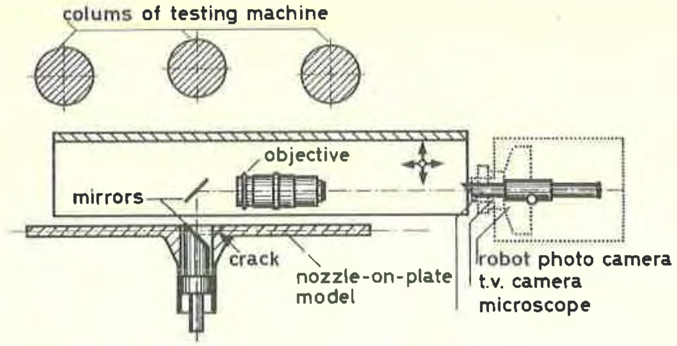


Fig. 4. Schematic arrangement for crack growth observations

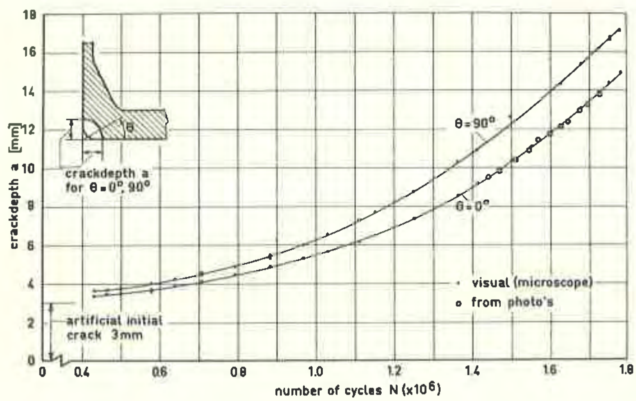


Fig. 5. Visual and photographical fatigue crack growth observations (1st N5/A508 model)

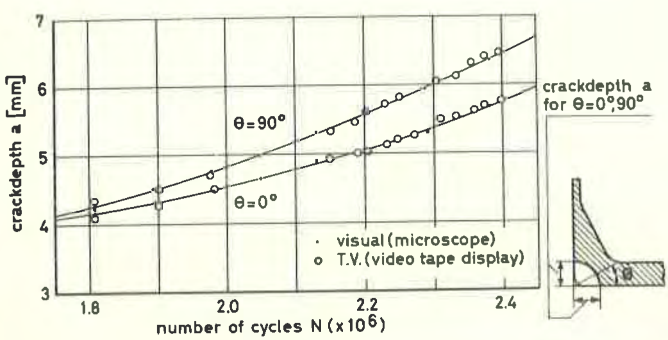


Fig. 6. Visual and TV fatigue crack growth observations (2nd N5/A508 model)

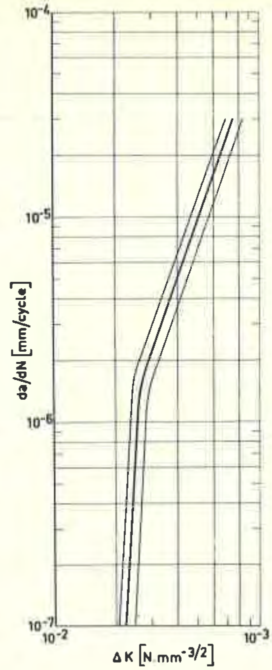


Fig. 7. Crack growth rate data for A508 Cl 2 material

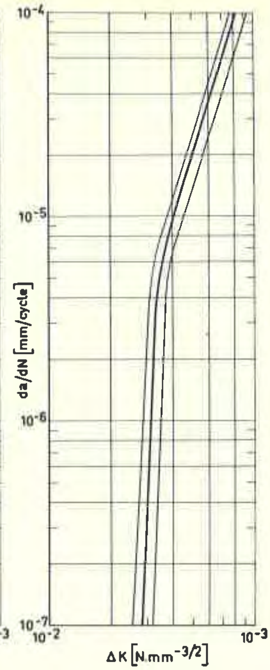


Fig. 8. Crack growth rate data for St. 42 material

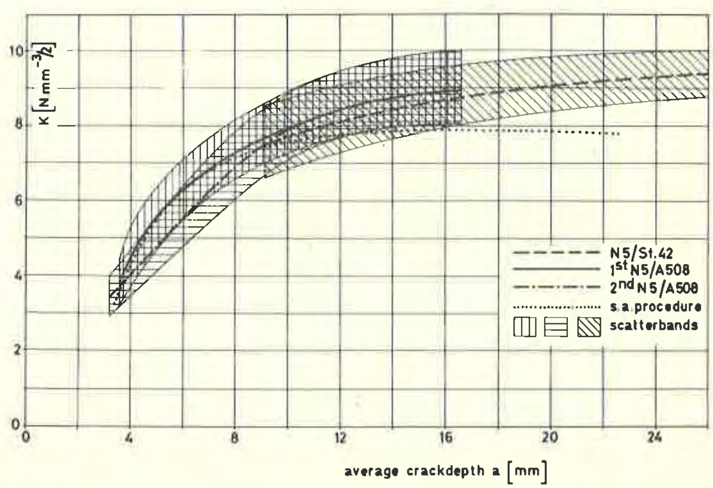


Fig. 9. Results from experiments and semi-analytical procedure

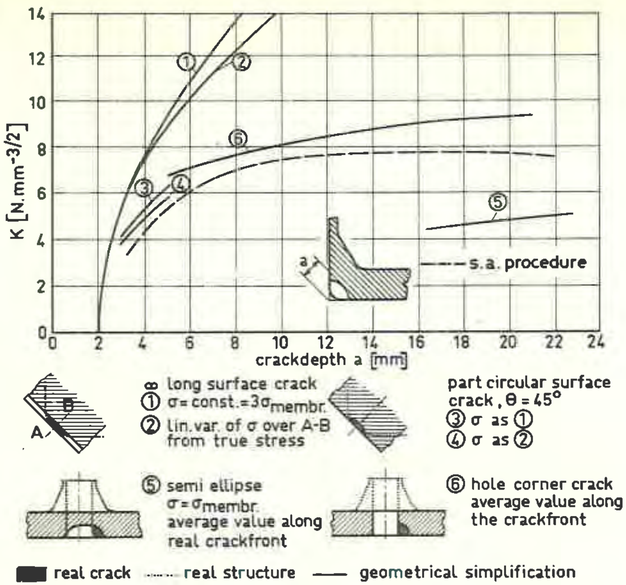
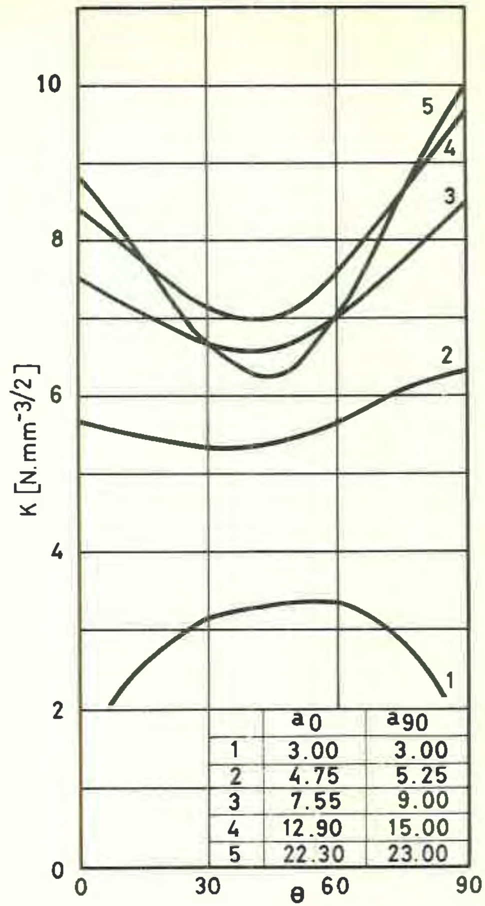


Fig. 10. Some analytical approximations



	a_0	a_{90}
1	3.00	3.00
2	4.75	5.25
3	7.55	9.00
4	12.90	15.00
5	22.30	23.00

Fig. 11. Typical K vs. θ plot from semi-an. procedure