Transient Stress Wave Propagation in HTGR Fuel Element Impacts

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SUMMARY

A study of transient stress wave propagation in graphite HTGR fuel elements has been undertaken as a step toward developing techniques for the evaluation of seismic impact loads. The objectives of the study were to identify appropriate numerical methods, to understand the influence of the geometry and the multiple holes on the response, and to determine the relative importance of high frequency response and lower mode vibrations.

A general review is made of the dynamic contact problem, and the methods available to model impact phenomena and stress wave propagation are evaluated. The important considerations are the method for time-integrating the equations of motion and the method for modeling the contact conditions. An explicit time integration is the most efficient means to capture the short-term response for contact problems, in which the boundary or forcing functions are rich in high frequency content. If the high frequency response dissipates in space and time, it may be appropriate to switch at some point to an implicit method, in which longer time steps may be used without sacrificing accuracy. To evaluate the tradeoff, a cost vs. accuracy study must be performed. A prerequisite for this is to eliminate the spurious response caused by the contact/release algorithm.

A series of four benchmark problems have been studied to determine the influence of the geometry and to evaluate the relative importance of the wave propagation and vibratory regimes. The four geometries consist of two-dimensional hexagonal cross sections, one of which is solid and three of which include varying numbers of holes (7, 19, 37). The geometries are scaled to preserve the shape of the ligaments.

Significant findings from the benchmark study are as follows:

1. Regardless of the number of holes, the stress wave disperses significantly in space and time, so high frequency response is important only for short times near the contact surface.
2. It appears that the peak tensile stress will be located near the center of the block and will be governed primarily by the low frequency modes.
3. The average rebound velocity is only a weak function of the number of holes.
4. The contact time is about 60% longer with holes than without, but again it is a weak function of the number of holes.

1. Introduction

The fuel elements of a High Temperature Gas-cooled Reactor (HTGR) are constructed from blocks of graphite that serve both as a neutron moderator and as a structure. The blocks are drilled with numerous holes (Fig. 1). The design loads consist of thermal and irradiation induced strains plus dynamic impacts due to a postulated seismic event. Paper D 1/8 [1] deals with the static stress analysis of the fuel element. It is shown that, owing to the geometry of the holes, this is a computationally demanding task.

This paper discusses the dynamic impact analysis of a multiple-holed structure. The dynamic analysis is more demanding, not only from the computational standpoint, but also because the requirements of the analysis method are not so well understood.

In the early 1970's the main approach to this problem was experimental, e.g. [2], [3]. These experiments, performed with unirradiated blocks, were insufficient to evaluate fatigue damage, postulated to occur with combined stresses in irradiated blocks. In the middle of the 1970's, analytical studies began to be formulated, e.g. [4], [5]. These were initially aimed at rationalizing the macroscopic experimental results. In the late 1970's the emphasis at General Atomic Company shifted to developing an analytical tool for a detailed stress analysis.

The difficulties encountered in our early analytical studies were twofold:

1. The relative importances of short-term wave propagation and long-term vibratory motion were not known;
2. The contact and release algorithms, coupled with the space-time discretization, caused a spurious response that did not diminish as the discretization was refined.

As a result, the analyses included an unknown amount of error.

To overcome these difficulties, a research program is being conducted at General Atomic Company that has the objective of developing and benchmarking methods for the impact analysis of multiple-holed structures. Current results from that research are reported herein.

Section 2 contains a general discussion of impact analysis methods. The choice of implicit vs. explicit time integration and the contact/release algorithm are discussed. These points are illustrated in the context of a one-dimensional bar problem. The spurious response, or "overshoot" caused by the contact conditions is demonstrated.

Section 3 describes some preliminary analyses of multiple-holed problems. These demonstrate the characteristics of stress waves and their dissipation as they propagate through multiple-holed structures, and they provide guidance in selecting an appropriate method.

It is concluded that, before the requirements of space-time discretization can be defined, it is first necessary to improve the contact/release algorithm.

2. Method of Analysis

2.1 Explicit vs. Implicit Time Integration

The objective is to solve the equation of motion in a multidimensional, solid continuum. For a complex geometry such as the fuel block, the finite element method was selected as the best approach. Following the usual variational formulation and spatial discretization, e.g. [6], one obtains

$$[M]\{\ddot{U}(t)\} + \{F^i(t)\} = \{F^e(t)\}, \tag{1}$$

where $[M]$ is the mass matrix,

$\{\ddot{U}\}$ is the discretized acceleration vector,

$\{F^e\}$ is the externally applied load vector, and

$\{F^i\}$ is an internal force vector due to the current stress state.

The time integration of eq. (1) can be performed in two different ways. In the explicit method the forces F^i and F^e are evaluated at the current time t, the current accelerations are computed with eq. (1) using a diagonal mass matrix, and the velocities and displacements are projected to the advanced time. In the implicit method the external forces F^e are projected forward in time, and the internal forces are computed such that eq. (1) is satisfied at the advanced time point. This is done by relating the internal forces to the displacements through the linearized form,

$$\{F^i(t)\} = [K(t)]\{U(t)\} + \{R(t)\}, \tag{2}$$

where $[K]$ is the stiffness matrix, and

$[R]$ is the equivalent force due to the initial strain.

Omitting the details, whcih are given in [6], the final result is that one must solve a system of linear equations,

$$[K^*(t,\Delta)]\{U(t+\Delta t)\} = \{F^*(t)\}, \tag{3}$$

to determine the displacements at the advanced time point.

There are numerous discussions in the literature on the advantages and disadvantages of the explicit and implicit solution methods, e.g. [7]. A few key points that are relevant to the fuel block problem are discussed below.

For a single time step the explicit method is always faster than the implicit method because no global matrix operations are required, and less data must be processed. This potential advantage of the explicit method increases with larger problems and higher dimensionality. On the other hand, the explicit method is numerically unstable unless the time step, Δt, is below a critical value, Δt_{cr}. Typically [7] one obtains

$$\Delta t_{cr} < \frac{\alpha \Delta x}{c},$$

where α ($\sim$1) is a constant depending on element type,

Δx is the minimum mesh spacing,

c is the speed of sound in the media.

As a result of these factors, it has been found [7] that the explicit method is more efficient for wave propagation problems, in which short time steps are necessary, and the implicit method is more efficient for structural response problems, in which longer time steps are sufficient to capture the lower frequency response.

The fuel block impact is in an intermediate regime in which neither method has clear advantages over the entire space-time domain. The initial contact is instantaneous, and a sharp stress wave that is rich in high frequencies emanates from the contact surface.

To examine this short-term behavior in detail, considering reflection and focusing of a wave as it passes through the holes, an explicit method is preferable. As will be shown later, the longer term response is of a vibratory nature in which the implicit method may be preferable.

2. The Bar Problem

A test problem consisting of a uniform slender bar impacting a rigid wall was studied to provide preliminary quantitative information on the accuracy vs. cost of the explicit and implicit integration methods. A typical finite element mesh is shown in Fig. 2. The space (Δx) and time (Δt) discretization was varied to perform comparisons.

In the exact solution to the problem a compressive stress wave, square in shape and constant in magnitude, emanates from the contact surface and propagates down the bar at the speed of sound. A tensile wave (summing to zero) is reflected, and when this reaches the contact surface the bar rebounds with uniform velocity and no vibrations. Since this is a pure wave propagation problem, the explicit method was expected to be more efficient.

The stress wave obtained in a typical finite element analysis is shown in Fig. 3. Two types of errors are defined on Fig. 3. These are an amplitude error, or overshoot, and a rise rate error, or slope. Figure 4 shows both error types as a function of spatial discretization, obtained using the HONDO II code [8] to perform an explicit time integration. It can be seen that the rise rate error is well behaved, approaching zero in the limit. The overshoot increases as the mesh is refined in space and time. It was also observed, Fig. 5, that as the mesh is refined the overshoot is compressed such that its volume integral approaches zero. Similar results were obtained using the MODSAP code [9] to perform an implicit integration, as shown in Fig. 6.

The effect of time discretization on the errors for both the explicit and implicit integration is shown in Fig. 7. The spatial mesh was held constant. It can be seen that for this class of problem the explicit method is more accurate and, therefore, much more efficient than the implicit method.

2.3 Contact Conditions

A study of the overshoot phenomenon has shown that it is caused by poorly posed contact conditions. In HONDO II and MODSAP the contact conditions are imposed through displacement constraints (in the case of a wall) or stiff springs (in the case of an interface). These have the characteristics that the high frequencies present in the abrupt constraint tend to excite the highest modes present in the discretized mesh.

A better formulation, in which the contact conditions were included in the variational principle (rather than imposed as constraints) was proposed by Hughes et al. in [10]. This formulation was implemented by Hallquist in the DYNA series of codes [11]. When DYNA3D was used to analyze the bar problem the contact overshoot approached zero in limit, as shown in Fig. 8. The reason for the success of Hughes' method is that the contact conditions are imposed by constraining the velocities rather than the displacements. The displacements are permitted to be in error, and correcting forces are imposed. These relaxed boundary conditions have the high frequencies already filtered out. It should be noted (Fig. 8) that there is still a problem with overshoot in the release algorithm of DYNA3D. We are presently investigating this.

3. Multiple-Hole Benchmark Analysis

A series of multiple-hole benchmark problems has been analyzed using the HONDO II code. One objective was to learn more about the physical characteristics of waves propagating through multiple-hole structures. These results are reported here. A second objective, to determine a cost vs. accuracy function for the explicit method, is still being pursued at this writing.

The cases studied consist of hexagonal cross section, one of which is solid, and three of which include varying numbers of holes (7, 19, 37). These models were subject to a flat-on-flat (rigid wall) impact at relative velocity 2V. The analysis was linear elastic for the sake of simplicity. The geometries were scaled to preserve the shape of the ligaments between holes. The finite element meshes contain three subdivisions of four-node isoparametric quadrilaterals across the thinnest ligament, as shown in Fig. 9. From the accuracy assessment in [1], we estimate that these meshes produce discretization errors of about 10% in the static stiffness. At this writing we have not attempted to achieve more accurate results because these would have been masked by the spurious overshoot caused by the contact conditions. It should be noted that wave effects would be more readily discernable with finer meshes.

Two frequently-used measures of the impact response are the "contact time" and the "coefficient of restitution." In attempting to evaluate these parameters, it was found that the customery definitions tend to be ambiguous. In the table below, contact time is defined in three ways. The "time of first total separation" is actually a false contact time caused by a slapping, or multiple contact behavior, of the higher modes. The "time of last total separation" is the time after which recontact never occurs. The "time of maximum kinetic energy" is another arbitrary definition. The contact times are normalized by dividing the actual times by the time for a sound wave to transverse the structure in both directions. In the impact of a uniform bar (pure wave propagation) this normalized contact time is 1.0. If the bar is approximated as a single degree of of freedom system with all of its mass on one end, the normalized contact time is $\pi/2 \approx 1.57$.

	Number of Holes in Hexagon			
	0	7	19	37
Time of first total separation	1.05	1.07	1.66	1.24
Time of last total separation	1.05	1.82	1.66	1.64
Time of maximum kinetic energy	1.05	1.49	1.40	1.40
Coefficient of restitution	.940	.864	.858	.853

The normalized contact times of around 1.6 indicate that the holes tend to lengthen the fundamental period and transform a problem that is initially in the wave propagation category into a problem that is in the vibratory motion category. The differences in the contact times, caused by the slapping motion, are apparently random and unpredictable. The coefficients of restitution indicate that a block with holes absorbs about 14% of its momentum (25% of its energy) in vibrations, whereas a block without holes absorbs only 6% of its momentum (12% of its energy). In most respects it appears that the response is only weakly dependent on the number of holes.

Figure 9 shows the distortion of the 37-hole mesh in the early stages of the wave propagation. It can be seen that the holes near the contact edge are distorted, while the rest of the block is moving at uniform velocity. We believe that the explicit method is preferable for modeling the short-term response during this transition phase, when the plane wave is entering the hole pattern.

Figure 10 shows the distortions of the 37-hole mesh after the rebound. From a study of the complete time history it appears that global structural modes, such as shown here, dominate the response during most of the impact event. Therefore, one might achieve improved efficiency by switching to an implicit integration, with longer time steps, after the wave passes the first few rows of holes.

Figure 11 shows the mesh of the solid hexagon and defines the z coordinate used in the following figures, in which the dispersion of the stress wave is examined.

Figure 12 shows the peak compressive stress as a function of the z coordinate for all the cases studied. The stresses are normalized such that the initial contact stress for a plane impact wave, $\sigma_z = \rho VC$, is equal to 1.0. In the solid hexagon the peak compressive stress is on the contact surface. The peak occurs about midway in time through the contact event, when the contact force has risen to a maximum. Unlike the bar, in which the contact force is a square wave, the contact force on the hexagon rises and falls in time. This is caused by the transfer of momentum from the corners of the hexagon, through the action of shear stresses. Thus, global response modes, resulting from a dispersion of the stress wave, are induced by the two-dimensionality of the hexagonal shape. In Fig. 12 it can be seen that the peak compressive stress in the solid hexagon is reduced from 100% at the contact surface to 40% at the center and to 5% at the far edge.

In the multiple-hole cases, also on Fig. 12, the peak compressive stress occurs about one quarter of the distance across the block. This is the distance needed to develop the structural response modes of the holes and the stress concentrations due to the hole distortions. Thereafter, as with the solid hexagon, the stress wave disperses. The holes modify the response because the ligaments are more flexible than the solid material, and this causes a lengthening of the natural period and the contact time. Moreover, a stress wave cannot pass through a pattern of holes unabated. Through the action of multiple reflections, the wave is dispersed as the structural response of the ligaments is excited. This is a short-term effect, however, and the hole pattern appears to act as an equivalent homogeneous material in the long term.

Figure 13 shows the peak tensile stress. In all cases this peak occurred after rebound, while the block was vibrating in its fundamental mode. Thus, the peak tensile stress is

situated in the center. The stresses are higher in the multiple-hole cases due to the stress concentrations caused by the hole distortion, as shown in Fig. 10. The peak stress magnitudes are only a weak function of the number of holes.

4. Conclusions

These analytical studies have provided insight into the characteristics of the impact response of HTGR fuel elements. From these results we conclude that one should use an explicit integration method to track the initial propagation of the stress wave as it enters the hole pattern. However, owing to dispersion of the stress wave, caused by the hexagonal geometry and the hole pattern, an implicit method may be more efficient for tracking the long-term response.

To quantify the cost vs. accuracy tradeoff, and to establish criteria for switching to the implicit method, it is necessary to improve the contact/release algorithm. Hallquist's implementation of Hughes' method in DYNA has corrected the contact overshoot. A problem still exists with release overshoot, which we are attempting to remedy.

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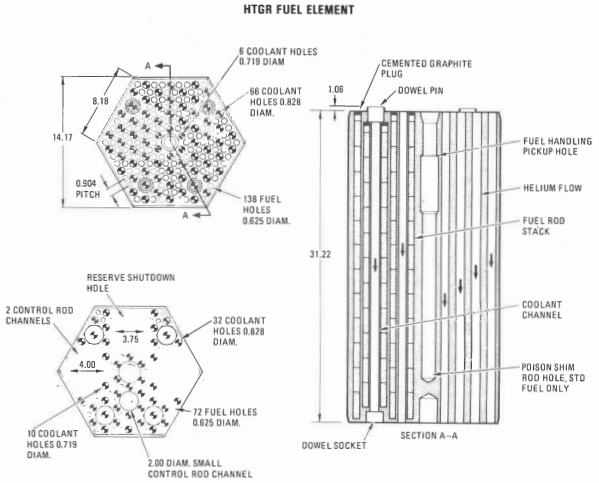

Fig. 1 HTGR Fuel Elements

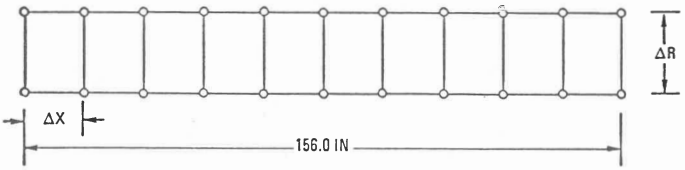

Fig. 2 Finite Element Model of Bar

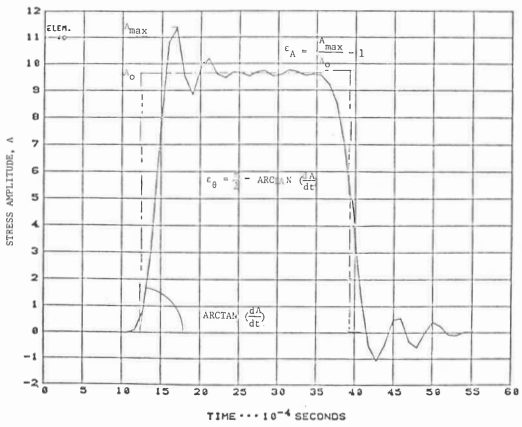

Fig. 3 Bar Impact Problem
Definition of Amplitude
and Rise Rate Errors

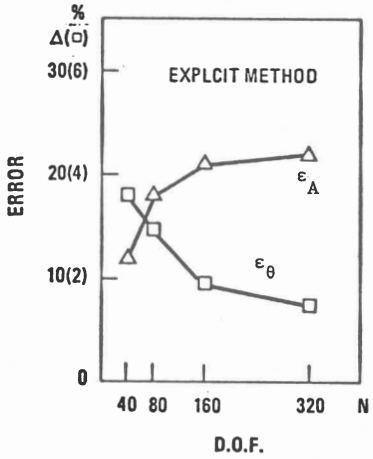

Fig. 4 Error vs. Spatial Discretization
Explicit Method (HONDO II)

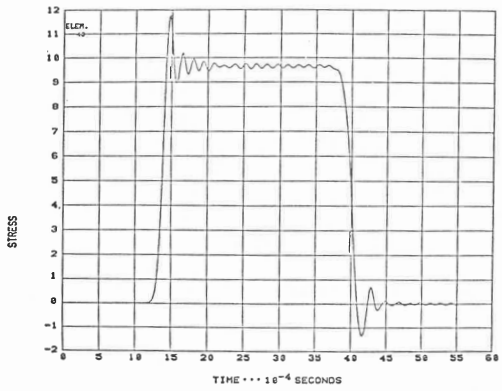

Fig. 5 Bar Impact Problem
Effect of Refined Mesh

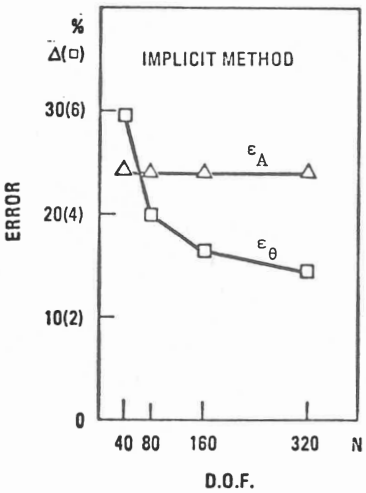

Fig. 6 Errors vs. Spatial Discretization
Implicit Method (MODSAP)

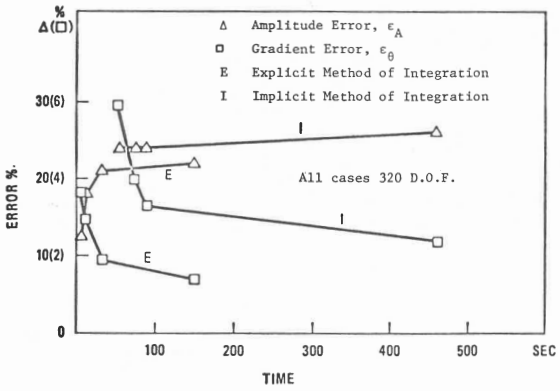

Fig. 7 Errors vs. Time Discretization
Explicit (HONDO II) and Implicit (MODSAP)

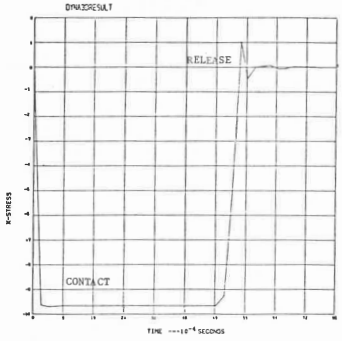

Fig. 8 Effect of Improved Contact
Conditions in DYNA3D

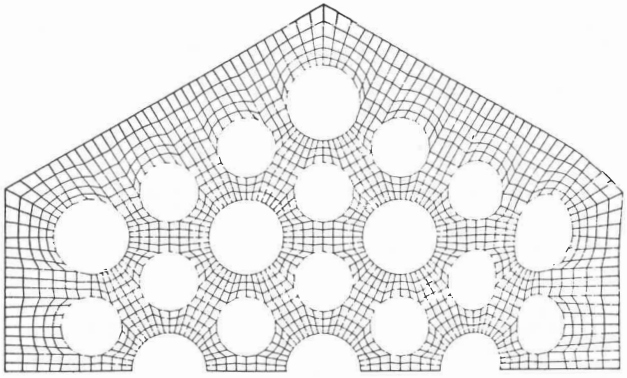

Fig. 9 Early Stages of Wave Propagation 37-Hole Benchmark Problem

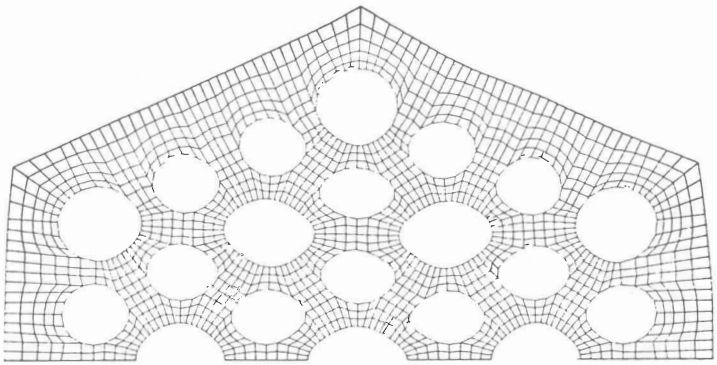

Fig. 10 Global Modes After Rebound 37-Hole Benchmark Problem

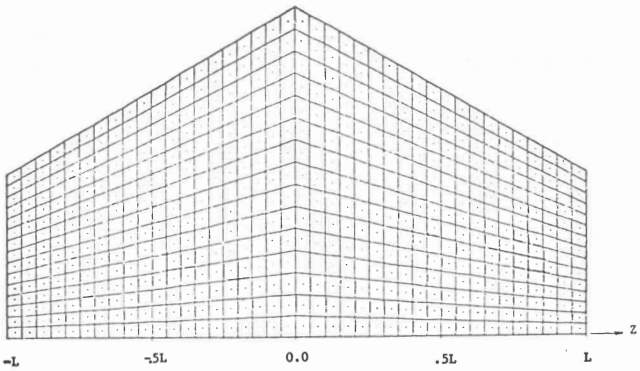

Fig. 11 Solid Hexagon Definition of Z Coordinate

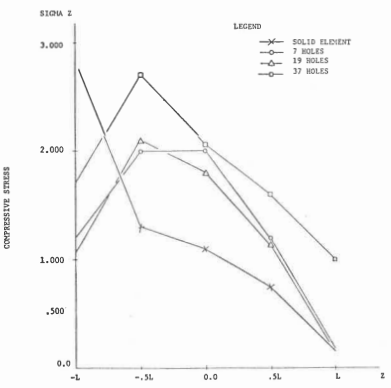

Fig. 12 Peak Compressive Stress
(Short-term Response)

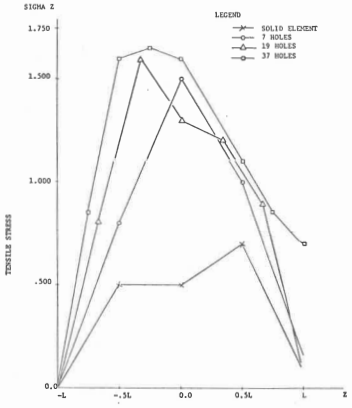

Fig. 13 Peak Tensile Stress
(Longer Term, Vibratory Response)