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The application of boundary element method to mixed mode fracture process using strain energy density criterion

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ABSTRACT: An efficient multi-region boundary element method, employing the quadratic quarter-point boundary elements, is developed for the analysis of the mixed mode fracture problem. The strain energy theory is used to characterize the crack behavior. The two-dimensional plane strain problems with the mixed mode crack are investigated to understand the fundamental aspects of the fracture process herein.

1 INTRODUCTION

Fracture process consists of three stages: crack initiation, subcritical crack growth and failure instability. The instability is considered as the structure failure due to rapid crack propagation. For brittle fracture problems, three stages are closed to one another and failure takes place abruptly with very short duration of crack growth.

The finite element method has been devoted to analyze the crack behaviors in fracture mechanics (Chen & Ting 1985). However, in dealing with the crack growth, the complete re-meshing of the domain must be required and makes the procedure impractical. To compensate the inadequacies of the finite element method, an efficient boundary element method is successfully employed to study the crack initiation, subcritical crack growth and failure instability in this work.

In a series of publications, the strain energy density criterion has been shown to be capable of characterizing the fundamental aspects of the fracture process (Sih 1973, 1974). Due to the complexity of the numerical procedure to simulate the crack growth, the investigations were restricted to mode I analysis. For the study of the fracture process in the mixed mode problems, there is little attention to be paid. The aim of this paper is thus to extend the application of the strain energy density criterion associated with the boundary element method to study the mixed mode fracture process.

A two-dimensional plate embedded with a slanted central crack subjected to uniform tensions at the top and the bottom ends is analyzed. The fracture process corresponding to various slanted crack angles and plate geometries subjected to monotonic increasing loading has been studied (Ting & Yang 1993). The variable loading will be investigated in this work. The path dependent nature of the fracture process from different loading conditions will be examined by the completeness of the fracture criterion and the effectiveness of the boundary element method.

2 STRAIN ENERGY DENSITY CRITERION

Based on Sih's theory, the strain energy density factor S for two dimensional elastic fracture problems in (r,θ) polar coordinate system (in Fig. 1) may be expressed in terms of K_I and K_{II} as:

(1) $$S = a_{11}K_I^2 + 2a_{12}K_IK_{II} + a_{22}K_{II}^2$$

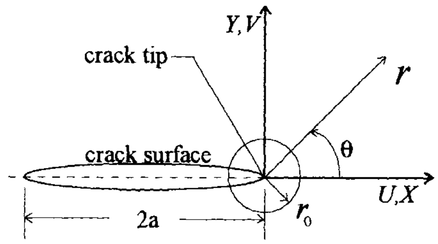

Figure 1. Crack tip region.

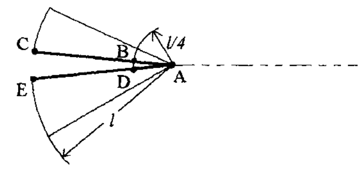

Figure 2. Element geometry for stress intensity factor computation.

with

$$16Ga_{11} = (1+\cos\theta)(\kappa-\cos\theta)$$
$$16Ga_{12} = [2\cos\theta-(\kappa-1)]\sin\theta$$
$$16Ga_{22} = (\kappa+1)(1-\cos\theta)+(1+\cos\theta)(3\cos\theta-1)$$

where K_I and K_{II} are the stress intensity factors for mode I and mode II, G is the modulus of rigidity, $\kappa = 3-4\nu$ or $\kappa = (3-\nu)/(1+\nu)$ for plane strain or plane stress conditions, respectively, and ν is the Poisson's ratio. The variation of S along the circular core region with radius r_0. For most engineering metal alloys, r_0 can be assumed as $10^{-3} \sim 10^{-2}$ *inches* (Gdoutos 1984, Theocaris 1981).

The basic hypotheses on the fracture process of two dimensional problems herein can be stated as follows:

1. The crack initiation takes place in the direction along which the strain energy density factor S possesses the maximum of the relative minimum values, S_{min}^{max}.

2. Instable failure is assumed to occur when the value S_{min}^{max} reaches its critical value S_c. S_c is considered to a material constant (Gillemot 1976, Sih & Macdonald 1974) and can be related with plane strain fracture toughness K_{IC} (Srawley 1969).

(2) $$S_c = \frac{(\kappa-1)}{8G}K_{IC}^2$$

3. The stable crack growth can be considered as a series of finite segments with different direction corresponding to progressively increasing load. The incremental length of the growth can then be calculated by the relationship between S and the strain energy density dW/dV.

(3) $$\left(\frac{dW}{dV}\right)_c = \frac{S}{r}$$ if $r < r_0$, crack initiation cannot occur.

where $(dW/dV)_c$ is the material constant which can be calculated from the true stress- true strain curve.

3 BOUNDARY ELEMENT METHOD DESCRIPTION

The boundary element method (BEM) has been extensively used in fracture mechanics. The first general numerical solution, Cruse and Van Buren (1971) were based on modeling the crack as an open notch. However, the singularity in the algebraic system of equations is obtained when the upper and lower crack surface are modelled in the same plane. Blandford, Ingraffea, and Liggett (1981) presented a muti-domain boundary integral equation for modeling each crack surface in a separate domain to avoid the

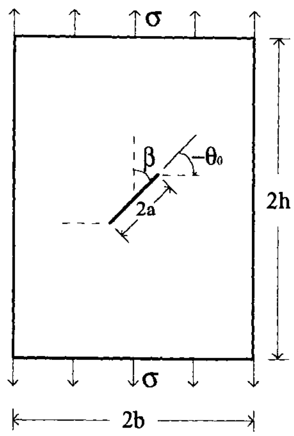

Figure 3. Plate with central slanted crack subject to uniform tension.

The material values are taken to the same as that used in Gdoutos and Sih's paper (1984) :

Young's modulus E	$= 3\times10^{7}$ psi
Poisson's ratio ν	$= 0.3$
$(dW/dV)_c$	$= 26684$ in-lb/in^3
S_c	$= 77$ lb/in
r_c	$= 2.885\times10^{-3}$ in
r_0	$= 1.2\times10^{-3}$ in
K_{IC}	$= 1.66965\times10^{5}$ psi$\sqrt{\text{in}}$

matrix singularity problem. In this work, the quadratic quarter-point boundary elements on each side of the crack tip(s) are used to accurately compute the stress intensity factor (SIF) (Martinez & Dominguez 1984, Sato ,Tanka & Nakamura 1984).

Denoting the displacement along the crack axis as U and the displacement normal to the crack axis as V are defined in figure 1. The definitions of the stress intensity factors K_I and K_{II} can be expressed as:

(4) $$K_I = \frac{\mu}{\kappa+1}\sqrt{\frac{2\pi}{l}}\left[4(V_B - V_D) + V_E - V_C\right]$$

(5) $$K_{II} = \frac{\mu}{\kappa+1}\sqrt{\frac{2\pi}{l}}\left[4(U_B - U_D) + U_E - U_C\right]$$

in which the points B, C, D, E are defined in Fig. 2.

4 RESULTS AND DISCUSSIONS

All of examples are plane strain problems and refer to a finite plate of side length 2b and height 2h with a central slanted crack length 2a subjected to a uniform tensile loading at both the top and the bottom ends as shown in Fig. 3.

Four different types of variable loading process are used based on Gdoutos and Sih's mode I analysis (1984) :

- Type I : Monotonic increase following a convex load curve.
- Type II: Monotonic increase following a concave load curve.
- Type III: Slight initial oscillation followed by a linear rise in load.
- Type IV: Large initial oscillation followed by a linear rise in load.

Type I :

The loading curve is given by $\sigma = \sigma(N)$ which consists of three linearly increasing parts as shown in Fig. 4. The first ten steps at $\Delta\sigma$=0.05 ksi, the second ten at $\Delta\sigma$=0.10 ksi and then by a sharp increment at $\Delta\sigma$=1.0 ksi are applied until the failure instability of the plate is reached.

During the first twenty steps, the crack increments with the various slanted angles β versus the loading steps are almost the similar. There exists the significant difference after $\Delta\sigma$=1.0 ksi among the various crack increments in Fig. 4. The rate of crack growth decrease with the increased slanted angles.

Type II:

In contrast with type I, the first ten steps at $\Delta\sigma$=1.0 ksi, the second ten at $\Delta\sigma$=0.1 ksi and finally incremental stress is decreased to $\Delta\sigma$=0.05 ksi until the failure instability of the plate is reached. In Fig. 5, the curves of the crack increment corresponding to differential angle β show significant contrast during the first ten steps, and follows the abruptly changes of the growth rate and propagate slowly.

To compare the results of the two types, every value of critical stress σ_c for different inclined angles of the Type I is larger than the Type II. On the contrary, every value of critical crack length a_c for different inclined angle of the Type I is smaller than the Type II.

Type III:

The loading curve also consists of three parts as shown in Fig. 6. The first part at $\Delta\sigma$=0.1 ksi for twenty steps, the second part at $\Delta\sigma$=-0.1 ksi for sixty steps and the last part at $\Delta\sigma$=0.1 ksi are taken to global instability.

From Fig. 6, the crack arrest can be observed for three inclined angles $\beta = 45^\circ$, 75° and 90° during the second loading range, and the crack starts to reinstate its motion at the value of the stress which is slightly lower than the original stress σ_i for crack initiation. Note that the crack arrest period for $\beta = 15^\circ$ during the second loading range has not found. Alternatively speaking, because it need a larger decrement of applied stress when $\beta = 15^\circ$.

Type IV:

In contrast with Type III, the first twenty steps at $\Delta\sigma$=0.1 ksi, the second fifty steps at $\Delta\sigma$=-0.5 ksi and then by a smaller increment at $\Delta\sigma$=0.05 ksi is kept throughout the loading process. In Fig. 7, the crack arrest period for $\beta = 15^\circ$ during the second loading range has found, but the range of $40 \le N \le 384$ is the shortest among the four inclined angles. When the crack restart to extend, the stress is again slightly less than σ_i and the rate of crack growth for $\beta = 15^\circ$ is the largest among the four inclined angles.

5 CONCLUSION

In this work, the strain energy density criterion has been successfully employed to predict the direction, the crack incremental length and the failure time of the mixed mode crack growth. In addition, muti-region boundary element method can be effectively modeled the mixed fracture problems. The combination of two methods in this paper provide a complete analysis procedure to study the fracture process of the mixed mode problem.

We also find that the behavior of the crack growth for mixed mode problem is the loading path dependent, and the critical stress σ_c is decreasing with the increasing slanted crack angles. Hence, The mode I can be used as the basis of conservative design.

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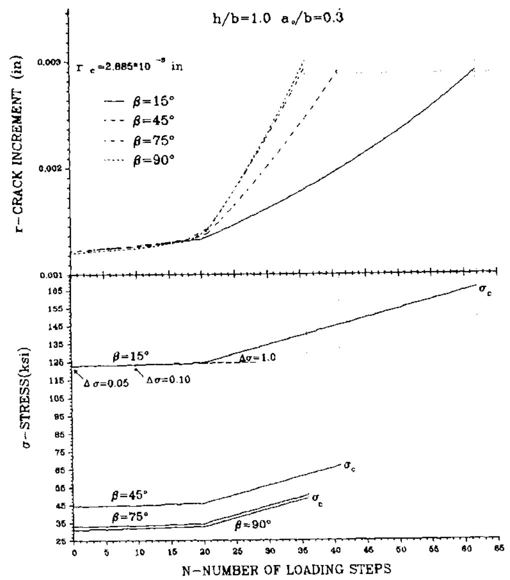

Figure 4. Variations of stress σ and crack increment r versus the loading steps N for various angles β under the rising load in convex form.

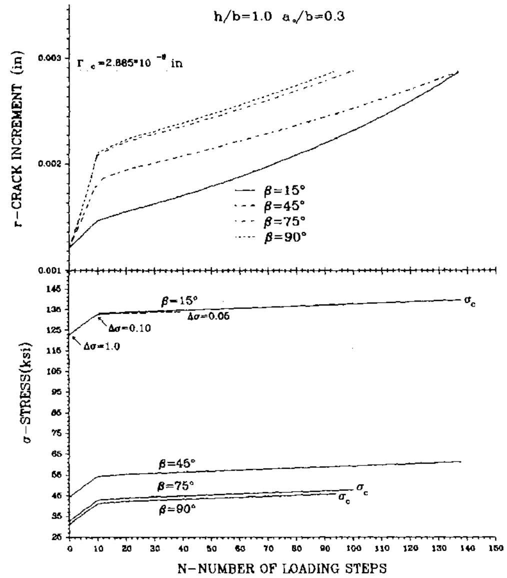

Figure 5. Variations of stress σ and crack increment r versus the loading steps N for various angles β under the rising load in concave form.

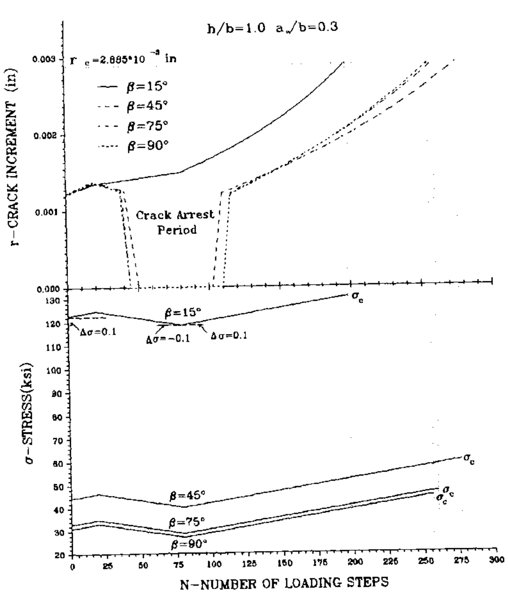

Figure 6. Variations of stress σ and crack increment r versus the loading steps N for various angles β under the rising load in slight initial oscillation form.

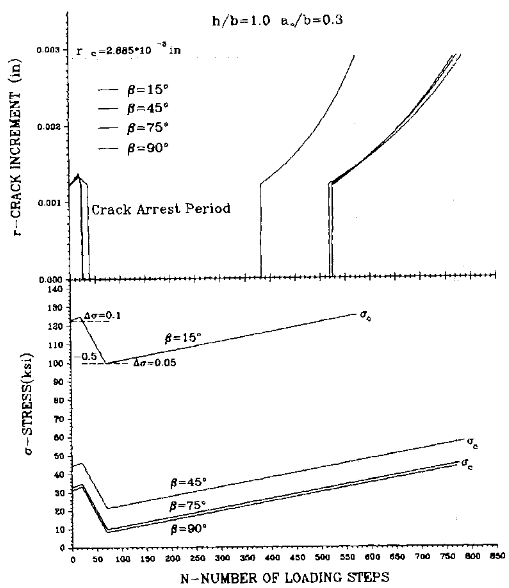

Figure 7. Variations of stress σ and crack increment r versus the loading steps N for various angles β under the rising load in large initial oscillation form.